Rewarding Learning

## General Certificate of Secondary Education

2008

## Additional Mathematics

## Paper 2 <br> Mechanics and Statistics

[G0302]


THURSDAY 15 MAY, MORNING

## TIME

2 hours.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet and the Supplementary Answer Booklet provided.
Answer all twelve questions.
At the conclusion of the examination attach the Supplementary Answer Booklet to your Answer Booklet using the treasury tag supplied.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 100 .
Figures in brackets printed down the right-hand side of pages indicate the approximate marks awarded to each question or part question.
You may use your calculator.
A copy of the formulae list is provided.
Take $\mathbf{g}=10 \mathrm{~m} / \mathbf{s}^{2}$ when required.

## Answer all twelve questions

1 A box of mass 15 kg is placed on a smooth plane inclined at $27^{\circ}$ to the horizontal. The box is prevented from sliding down the plane by a force $\mathbf{P}$ acting along the line of greatest slope as shown in Fig. 1.


Fig. 1
(i) Copy Fig. 1 and mark on it all the forces acting on the box.
(ii) Calculate the normal reaction between the box and the plane.
(iii) Calculate the least force $\mathbf{P}$ acting along the plane which is required to prevent the box from sliding down the plane.

2 As part of a fitness promotion, each member of a Health and Leisure Centre was asked to record the total time spent training in the gym over a four-week period. The times were recorded to the nearest hour and a summary of the results is given in Table 1.

Table 1

| Total <br> time (hours) | $4-11$ | $12-19$ | $20-29$ | $30-39$ | $40-59$ | $60-79$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> members | 96 | 60 | 112 | 75 | 98 | 54 |

Using Fig. 2 in your Supplementary Answer Booklet draw a histogram to represent this information. Label each axis clearly.

3 A box of mass 9 kg initially at rest is pulled in a straight line along a rough horizontal plane by a light inextensible rope. The rope is parallel to the plane. The box moves with constant acceleration.

After travelling 24 metres along the plane the box is moving with velocity $4 \mathrm{~m} / \mathrm{s}$.
(i) Show that the acceleration of the box is $\frac{1}{3} \mathrm{~m} / \mathrm{s}^{2}$.

The tension in the rope is 30 N , as shown in Fig. 3.


Fig. 3
(ii) Calculate the force due to friction opposing the motion of the box.
(iii) Calculate the coefficient of friction between the box and the plane.

4 Year 9 students were shown an angle and asked to estimate its size to the nearest degree.
Table 2 summarises the results.
Table 2

| Estimate <br> (degrees) | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ | $60-69$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> students | 22 | 28 | 47 | 60 | 64 | 32 |

Calculate an estimate of the median for their results.

5 (Throughout this question $\mathbf{i}$ and $\mathbf{j}$ denote unit vectors parallel to a set of standard $x-y$ axes)
A package of mass 5 kg is initially at rest at the origin O on a smooth horizontal plane. It is acted upon by a force $\mathbf{F}=(5 \mathbf{i}-12 \mathbf{j}) \mathrm{N}$, causing the package to move along the plane.
(i) Calculate the acceleration of the package in vector form.

After 3 seconds the package is at a point A .
(ii) Find the displacement vector $\mathbf{O A}$.

The force $\mathbf{F}$ is the resultant of two forces $\mathbf{F}_{\mathbf{1}}$ and $\mathbf{F}_{\mathbf{2}}$ where $\mathbf{F}_{\mathbf{1}}=(4 \mathbf{i}-7 \mathbf{j}) \mathrm{N}$.
(iii) Find the force $\mathbf{F}_{\mathbf{2}}$ in vector form.

6 In her end-of-year examinations Laura took six written modules and four practical modules.
The mean mark for her six written modules was 54 . The mean mark for her four practical modules was 64 .
(i) Calculate the mean mark for all ten modules.

The standard deviation of the marks in her six written modules was 18.2
The standard deviation of the marks in her four practical modules was also 18.2
(ii) Calculate the standard deviation of the marks in all ten modules.

7 In a complex of 40 new houses, 23 houses have a balcony. Sixteen houses have both a fifth bedroom and a balcony. Eleven houses have neither a fifth bedroom nor a balcony.
(i) Illustrate this information on a clearly labelled Venn diagram.
(ii) Hence or otherwise calculate the probability that a house chosen at random has a fifth bedroom.
(iii) Given that a house has no balcony, what is the probability that it has a fifth bedroom?

8 A non-uniform beam AB is 8 metres long and has mass 18 kg . It is suspended from the ceiling of a warehouse by two vertical cables attached to the beam, one at C and one at D , where $\mathrm{AC}=1.5 \mathrm{~m}$ and $\mathrm{AD}=6 \mathrm{~m}$. The beam is hanging in equilibrium in a horizontal position as shown in Fig. 4.


Fig. 4
The tensions in the two cables are equal.
(i) Calculate the tension in each cable.

The centre of mass of the beam is $x$ metres from the end $\mathbf{A}$.
(ii) Show that the value of $x$ is 3.75

A package is now suspended from the beam. The tension in the cable at C becomes zero and the beam remains in equilibrium in a horizontal position.
(iii) State the point at which this package should be suspended so that the value of its mass Mkg is a minimum.
(iv) Find this value of M.

9 A bus and a car travel on a straight horizontal road between two sets of traffic lights at X and Y .
They both start from rest at X , travel the distance XY in the same time and both finish at rest at Y.

The bus moves from rest at X with uniform acceleration $1.25 \mathrm{~m} / \mathrm{s}^{2}$ for 16 seconds until it reaches a maximum speed of $\mathrm{V} \mathrm{m} / \mathrm{s}$. It travels at this speed for 24 seconds and then comes to rest at Y with uniform retardation $2.5 \mathrm{~m} / \mathrm{s}^{2}$.
(i) Using Fig. 5 in your Supplementary Answer Booklet sketch the speed-time graph for the bus.

Calculate
(ii) the maximum speed reached by the bus,
(iii) the time taken by the bus to travel from X to Y ,
(iv) the total distance XY travelled by the bus.

The car accelerates uniformly from rest at X until it reaches its maximum speed. The brakes are then applied and the car retards uniformly to rest at Y . The magnitude of the retardation is three times the magnitude of the acceleration.
(v) On Fig. 5 in your Supplementary Answer Booklet add a sketch of the speed-time graph for the car.

Calculate
(vi) the time for which the car accelerates,
(vii) the greatest speed reached by the car.

10 For the past few years a college has been running evening classes in Art and Craft. The numbers attending these classes are summarised in Table 3.

Table 3

|  | Term 1 | Term 2 | Term 3 |
| :---: | :---: | :---: | :---: |
| 2004 | 246 | 212 | 142 |
| 2005 | 222 | 194 | 124 |
| 2006 | 198 | 170 | 112 |
| 2007 | 177 | 149 | 94 |

These data have been plotted on the graph given in Fig. 6 in your Supplementary Answer Booklet.
(i) Calculate appropriate moving averages to smooth the data.
(ii) Plot these averages on the graph and draw the trend line.
(iii) Showing clearly where any reading is taken, use your trend line to calculate an estimate of the number of students likely to attend the Art and Craft classes in Term 1 in 2008.

The cost of running the Art and Craft classes in term 1 of 2008 is $£ 5000$. The charge to each student is $£ 55$.
(iv) Calculate the percentage profit based upon the estimated number of students likely to take the classes in term 1 of 2008.

11 A van of mass 1250 kg is towing a trailer of mass 550 kg by means of a light horizontal tow bar. The tractive force produced by the van's engine is 3840 N . The van and trailer are travelling along a straight horizontal road as shown in Fig. 7.


Fig. 7
The van and trailer accelerate uniformly from rest to $21 \mathrm{~m} / \mathrm{s}$ in 15 seconds.
(i) Calculate this acceleration of the van and trailer,
(ii) Show that the total resistance to the motion of the van and trailer is 1320 N .

The resistance to the motion of the van is 0.704 N per kg of mass.
Calculate, in Newtons,
(iii) the resistance to the motion of the van,
(iv) the resistance to the motion of the trailer,
(v) the magnitude of the tension in the towbar.

The van and trailer then travel at a constant speed of $21 \mathrm{~m} / \mathrm{s}$ and after some time the towbar breaks.
(vi) Assuming that the resistance to the motion of the trailer remains the same calculate the additional distance travelled by the trailer before it comes to rest.

12 Nikki recorded the number of hours of labour and the corresponding total cost for each of ten cars serviced in one day at her local garage. The results are shown in Table 4.

Table 4

| Car | A | B | C | D | E | F | G | H | I | J |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number <br> of hours | 5.5 | 3.5 | 0.5 | 2.5 | 2.5 | 2 | 3 | 4 | 4.5 | 1.5 |
| Total <br> cost $(£)$ | 235 | 220 | 88 | 129 | 163 | 107 | 142 | 230 | 145 | 164 |

(i) Find the rank order for the number of hours and the rank order for the total costs.
(ii) Calculate Spearman's coefficient of rank correlation.
(iii) What significance, if any, do you attach to the value you obtained in (ii)?

The data from Table 4 are plotted on the graph given in Fig. 8 in your Supplementary Answer Booklet.
(iv) Draw the line of best fit on this graph.
(v) Determine the equation of the line of best fit which you have drawn.
(vi) Suggest a suitable meaning for the value of the gradient in your line in part (v).


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## SUPPLEMENTARY <br> ANSWER BOOKLET

2 Using the information from Table 1, draw a histogram in Fig. 2.
Label each axis clearly.

| $\square$ |  |  | - | $T$ | - | $\square$ |  | - | $\square$ |  | T |  |  |  |  |  |  |  |  |  |  |  | T |  |  |  |  | T |  |  |  |  |  |  |  |  |  |  |  |  |
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Fig. 2

9 (i) Sketch the speed-time graph for the bus in Fig. 5.
(v) Add a sketch of the speed-time graph for the car in Fig. 5.


Fig. 5

10 Plot the moving averages in Fig. 6 and draw the trend line.


Fig. 6

12 Draw the line of best fit through the data points shown in Fig. 8.


Fig. 8

