GCSE

Additional Mathematics

Summer 2010

Mark Schemes

Issued: October 2010

NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE)
AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)

MARK SCHEMES (2010)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

CONTENTS

Rewarding Learning

General Certificate of Secondary Education
2010

Additional Mathematics

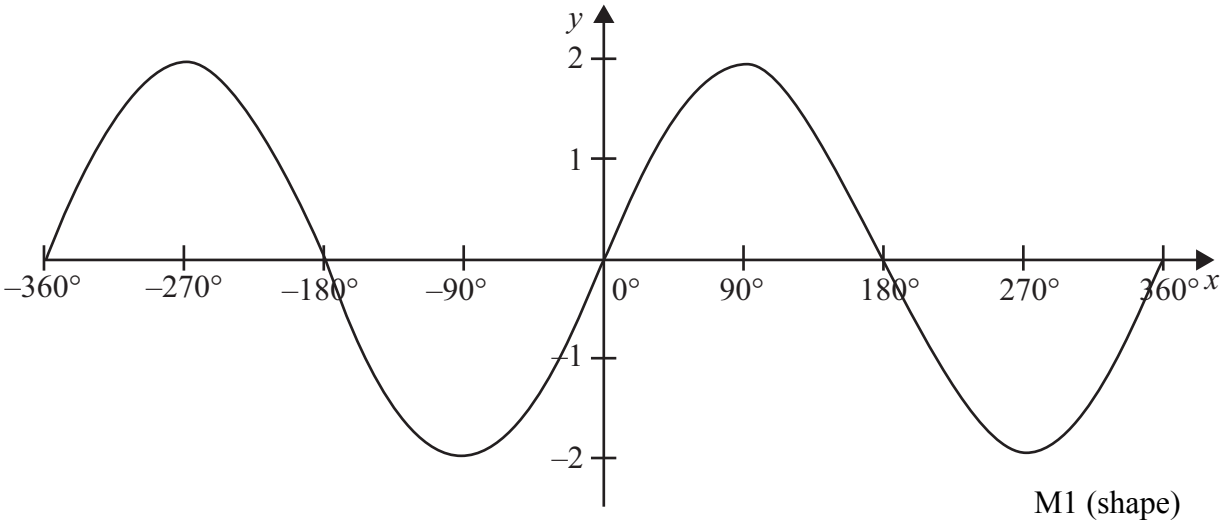
Paper 1
Pure Mathematics

[G0301]

MONDAY 17 MAY, AFTERNOON

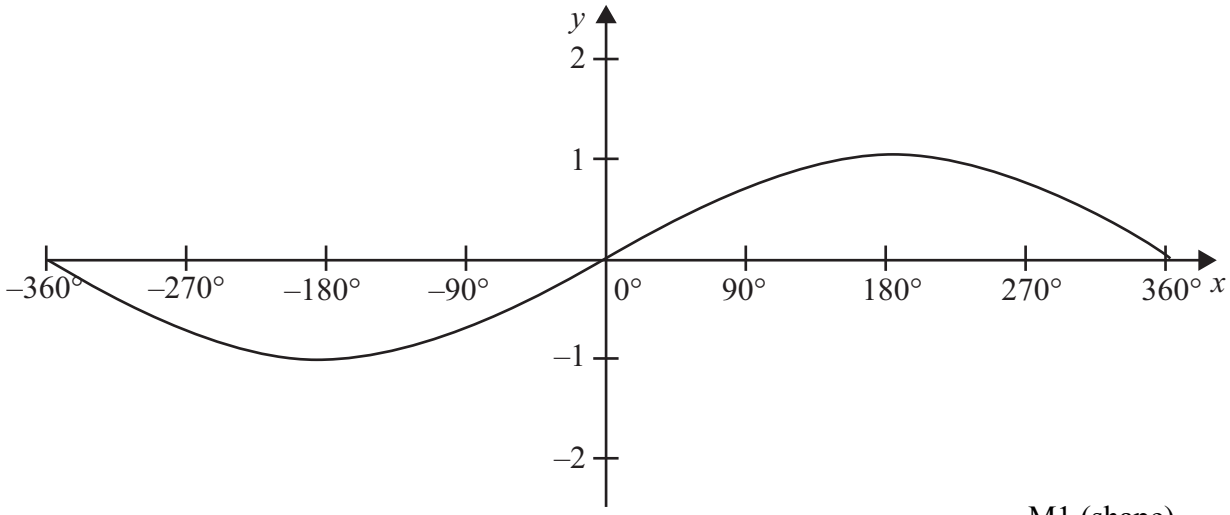
MARK SCHEME

1 **(i)** $y = 2\sin x$



M1 (shape)
W1 (y values)

(ii) $y = \sin\left(\frac{1}{2}x\right)$



M1 (shape)
W1 (y values) 4

2 **(i)** $\tan x = 3$

$\therefore x = 71.57°$ **or** $-108.43°$ to 2 d.p.
(72° **or** –108° to nearest degree) 2 × MW1

(ii) $\tan\left(\frac{x}{2} - 10°\right) = 3$

$\therefore \frac{x}{2} - 10 = 71.57$ **or** -108.43 M1

$\therefore \frac{x}{2} = 81.57$ **or** -98.43

$\therefore x = 163.14°$ **or** $-196.86°$ to 2 d.p.
(163° **or** –197° to nearest degree) 2 × MW1 5

AVAILABLE MARKS

			AVAILABLE MARKS
3	**(i)** $A^{-1} = \frac{1}{20-18}\begin{bmatrix}5 & 3\\ 6 & 4\end{bmatrix} = \frac{1}{2}\begin{bmatrix}5 & 3\\ 6 & 4\end{bmatrix}$	MW1, MW1	
	(ii) $\begin{bmatrix}4 & -3\\ -6 & 5\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}18\\ -29\end{bmatrix}$	M1	
	$\begin{bmatrix}x\\ y\end{bmatrix} = A^{-1}\begin{bmatrix}18\\ -29\end{bmatrix}$		
	$= \frac{1}{2}\begin{bmatrix}5 & 3\\ 6 & 4\end{bmatrix}\begin{bmatrix}18\\ -29\end{bmatrix}$	M2	
	$= \frac{1}{2}\begin{bmatrix}3\\ -8\end{bmatrix} = \begin{bmatrix}\frac{3}{2}\\ -4\end{bmatrix}$		
	$\therefore x = \frac{3}{2}, y = -4$	W1	6
4	**(a)** $y = 2x^5 - \frac{2}{5}x^{-2}$		
	$\frac{dy}{dx} = 10x^4 + \frac{4}{5}x^{-3}$		
	$= 10x^4 + \frac{4}{5x^3}$	2 × MW1	
	(b) $\int\left(6x^2 - \frac{3}{x^4} + 1\right)dx$		
	$= 2x^3 + x^{-3} + x + c$		
	$= 2x^3 + \frac{1}{x^3} + x + c$	4 × MW1	6

			AVAILABLE MARKS
5	**(i)** $x = 1$		
	$y = 2 + 4 - 7 = -1$	MW1	
	$\frac{dy}{dx} = 6x^2 + 4$		
	$x = 1$ so $m = 6 + 4 = 10$	MW1	
	Either $y - (-1) = 10(x - 1)$ **or** $y = 10x + c$	M1	
	$y + 1 = 10x - 10$ $\quad -1 = 10 + c$		
	$-11 = c$		
	$y = 10x - 11$ $\quad y = 10x - 11$	W1	
	(ii) $6x^2 + 4 = 10$	M1	
	$6x^2 = 6$		
	$x^2 = 1$		
	$x = 1$ **or** -1	W1	
	$x = -1$		
	$y = -2 - 4 - 7 = -13$		
	$(-1, -13)$	W1	7
6	**(i)** $\frac{x+2}{2x+3} - \frac{x-1}{3x-1}$		
	$= \frac{(x+2)(3x-1) - (2x+3)(x-1)}{(2x+3)(3x-1)}$	M2	
	$= \frac{(3x^2 + 5x - 2) - (2x^2 + x - 3)}{(6x^2 + 7x - 3)}$		
	$= \frac{x^2 + 4x + 1}{6x^2 + 7x - 3}$	W1, W1 (terms in numerator)	
	(ii) Equation can be written as		
	$\frac{x+2}{2x+3} - \frac{x-1}{3x-1} = \frac{1}{2}$		
	$\therefore \frac{x^2 + 4x + 1}{6x^2 + 7x - 3} = \frac{1}{2}$		
	$\therefore 2(x^2 + 4x + 1) = 6x^2 + 7x - 3$	M2	
	$\therefore 2x^2 + 8x + 2 = 6x^2 + 7x - 3$		
	$\therefore 4x^2 - x - 5 = 0$	W1	
	$\therefore (4x - 5)(x + 1) = 0$		
	$\therefore x = \frac{5}{4}$ **or** $x = -1$	W1	8

			AVAILABLE MARKS
7	**(a)** Take logs: $\log 9^{(\frac{2}{5}x-3)} = \log 7$	M1	
	$\left(\frac{2}{5}x - 3\right)\log 9 = \log 7$	M1	
	$\frac{2}{5}x = \frac{\log 7}{\log 9} + 3$		
	$x = \frac{5}{2}\left(\frac{\log 7}{\log 9} + 3\right)$	M1	
	$= 9.714$ (to 3 d.p) **or** 9.71 (to 3 s.f.)	W1	
	(b) $y^{\frac{1}{2}} = 5$		
	$\therefore y = 5^2 = 25$	MW1	
	(c) **(i)** $\log_3 1.6 = \log_3\left(\frac{8}{5}\right)$		
	$= \log_3 8 - \log_3 5$		
	$= b - a$	MW1	
	(ii) $\log_3 120 = \log_3(3 \times 5 \times 8)$		
	$= \log_3 3 + \log_3 5 + \log_3 8$	M1	
	$= 1 + a + b$	MW1	8

8 **(i)** $\hat{JKX} = 180 - 32.90 = 147.10°$

$\hat{JXK} = 32.90 - 21.20 = 11.70°$ MW1

(ii) $\frac{JX}{\sin 147.1} = \frac{1}{\sin 11.7}$ M2

$JX = \frac{\sin 147.1}{\sin 11.7} = 2.68$ km W1

(iii) $\hat{HJX} = 180 - 21.20 = 158.80°$ MW1

(iv) $HX^2 = 3^2 + 2.68^2 - 2 \times 3 \times 2.68 \cos 158.8$ M2

$\therefore HX = 5.58$ km W1

(v) $\frac{\sin \hat{XHJ}}{2.68} = \frac{\sin 158.8}{5.58}$ M1

$\therefore \sin \hat{XHJ} = 0.1737$

$\therefore \hat{XHJ} = 10.00°$ W1

(vi) Let h = height of helicopter as it passes over Julie's house.

$\frac{h}{3} = \tan 10$ M1

$\therefore h = 3 \tan 10 = 0.53$ km $= 530$ m W1

AVAILABLE MARKS

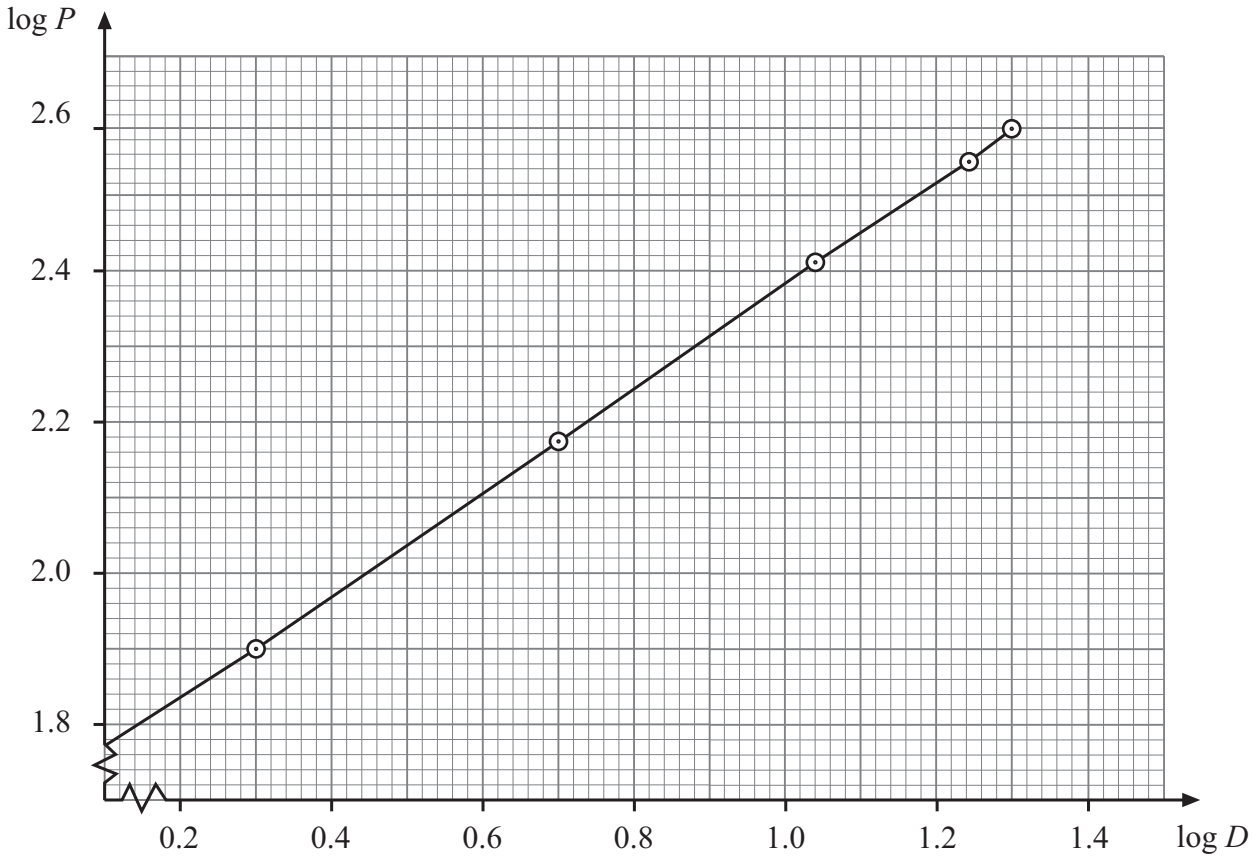
12

9 **(i)** $P = a D^b$

$\therefore \log P = b \log D + \log a$ M1

$\log P$	$\log D$
1.900	0.301
2.179	0.699
2.419	1.041
2.551	1.230
2.600	1.301

M1
W1



			AVAILABLE MARKS
	Straight line graph, so formula holds.	W1 for labels W1 for points W1 for straight line	
(ii)	$b = \frac{2.600 - 1.900}{1.301 - 0.301} = 0.7$	M1W1	
	$P = aD^{0.7}$		
	$79.50 = a(2^{0.7})$		
	$79.50 = 1.6245a$		
	$a = 48.9$	M1W1	
(iii)	$P = 48.9\ (28^{0.7}) = £503.87$	MW1	
	Assume the relationship holds for more than 20 days	M1	
(iv)	$48.9D^{0.7} = 330$		
	$D^{0.7} = 6.7485$ **or** $0.7 \log D = \log 6.7485$	MW1	
	$D = 15.296$ $\log D = 1.18458$		
	$D = 15.296$		
	15 days	MW1	14

10 **(i)** $12x + 8y + 20z = 320$

$\therefore\ 3x + 2y + 5z = 80 \quad ①$ — MW1

(ii) $25x + 30y + 15z = 660$

$\therefore\ 5x + 6y + 3z = 132 \quad ②$ — MW1

(iii) $18\left(\frac{2}{3}\right)x + 12y + 27z = 393$ — MW1

$\therefore\ 12x + 12y + 27z = 393$

$\therefore\ 4x + 4y + 9z = 131 \quad ③$ — MW1

(iv) $① \times 3 - ② \rightarrow 4x + 12z = 108$

$\therefore\ x + 3z = 27 \quad ④$ — M1W1

$① \times 2 - ③ \rightarrow 2x + z = 29 \quad ⑤$ — M1W1

$④ \times 2 - ⑤ \rightarrow 5z = 25$ — M1W1

$\therefore\ z = 5$ — (2 eqns → 1 eqn)

$\therefore\ x = 27 - 3z = 12$

$\therefore\ y = \dfrac{80 - 3x - 5z}{2} = 9.5$ — M1 (back sub)

$\therefore$ masses are

coin	mass (g)
£2	12
£1	9.5
50p	8
20p	5

W1

(v) Let n = number of counterfeit coins.

Then $10\,n + (20 - n)12 = 228$ — M1

$\therefore\ -2\,n = -12$

$\therefore\ n = 6$

i.e. 6 counterfeit coins — W1

14

AVAILABLE MARKS

11 (i) Crosses x-axis when $y = 0$

AVAILABLE MARKS

$14x + 3x^2 - 2x^3 = 0$ M1

$2x^3 - 3x^2 - 14x = 0$

$x(2x^2 - 3x - 14) = 0$

$x(2x - 7)(x + 2) = 0$

$x = 0$ **or** $\frac{7}{2}$ **or** -2

So points are $(0, 0)$, $(3\frac{1}{2}, 0)$ and $(-2, 0)$ W1

(ii) $\frac{dy}{dx} = 14 + 6x - 6x^2$ MW1

$14 + 6x - 6x^2 = 0$ M1

$6x^2 - 6x - 14 = 0$

$3x^2 - 3x - 7 = 0$

$x = \frac{3 \pm \sqrt{9 + 84}}{6} = \frac{3 \pm \sqrt{93}}{6}$

$x = \frac{3 \pm 9.644}{6}$ M1

$x = 2.11$ **or** -1.11 W1W1

$\therefore$ points are $(2.11, 24.11)$ and $(-1.11, -9.11)$ W1

(iii) $\frac{d^2y}{dx^2} = -12x + 6$ M1

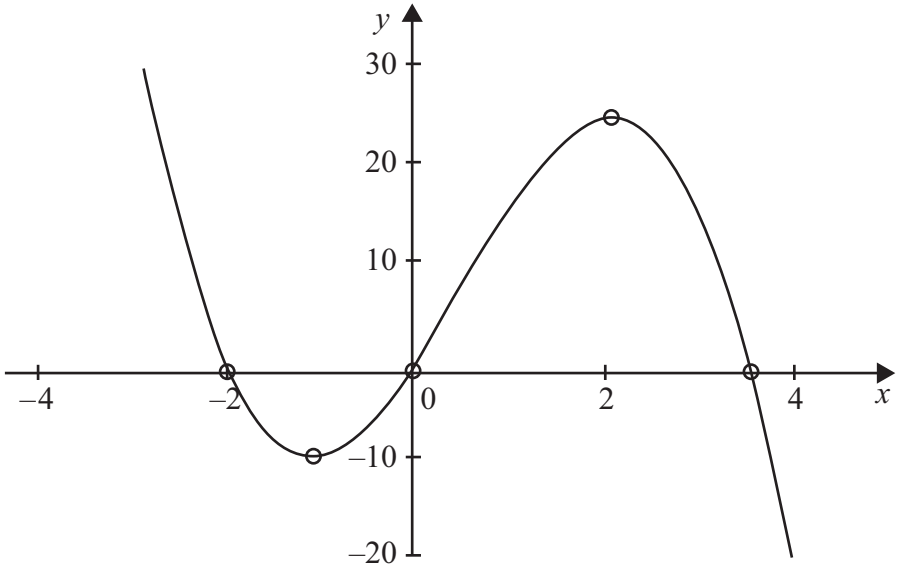
$x = 2.11$

$\frac{d^2y}{dx^2} = -19.32$ Maximum TP $(2.11, 24.11)$

$x = -1.11$

$\frac{d^2y}{dx^2} = 19.32$ Minimum TP $(-1.11, -9.11)$ MW1

(iv)



W1 (shape)
W1 (max and min)
W1 (intercepts on axes)

(v) $\text{area} = -\int_{-2}^{0} (14x + 3x^2 - 2x^3)\,dx$ M1

$= -\left[7x^2 + x^3 - \frac{x^4}{2}\right]_{-2}^{0}$ MW1

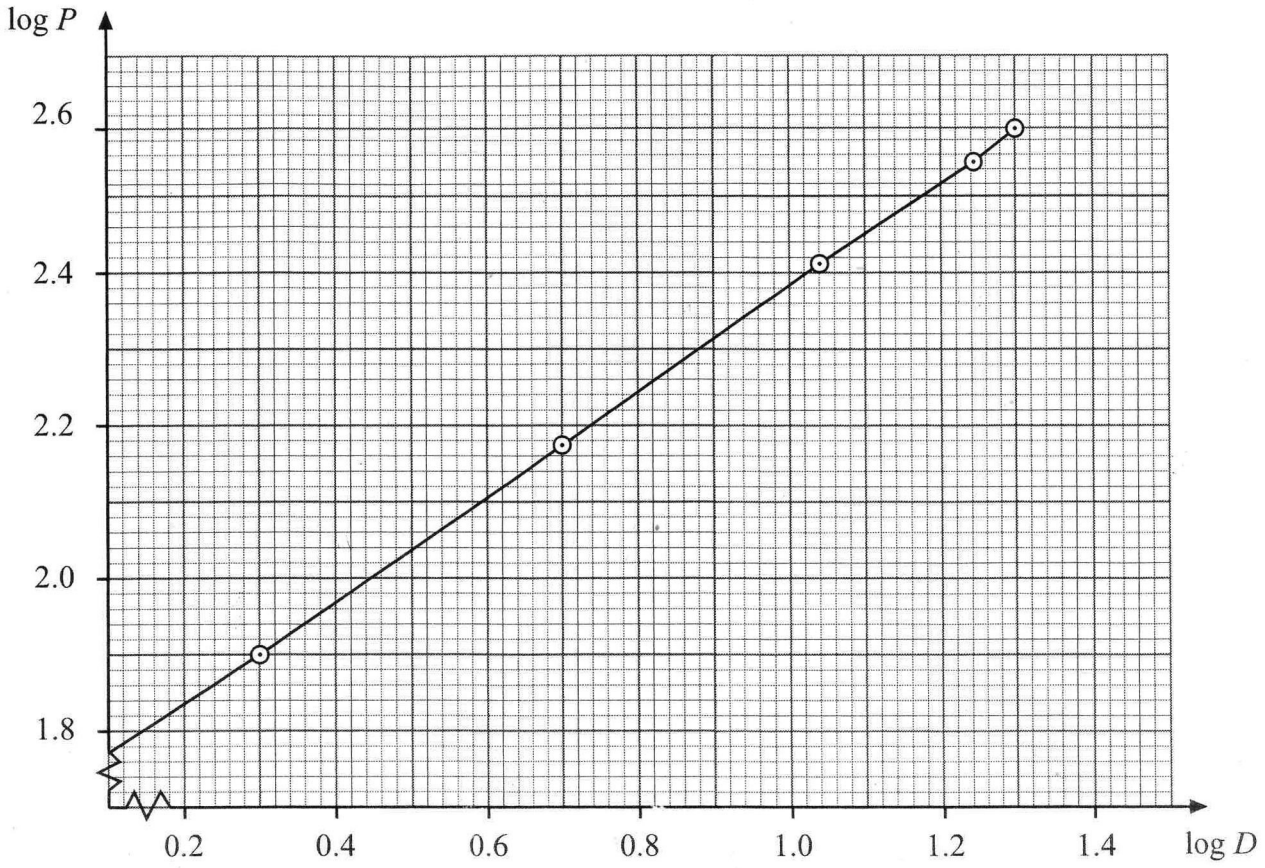
$= -(0) + (28 - 8 - 8)$

$= 12$ W1 16

Total **100**

AVAILABLE MARKS

GCSE ADDITIONAL MATHEMATICS
SUMMER 2010
PAPER 1
OVERLAY QUESTION 9



Rewarding Learning

General Certificate of Secondary Education
2010

Additional Mathematics

Paper 2
Mechanics and Statistics

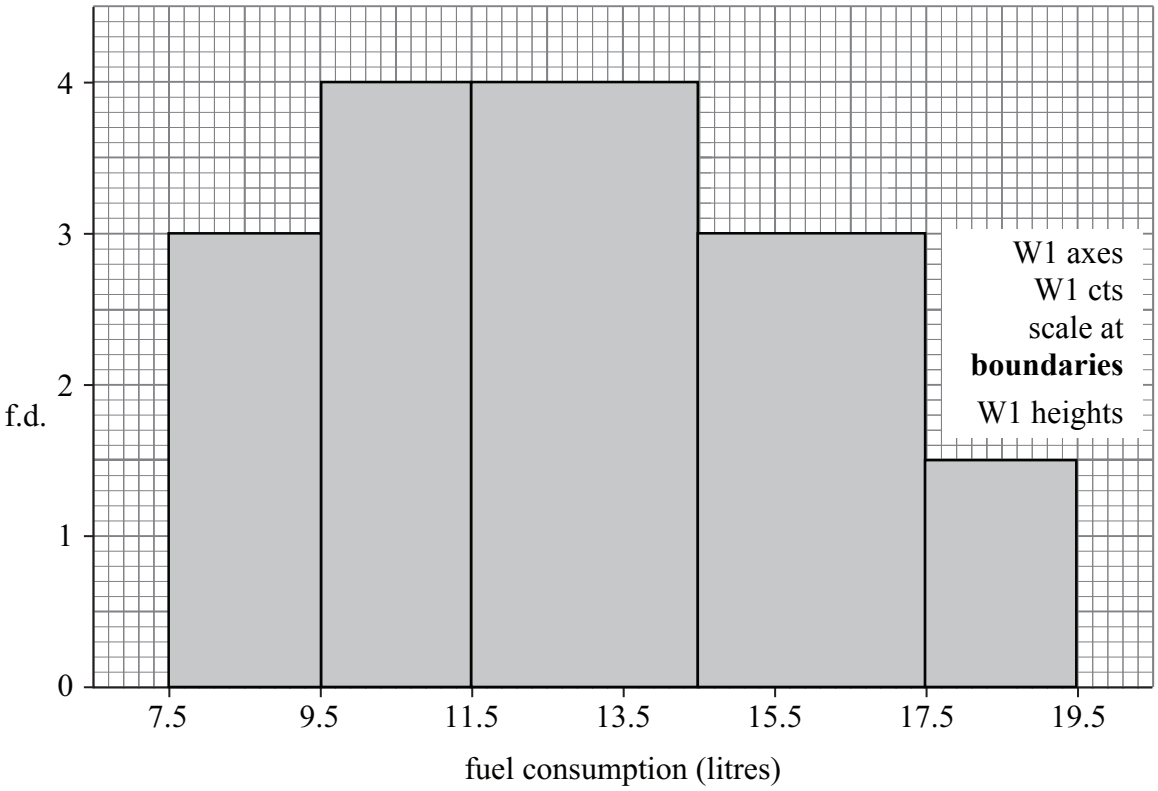
[G0302]

WEDNESDAY 19 MAY, MORNING

MARK SCHEME

		AVAILABLE MARKS
1	**(i)** $2\mathbf{i} - 3\mathbf{j} + 7\mathbf{i} - 5\mathbf{j} + a\mathbf{i} + b\mathbf{j} = 0\mathbf{i} + 0\mathbf{j}$ MW1	
	$\Rightarrow 2 + 7 + a = 0$	
	$\Rightarrow a = -9$ W1	
	and $-3 - 5 + b = 0$	
	$\Rightarrow b = 8$ W1	
	(ii) $8\mathbf{i} - 6\mathbf{j} = 4a$	
	$\Rightarrow a = (2\mathbf{i} - 1.5\mathbf{j})$ m/s^2 MW1	4

2 class width 2 2 3 3 2 M1W1
f.d. 3 4 4 3 1.5



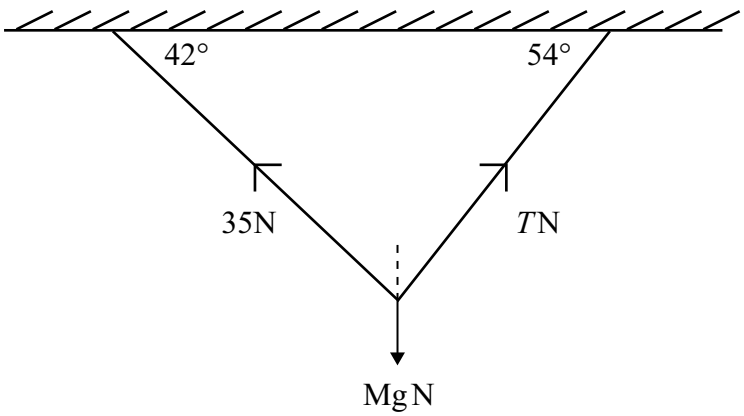
5

3 median class $\frac{(131 + 1)}{2}$: 66th : class $2 \le t < 4$ MW1

Median $= 2 + \frac{66 - 35}{36} \times 2 = 3.72$ mins

for 2+ MW1
for 66 – 35 MW1
for 2/36 MW1
ans MW1 5

4



AVAILABLE MARKS

(i) Resolve horizontally

$35\cos 42 = T\cos 54$ MW1

$\Rightarrow T = \dfrac{35\cos 42}{\cos 54}$ MW1

$\Rightarrow T = \underline{44.25\,\text{N}}$ W1

(ii) Resolve vertically

$Mg = 35\sin 42° + 44.25\sin 54°$ MW1

$\Rightarrow Mg = 23.42 + 35.80$ W1

$\Rightarrow Mg = 59.22$

$\Rightarrow M = \underline{5.92}\ \text{kg}$ W1 — 6

5 **(a)** **(i)** $4.3^2 = 18.49$ MW1

(ii) Mean $= (22.8 + 10) \times 2 = 65.6$ MW1

Standard deviation $= 4.3 \times 2 = 8.6$ M1W1

(b) Median $= \dfrac{(62 + 72)}{2} = 67$

Mean $\dfrac{514}{8} = 64.25$

Mode $= 72$ M1W1

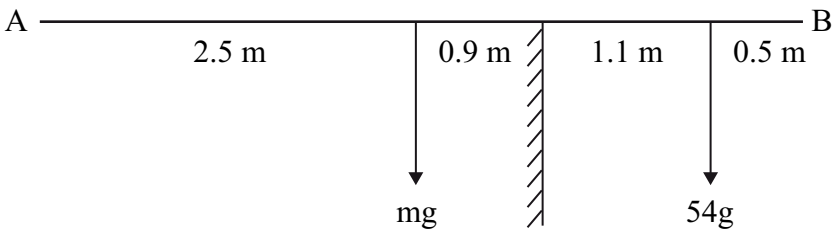
So 72 gives her highest score, i.e. mode MW1 — 7

				AVAILABLE MARKS
6	**(i)**	At the greatest height $v = 0$ using $v^2 = u^2 + 2as$ $0 = 6^2 + 2(-10)\, s$ $\Rightarrow s = \frac{36}{20} = 1.8$ m	MW1	
		$\Rightarrow$ Greatest height above the ground = 8 + 1.8 = 9.8 m	W1	
	(ii)	Let v = speed as ball strikes the ground $u = 0$ as ball starts falling $\Rightarrow v^2 = 0 + 2(10)\,(9.8)$ $\Rightarrow v^2 = 196$ $\Rightarrow v = 14$ m/s	MW1 W1	
	(iii)	t_1 = time to reach the greatest height using $v = u + at$ $0 = 6 + (-10)t_1$ $\Rightarrow t_1 = 0.6$ secs t_2 = time from greatest height to the ground Again using $v = u + at$ $14 = 0 + 10t_2$ $\Rightarrow t_2 = 1.4$ secs Total time from the instant the ball is thrown until it reaches the ground = $t_1 + t_2$ $\Rightarrow$ Total time = 0.6 + 1.4 = 2 secs	MW1 MW1 W1	7
7	**(i)**	$(1/10)^3 = 1/1000$ (0.001)	M1W1	
	(ii)	P (correct) = (1/10) × (1/9) × (1/8) = 1/720	M1W1	
	(iii)	P (wrong) = 1–1/720 = 719/720	MW1	
	(iv)	P (2nd correct) = (719/720) × (1/719) = 1/720	M1, M1W1 W1	9

8 **(i)** When the plank is on the point of tilting, the reaction at A is zero. M1

AVAILABLE MARKS

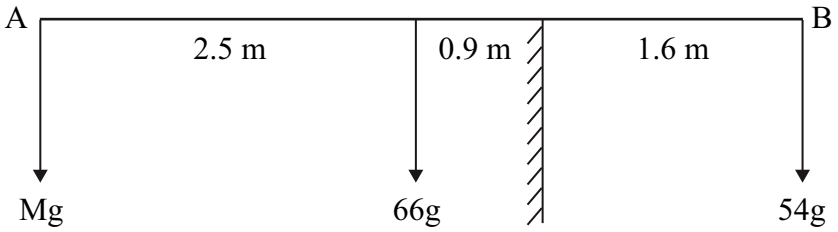
(ii)



Take moments at edge of roof
Let m kg equal the mass of plank
$mg(0.9) = 54g(1.1)$ MW2

$\Rightarrow m = \dfrac{54 \times 1.1}{0.9} = 66\,\text{kg}$ W1

(iii)



Again, take moments at edge of roof
$Mg(3.4) + 66g(0.9) = 54g(1.6)$ MW3

$\Rightarrow \text{M} = \dfrac{54(1.6) - 66(0.9)}{3.4}$ MW1

$\Rightarrow \text{M} = \dfrac{86.4 - 59.4}{3.4}$

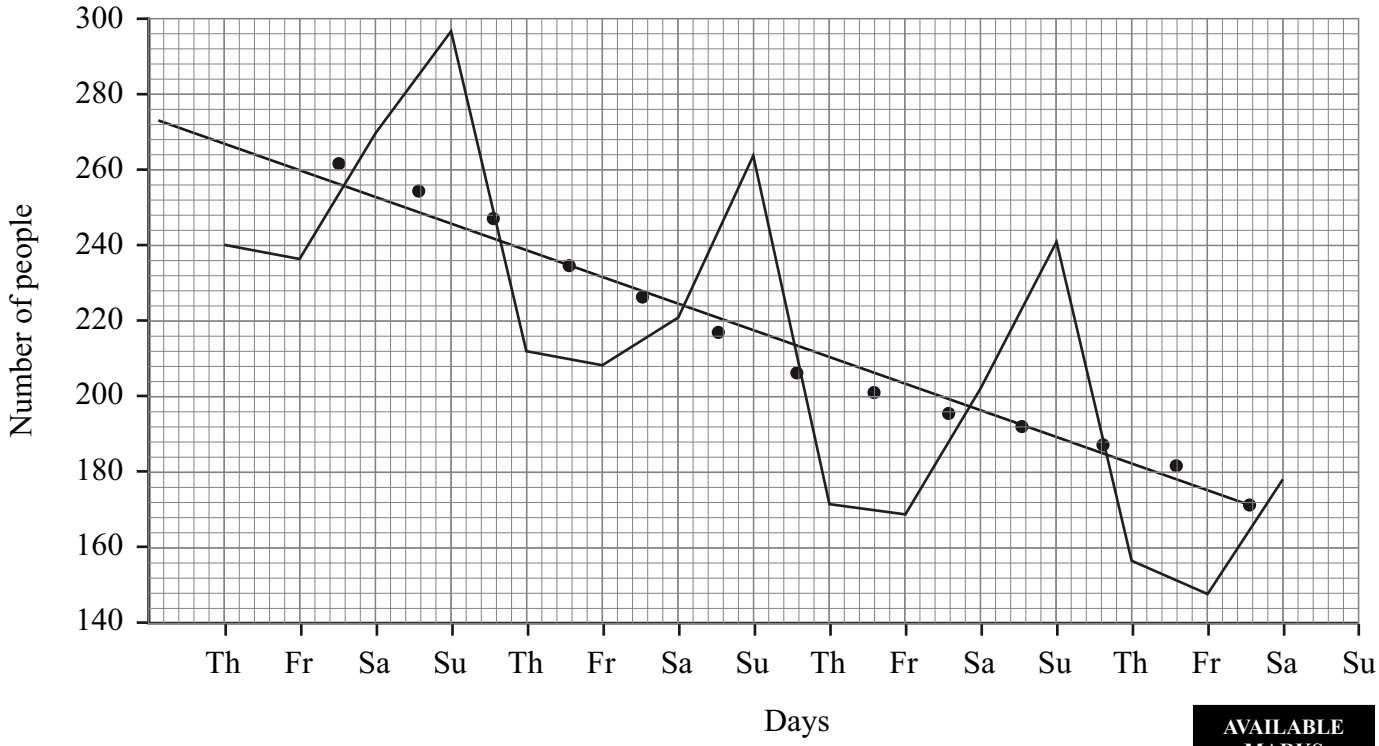
$\Rightarrow \text{M} = \dfrac{27}{3.4} = 7.9\,\text{kg}$ (to 1 dec. place) W1 9

9 **(i)** Averages

261
254
246.75
234.5
226
216
206
201.25
195.75
192
187
181.25 M2W1

AVAILABLE MARKS

(ii)



AVAILABLE MARKS

set M2
line W1

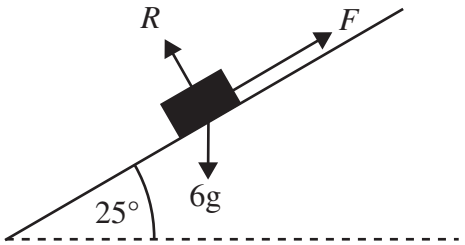
(iii) $\frac{x + 179 + 148 + 157}{4} = 172$ M1W1

reading

$x = 204$ M1W1

(iv) Cyclical M1 11

10 (i)



MW2

(ii) Resolve perpendicular to the plane

$R = 6\,g\cos 25°$ MW1

$\Rightarrow R = 54.38\,N$

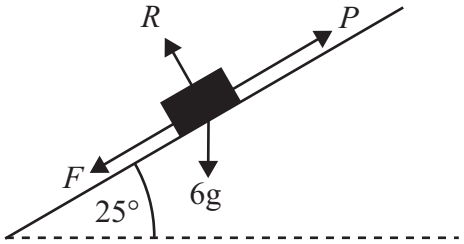
(iii) $F = 6g\sin 25°$

$F_{max} = \mu R$

$\Rightarrow \mu R = 6g\sin 25°$ MW1

$\Rightarrow \mu = \dfrac{6g\sin 25°}{6g\cos 25°} = 0.47$ (to 2 dec. places) W1

(iv)



MW1

(v) Resolve perpendicular to the plane

$R = 6g\cos 25°$

Resolve along the plane

$P = \mu R + 6g\sin 25°$

$\Rightarrow P = 0.47(6g\cos 25°) + 6g\sin 25°$ MW1

$\Rightarrow P = 25.36 + 25.36$

$\Rightarrow P = 50.72\,N$ W1

(accept $P = 50.92\,N$ (obtained by using μ to 2 d.p.))

(vi) Let the new P be called P_1

$P_1 - (\mu R + 6g\sin 25°) = ma = 6 \times 3.5$ MW1 MW1

$\Rightarrow P_1 = 21 + 50.72\,N = 71.72\,N$ W1

(accept $P_1 = 71.92\,N$ (obtained by using μ to 2 d.p.))

AVAILABLE MARKS

11

11 **(i)**

	7.5	2	6	7.5	4	3	1	5
	6.5	3	5	8	6.5	2	1	4
d^2	1	1	1	0.25	6.25	1	0	1

Ranks W2

(ii) $r = 1 - \dfrac{6(11.5)}{8(63)}$ Σd^2 M1W1

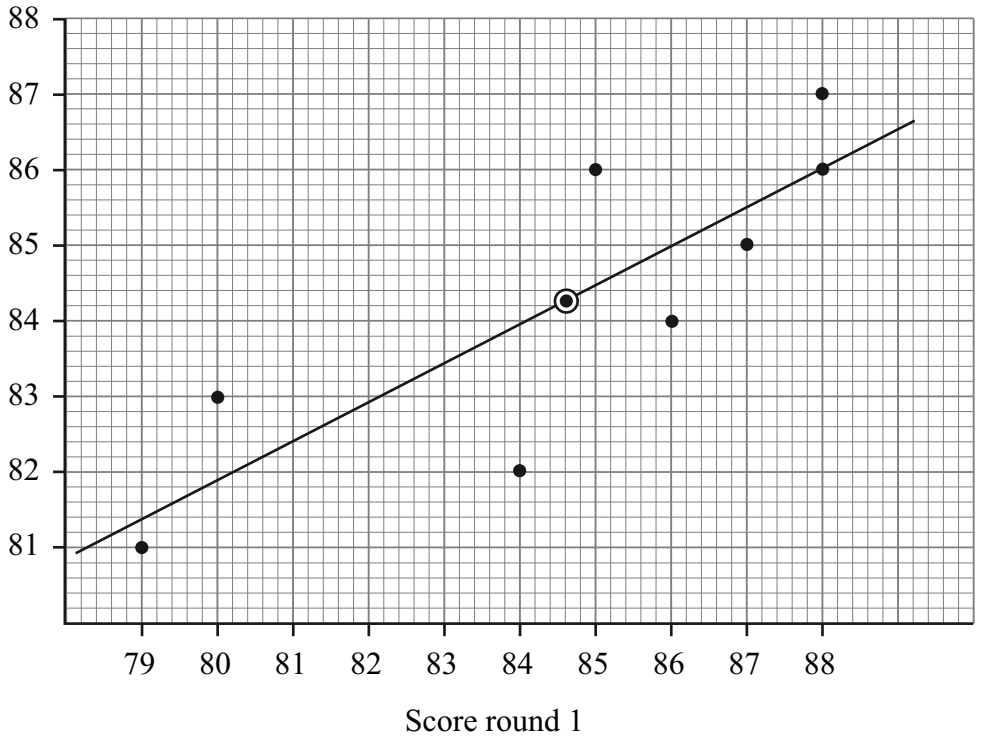
$= 0.863$ M1W1

(iii) positive correlation M1

(iv) mean round 1 = 84.625 mean round 2 = 84.25 MW1

MW1 thru means

MW1 slope



(v) gradient $= \dfrac{86 - 81.9}{88 - 80} = 0.51$ correct method for gradient M1

$y = mx + c$

$84.25 = 0.51 \times 84.625 + c$ correct method for intercept M1

$\therefore$ c = 41.1

So eqn for line of best fit is

$y = 0.51x + 41.1$ equation MW1 13

AVAILABLE MARKS

12 **(i)** using $v = u + at$

$18 = 0 + 3t$ MW1

$\Rightarrow t = \frac{18}{3} = 6$ secs W1

(ii) using $s = ut + \frac{1}{2}at^2$ MW1

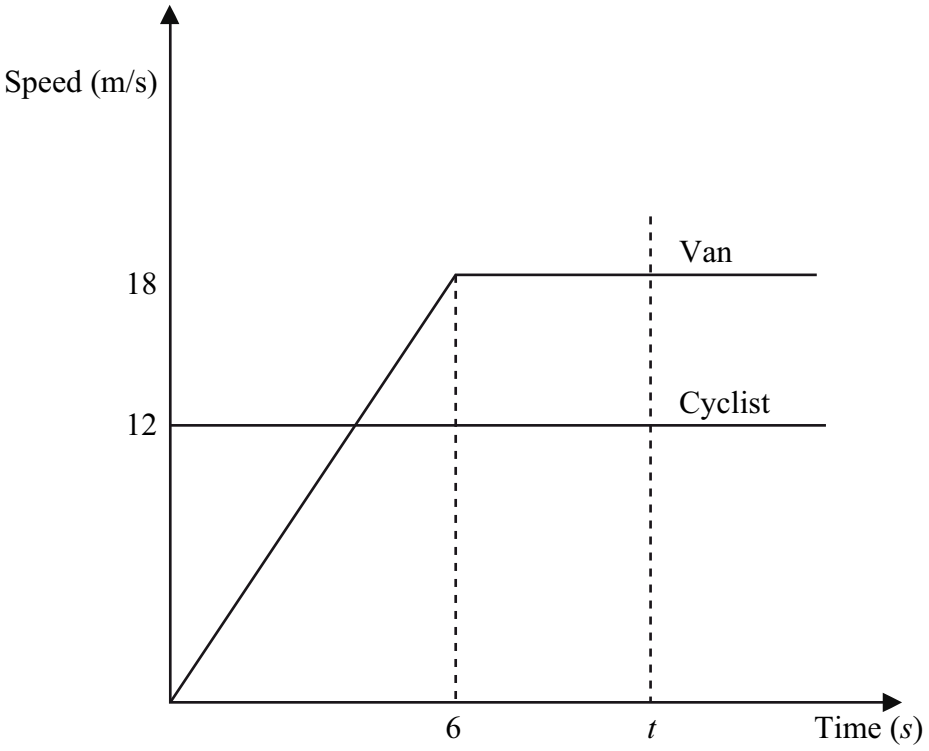
$s = 0 + \frac{1}{2}.3.36 = 54$ m = distance travelled by van W1

(iii) distance travelled by cyclist in 6 secs $= 6 \times 12 = 72$ m MW1

$\Rightarrow$ distance travelled by cyclist is greater than that travelled by the van in the first 6 secs.

$\Rightarrow$ van does not overtake the cyclist before it reaches its maximum speed W1

(iv)



W2

(v) **(a)** Distance travelled by the van $= \frac{1}{2}(t + (t - 6)).18$ MW1

$= 9\ (2t - 6)$

$= 18t - 54$

(b) Distance travelled by cyclist $= 12t$ MW1

(vi) These 2 distances are the same

$\Rightarrow 18t - 54 = 12t$ M1

$\Rightarrow 6t = 54$

$\Rightarrow t = 9$ secs W1

$\Rightarrow$ the distance between S and T is the distance travelled in 9 secs

$\Rightarrow$ ST $= 12 \times 9 = 108$ m

or

$\Rightarrow$ ST $= (18 \times 9) - 54 = 108$ m W1 13

Total **100**

AVAILABLE MARKS
13
100