Rewarding Learning

## General Certificate of Secondary Education

2012

## Additional Mathematics

## Paper 1 <br> Pure Mathematics

[G0301]

MONDAY 28 MAY, MORNING

## TIME

2 hours.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet and the Supplementary Answer Booklet provided.
Answer all eleven questions.
At the conclusion of this examination attach the Supplementary Answer Booklet to your Answer Booklet using the treasury tag supplied.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 100 .
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
A copy of the formulae list is provided.

## Answer all eleven questions

1 (i) Using the axes and scales in Fig. 1 in your Supplementary Answer Booklet, sketch the graph of $y=\tan x$ for $-75^{\circ} \leqslant x \leqslant 75^{\circ}$.
(ii) Hence, using the axes and scales in Fig. 2 in your Supplementary Answer Booklet, sketch the graph of $y=\tan \left(\frac{1}{2} x\right)$ for $-150^{\circ} \leqslant x \leqslant 150^{\circ}$.

2 (i) Solve the equation

$$
\sin \theta=-0.4
$$

for $0^{\circ} \leqslant \theta<360^{\circ}$.
(ii) Hence solve the equation

$$
\begin{equation*}
\sin \left(2 x+60^{\circ}\right)=-0.4 \tag{3}
\end{equation*}
$$

for $0^{\circ} \leqslant x<180^{\circ}$.

3 (i) Find $\mathbf{A}^{-1}$ where $\mathbf{A}=\left[\begin{array}{cc}5 & -4 \\ -3 & 2\end{array}\right]$
(ii) Hence, using a matrix method, solve the following simultaneous equations for $x$ and $y$.

$$
\begin{align*}
5 x-4 y & =-10 \\
-3 x+2 y & =4 \tag{4}
\end{align*}
$$

4 (a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ if $y=\frac{3}{2} x^{6}-\frac{1}{2 x^{3}}+2$
(b) Find $\int\left(7 x^{6}-\frac{3}{x^{3}}\right) \mathrm{d} x$

5 (i) Find the equation of the tangent to the curve $y=6 x^{3}-2 x^{4}$ at the point $(1,4)$.
(ii) Find the coordinates of P , the point of intersection, of this tangent and the straight line $y-26 x-18=0$
(iii) Show that this point P lies on the line $y-14 x=0$

6 (i) Show that

$$
\frac{5 x+1}{2 x+3}-\frac{3 x-7}{1-x}
$$

can be written as

$$
\begin{equation*}
\frac{11 x^{2}-9 x-22}{2 x^{2}+x-3} \tag{4}
\end{equation*}
$$

(ii) Hence, or otherwise, solve the equation

$$
\begin{equation*}
\frac{5 x+1}{2 x+3}-\frac{3 x-7}{1-x}=4 \tag{4}
\end{equation*}
$$

7 (a) Solve the equation

$$
\begin{equation*}
8^{(4 x-3)}=50 \tag{4}
\end{equation*}
$$

(b) (i) Show that $\log _{10}\left(10 x^{3}\right)=1+3 \log _{10} x$.
(ii) Hence, given the equation

$$
\begin{equation*}
\frac{1}{2} \log _{10} y=1+3 \log _{10} x \tag{2}
\end{equation*}
$$

express $y$ in terms of $x$.

8 Two cowboys, Frank and Jesse, were riding along separate straight trails towards a ranch R, passing over flat terrain.

Frank crossed over a straight railway line PQ at a point X and continued on to the ranch R , where $\mathrm{XR}=4.50 \mathrm{~km}$.

A station $S$ on the railway line is 9.50 km from R and the size of the angle SRX is $60^{\circ}$, as shown in Fig. 3.


Fig. 3
(i) Calculate the distance SX.
(ii) Calculate the size of the angle $\mathrm{X} \hat{\mathrm{S}}$ R correct to 2 decimal places.

Jesse crossed the railway line at a point $Y$ and continued on to the ranch $R$ where he met Frank. The distance XY is 3.15 km .
(iii) Find the size of the angle $\mathrm{R} \hat{X} Y$ correct to 2 decimal places.
(iv) Calculate the distance YR.

While at the ranch the cowboys noticed the smoke of a train passing through the station S heading in the direction of Q. Jesse knew that falling rocks had blocked the line at a point Z . The cowboys left immediately to ride back to the line to warn the driver. Frank rode along the trail RX and Jesse along the trail RY.

Both cowboys rode at $20 \mathrm{~km} / \mathrm{h}$ and the train travelled at $40 \mathrm{~km} / \mathrm{h}$.
(v) Determine which, if either, of the cowboys would arrive at the line in time to warn the driver. You must show working to justify your answer.

9 Carly noted the cost $C$ (in pounds) and the age $A$ (in months) of five FAYE laptops. The data are given in Table 1.

Table 1

| Age <br> A (months) | Cost <br> $£ C$ |
| :---: | :---: |
| 5 | 206.54 |
| 9 | 139.78 |
| 14 | 101.69 |
| 23 | 71.13 |
| 34 | 53.68 |

She believes that a relationship of the form

$$
C=p A^{q}
$$

exists between $C$ and $A$, where $p$ and $q$ are constants.
(i) Using Fig. 4 in your Supplementary Answer Booklet, verify this relationship by drawing a suitable straight line graph, using values correct to 3 decimal places. Label the axes clearly.
(ii) Hence, or otherwise, obtain values for $p$ and $q$. Give your answers correct to 2 significant figures.

Carly buys a one year old FAYE laptop.
(iii) Use the formula $C=p A^{q}$ with the values you obtained for $p$ and $q$ to calculate the price Carly paid for her laptop.

Carly plans to sell her laptop when it is worth half what she paid for it.
(iv) Determine how long Carly should keep her laptop.

10 Matthew, Emma and Simon decided to invest some money in low, medium and high risk accounts.

Matthew invested $£ 5000, £ 3000$ and $£ 2000$ in the low, medium and high risk accounts respectively, and his expected interest after one year is $£ 500$.

Let $x, y$ and $z$ represent the percentage interest rates for the low, medium and high risk accounts respectively.
(i) Show that $x, y$ and $z$ satisfy the equation

$$
\begin{equation*}
5 x+3 y+2 z=50 \tag{1}
\end{equation*}
$$

Emma invested $£ 2000, £ 2000$ and $£ 7000$ in the low, medium and high risk accounts respectively. Her expected interest after one year is $£ 860$.
(ii) Show that $x, y$ and $z$ also satisfy the equation

$$
\begin{equation*}
2 x+2 y+7 z=86 \tag{1}
\end{equation*}
$$

Simon invested $£ 8000, £ 2000$ and $£ 10000$ in the low, medium and high risk accounts respectively. His expected interest after one year is $£ 1340$.
(iii) Show that $x, y$ and $z$ also satisfy the equation

$$
\begin{equation*}
4 x+y+5 z=67 \tag{1}
\end{equation*}
$$

(iv) Solve these equations, showing clearly each stage of your solution, to find the percentage interest rates for the low, medium and high risk accounts.

Penny wants to invest a total of $£ 10000$ in the medium and high risk accounts such that she can expect to earn a total of $£ 800$ in interest after one year.

Let $a$ and $b$ represent the amounts she should invest in the medium and high risk accounts respectively.
(v) Write down two equations satisfied by $a$ and $b$.
(vi) Solve these equations to determine how much she should invest in each account.

11 A curve is defined by the equation

$$
y=2 x^{3}-3 x^{2}-5 x
$$

(i) Find the coordinates of the points where this curve crosses the $x$-axis.
(ii) Find the coordinates of the turning points of this curve. Give your answers to 2 decimal places.
(iii) Identify each turning point as either a maximum or a minimum point. You must show working to justify your answers.
(iv) Using your answers from parts (i) to (iii) sketch this curve using Fig. 5 in your Supplementary Answer Booklet.
(v) Find the area enclosed by the $x$-axis and the part of the curve which lies above the $x$-axis.
Centre Number
$\square$
Candidate Number

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Paper 1
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## SUPPLEMENTARY ANSWER BOOKLET

1 (i) Sketch the graph of $y=\tan x$, for $-75^{\circ} \leqslant x \leqslant 75^{\circ}$, on the axes in Fig. $\mathbf{1}$ below.


Fig. 1
(ii) Sketch the graph of $y=\tan \left(\frac{1}{2} x\right)$, for $-150^{\circ} \leqslant x \leqslant 150^{\circ}$, on the axes in Fig. 2 below.


Fig. 2

9 Draw a suitable straight line graph using the axes and scales in Fig. 4 below. Label the axes.


Fig. 4

11 Sketch the graph of $y=2 x^{3}-3 x^{2}-5 x$ in Fig. 5 below.


Fig. 5

