General Certificate of Secondary Education
2014

## Additional Mathematics

Paper 1<br>Pure Mathematics

[G0301]

## MONDAY 9 JUNE, MORNING

## TIME

2 hours.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet and the Supplementary Answer Booklet provided.
Answer all eleven questions.
At the conclusion of this examination attach the Supplementary Answer Booklet to your Answer Booklet using the treasury tag supplied.

## INFORMATION FOR CANDIDATIES

The total mark for this paper is 100 .
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
A copy of the formulae list is provided.

## Answer all eleven questions

1 (i) Using the axes and scales in Fig. 1 in your Supplementary Answer Booklet, sketch the graph of $y=\sin \left(\frac{1}{2} x\right)$ for $-360^{\circ} \leqslant x \leqslant 360^{\circ}$.
(ii) Hence, using the axes and scales in Fig. 2 in your Supplementary Answer Booklet, sketch the graph of $y=\sin \left(\frac{1}{2} x\right)+1$ for $-360^{\circ} \leqslant x \leqslant 360^{\circ}$.

2 (i) Solve the equation

$$
\tan \theta=0.6
$$

$$
\begin{equation*}
\text { for }-180^{\circ} \leqslant \theta \leqslant 180^{\circ} \text {. } \tag{2}
\end{equation*}
$$

(ii) Hence solve the equation

$$
\tan \left(\frac{1}{2} x-60^{\circ}\right)=0.6
$$

$$
\begin{equation*}
\text { for }-360^{\circ} \leqslant x \leqslant 360^{\circ} \text {. } \tag{3}
\end{equation*}
$$

3 (i) Find $\mathbf{A}^{-1}$ where $\mathbf{A}=\left[\begin{array}{ll}4 & -3 \\ 2 & -9\end{array}\right]$
(ii) Hence, using a matrix method, solve the following simultaneous equations for $x$ and $y$.

$$
\begin{align*}
& 4 x-3 y=6 \\
& 2 x-9 y=8 \tag{4}
\end{align*}
$$

4 (a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ if $y=4 x^{3}-\frac{3}{4} x^{2}+\frac{4}{3 x^{2}}$
(b) Find $\int\left(4 x-\frac{2}{x^{3}}\right) \mathrm{d} x$

5 A curve is defined by the equation $y=x^{3}-3 x^{2}-18 x$
(i) Show that the tangent to this curve at the point $(4,-56)$ is parallel to the straight line $l$ with equation $y-6 x=2$
(ii) Find the coordinates of the other point on this curve at which the tangent is parallel to the straight line $l$.

6 (i) Show that

$$
\frac{3 x+8}{x+3}-\frac{6 x}{2 x+1}
$$

can be written as

$$
\begin{equation*}
\frac{x+8}{2 x^{2}+7 x+3} \tag{4}
\end{equation*}
$$

(ii) Hence, or otherwise, solve the equation

$$
\begin{equation*}
\frac{3 x+8}{x+3}-\frac{6 x}{2 x+1}=-2 \tag{4}
\end{equation*}
$$

7 (a) Solve the equation

$$
\begin{equation*}
3^{2-3 x}=4 \tag{4}
\end{equation*}
$$

giving your answer correct to 3 decimal places.
(b) Solve the equation

$$
\begin{equation*}
\log _{2 x} 36=2 \tag{2}
\end{equation*}
$$

(c) If $\log _{3} 4=m$ and $\log _{3} 5=n$, express the following in terms of $m$ and $n$
(i) $\log _{3} 20$
(ii) $\log _{3} 60$

8 A charity has organised a sponsored abseil from the roof A of the Majestic Hotel, AB. Fundraisers will abseil from the roof A to a balcony X, vertically below A.

An observer is at the point D on the roof of the Grand Hotel, DE, which is of height 25 m .
Another observer is at the point C on the horizontal ground, $\mathrm{BE}, 20 \mathrm{~m}$ from the point E .
The angles $A \hat{C D}$, $A \hat{D D}$ and $A \hat{D} C$ were measured as $84^{\circ}, 34^{\circ}$ and $71^{\circ}$ respectively, as shown in Fig. 3.


Fig. 3
(i) Find the size of the angle CAD.
(ii) Calculate the distance CD.
(iii) Calculate the distance AD .
(iv) Given that the distance XD is 73.25 m , calculate the distance abseiled, AX .

The fastest abseiling time for a girl was two and a half minutes.
(v) Calculate her mean abseiling speed in metres per second.

9 Sarah measured the mass $M$ (grams) and the height $H(\mathrm{~cm})$ of five plants of the same type. The results are given in Table 1.

## Table 1

| Height <br> $H(\mathrm{~cm})$ | Mass <br> $M(\mathrm{~g})$ |
| :---: | :---: |
| 20 | 185 |
| 30 | 222 |
| 50 | 279 |
| 80 | 345 |
| 124 | 420 |

Sarah believes that a relationship of the form

$$
M=p H^{q}
$$

exists between $M$ and $H$, where $p$ and $q$ are constants.
(i) Using Fig. 4 in your Supplementary Answer Booklet, verify this relationship by drawing a suitable straight line graph, using values correct to 3 decimal places. Label the axes clearly.
(ii) Hence, or otherwise, obtain values for $p$ and $q$. Give your answers to 2 significant figures.
(iii) Use the formula $M=p H^{q}$ with the values you obtained for $p$ and $q$ to find the mass of a plant with a height of 64 cm .
(iv) Use the formula $M=p H^{q}$ with the values you obtained for $p$ and $q$ to find the height of a plant which has a mass of $\frac{1}{2} \mathrm{~kg}$. State any assumption which you make.

10 A wholesaler provides bags of coal, bags of logs and bags of peat briquettes, priced at $£ x, \mathfrak{£} y$ and $£ z$ per bag respectively, to three local garages.

Garage A purchases 150 bags of coal, 100 bags of logs and 50 bags of peat briquettes at a total cost of $£ 2250$
(i) Show that $x, y$ and $z$ satisfy the equation

$$
\begin{equation*}
3 x+2 y+z=45 \tag{1}
\end{equation*}
$$

Garage B purchases 195 bags of coal, 170 bags of logs and 75 bags of peat briquettes at a total cost of $£ 3195$
(ii) Show that $x, y$ and $z$ also satisfy the equation

$$
\begin{equation*}
39 x+34 y+15 z=639 \tag{1}
\end{equation*}
$$

Garage C purchases 150 bags of coal, 75 bags of logs and 60 bags of peat briquettes at a total cost of $£ 2130$
(iii) Show that $x, y$ and $z$ also satisfy the equation

$$
\begin{equation*}
10 x+5 y+4 z=142 \tag{1}
\end{equation*}
$$

(iv) Solve these equations, showing clearly each stage of your solution, to find the cost of a bag of coal, a bag of logs and a bag of peat briquettes.

As a special offer, the wholesaler is offering $25 \%$ off the price of a bag of coal and $25 \%$ off the price of a bag of logs.

Garage C puts in a new order to avail of the special offer. This order includes 75 bags of logs and 60 bags of peat briquettes. The wholesaler is asked to complete the order with bags of coal, so that the total cost is again $£ 2130$
(v) Calculate how many bags of coal are in the new order for garage C .

11 A curve is defined by the equation

$$
y=x^{3}-5 x^{2}+8 x
$$

(i) Verify that the curve crosses the $x$-axis at the origin.
(ii) Show that there are no other points at which the curve crosses the $x$-axis.
(iii) Find the coordinates of the turning points of the curve.
(iv) Identify each turning point as either a maximum or a minimum point. You must show working to justify your answers.
(v) Using your answers from parts (i) to (iv) sketch the curve using Fig. 5 in your Supplementary Answer Booklet.
(vi) Find the area enclosed by the curve, the positive $x$-axis and the straight line $x=a$, where $a$ is the $x$-coordinate of the minimum point.


General Certificate of Secondary Education

Additional Mathematics

Paper 1<br>Pure Mathematics

[G0301]

MONDAY 9 JUNE, MORNING

## SUPPLEMENTARY <br> ANSWER BOOKLET

1 (i) Sketch the graph of $y=\sin \left(\frac{1}{2} x\right)$ for $-360^{\circ} \leqslant x \leqslant 360^{\circ}$, on the axes in Fig. 1 below.


Fig. 1
(ii) Sketch the graph of $y=\sin \left(\frac{1}{2} x\right)+1$ for $-360^{\circ} \leqslant x \leqslant 360^{\circ}$, on the axes in Fig. 2 below.


Fig. 2

9 Draw a suitable straight line graph using the axes and scales in Fig. 4 below. Label the axes.


Fig. 4

11 Sketch the graph of $y=x^{3}-5 x^{2}+8 x$ in Fig. 5 below.


Fig. 5

