New Specification

Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2014

Further Mathematics

Unit 1

Pure Mathematics

[GMF11]

MONDAY 9 JUNE, MORNING

GMF11

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Complete in blue or black ink only. Do not write with a gel pen.
All working should be clearly shown since marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all sixteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

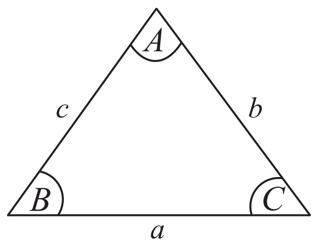
Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \dfrac{1}{2} ab \sin C$

Differentiation: If $y = ax^n$ then $\dfrac{dy}{dx} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log(ab) = \log a + \log b$

$\log\left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

STATISTICS

Statistical measures: Mean $= \dfrac{\Sigma fx}{\Sigma f}$ $\qquad$ Median $= L_1 + \dfrac{\left\{\frac{N}{2} - (\Sigma f)_1\right\} c}{f_{median}}$

where

L_1 is lower class boundary of the median class

N is total frequency

$(\Sigma f)_1$ is the sum of the frequencies up to but not including the median class

f_{median} is the frequency of the median class

c is the width of the median class

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - (\overline{x})^2}$ $\quad$ where $\overline{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

1 Matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} -1 & 7 \\ 4 & -2 \end{bmatrix}$$

Express as a single matrix:

(i) $\mathbf{A} + \mathbf{B}$

Answer ______________________________ [2]

(ii) $\mathbf{A}^2$

Answer ______________________________ [2]

Examiner Only	
Marks	Remark

(iii) Hence find the 2×2 matrix $\mathbf{X}$ which satisfies the equation

$$\mathbf{X} + \mathbf{B} = \mathbf{A}^2$$

Answer ______________________________ [2]

Examiner Only	
Marks	Remark

[Turn over

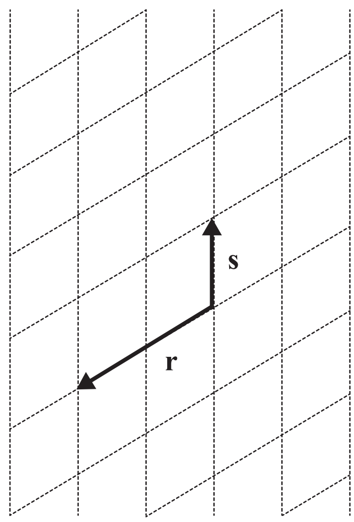
Examiner Only	
Marks	Remark

2 **(a)** Simplify the vector expression

$$\begin{bmatrix} -2 \\ 7 \end{bmatrix} + 3\begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

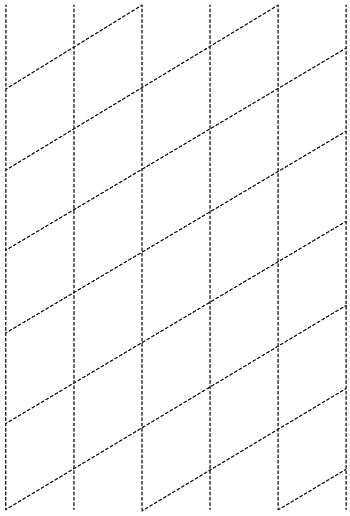
Answer ____________________________ [1]

(b) The vectors **r** and **s** are shown below.



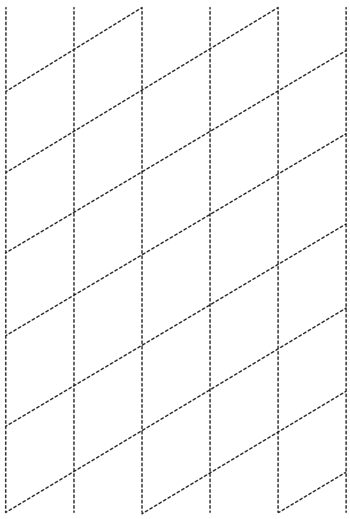
On the grids below, show

(i) the vector **s** − **r**



[1]

(ii) the vector **r** + 3**s**



[1]

Examiner Only	
Marks	Remark

[Turn over

Examiner Only	
Marks	Remark

3 Solve the equation $x^2 - 4x - 2 = 0$ by completing the square.

Give your answer in the form $a \pm \sqrt{b}$ where a and b are whole numbers.

Answer ______________________________ [4]

Examiner Only	
Marks	Remark

4 If $y = 4x^3 + \dfrac{4}{3x^2}$

(i) find $\dfrac{dy}{dx}$

Answer ______________________________ [2]

(ii) **Hence** find $\dfrac{d^2y}{dx^2}$

Answer ______________________________ [2]

[Turn over

Examiner Only	
Marks	Remark

5 Find $\int_{-4}^{1} (4x - \frac{2}{x^3} + 6)\, dx$

Answer ________________ [5]

Examiner Only	
Marks	Remark

6 **(i)** Solve the equation

$$\tan \theta = 0.6$$

for $-180° \leqslant \theta \leqslant 180°$

Answer ______________________________ [2]

(ii) **Hence** solve the equation

$$\tan (\tfrac{1}{2} x - 60°) = 0.6$$

for $-360° \leqslant x \leqslant 360°$

Answer ______________________________ [3]

[Turn over

7 Matrices **P** and **Q** are given by

$$\mathbf{P} = \begin{bmatrix} 4 & -3 \\ 2 & -9 \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

(i) Find $\mathbf{P}^{-1}$

Answer ______________________________ [2]

(ii) **Hence** find the matrix **X** where $\mathbf{PX} = \mathbf{Q}$

Answer ______________________________ [3]

Examiner Only	
Marks	Remark

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(Questions continue overleaf)

[Turn over

8 Simplify fully the following expressions:

Examiner Only	
Marks	Remark

(i) $\dfrac{4x^2-9}{(x-1)^2} \times \dfrac{x^2-x}{2x+3}$

Answer ______________________________ [3]

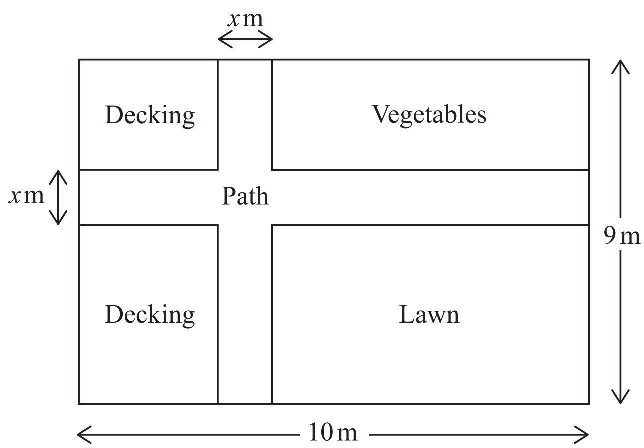
(ii) $\dfrac{3x+8}{x+3} - \dfrac{6x}{2x+1}$

Examiner Only	
Marks	Remark

Answer ______________________________ [4]

[Turn over

9 A landscape gardener is working on a rectangular garden of width 9 m and length 10 m. He wants to divide the garden by a path of width x m into lawn, vegetables and decking areas, as shown in the diagram below.



The area of the path is **one fifth** of the area of the garden.

Form an equation in terms of x and solve it to find the width of the path.

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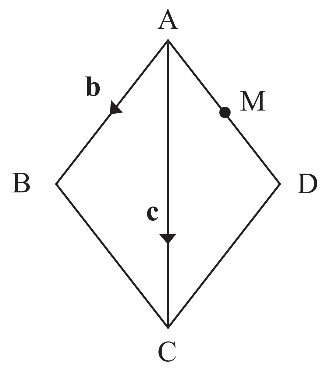
Examiner Only	
Marks	Remark

Answer _________ m [5]

[Turn over

10 In the rhombus ABCD below, $\overrightarrow{AB}$ represents the vector **b** and $\overrightarrow{AC}$ represents the vector **c**.

M is the midpoint of AD.



(i) Express each of the following vectors in terms of **b** and **c**, simplifying your answers as far as possible:

(a) $\overrightarrow{BC}$

Answer ______________________________ [1]

(b) $\overrightarrow{BD}$

Answer ______________________________ [1]

Examiner Only	
Marks	Remark

Examiner Only	
Marks	Remark

(c) $\overrightarrow{MB}$

Answer ______________________________ [1]

(ii) The point F is on AB produced such that AF = 3 AB.

(a) Find the vector $\overrightarrow{CF}$

Answer ______________________________ [1]

(b) **Hence** show that CF is parallel to MB and twice as long.

[1]

11 **(a)** Solve the equation

$$3^{2-3x} = 4$$

Answer ____________________ [4]

(b) Solve the equation

$$\log_{2x} 36 = 2$$

Answer ____________________ [2]

Examiner Only	
Marks	Remark

(c) If $\log_3 4 = m$ and $\log_3 5 = n$, express in terms of m and n

(i) $\log_3 20$

Answer ____________________ [1]

(ii) $\log_3 60$

Answer ____________________ [2]

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Marks	Remark

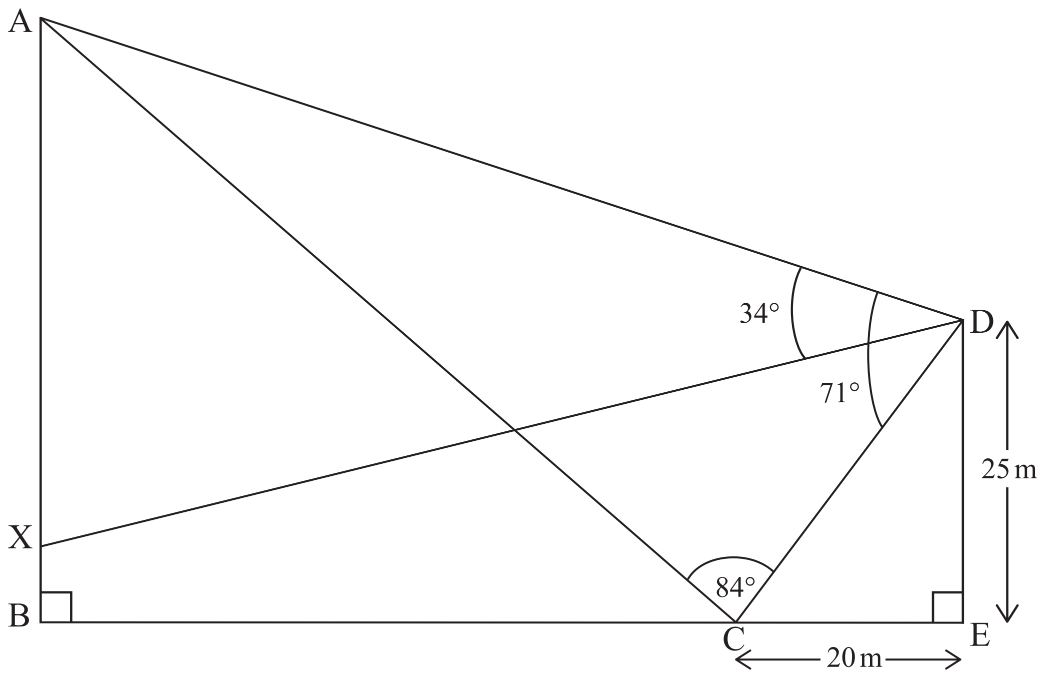
Examiner Only	
Marks	Remark

12 A charity has organised a sponsored abseil from the roof A of the Majestic Hotel, AB. Fundraisers will abseil from the roof A to a balcony X, vertically below A.

An observer is at the point D on the roof of the Grand Hotel, DE, which is of height 25 m.

Another observer is at the point C on the horizontal ground, BE, 20 m from the point E.

The angles $A\hat{C}D$, $A\hat{D}X$ and $A\hat{D}C$ were measured as 84°, 34° and 71° respectively, as shown in the diagram below.



(i) Find the size of the angle $C\hat{A}D$.

Answer ________________° [1]

(ii) Calculate the distance CD.

Answer ________________ m [2]

(iii) Calculate the distance AD.

Answer ____________________ m [3]

(iv) Given that the distance XD is 73.25 m, calculate AX, the distance abseiled.

Answer ____________________ m [3]

Examiner Only	
Marks	Remark

[Turn over

13 A curve is defined by the equation $y = x^3 - 3x^2 + 2x$

(i) Find the equation of the straight line T which is the tangent to this curve at the point (3,6).

Answer ______________________________ [3]

Examiner Only	
Marks	Remark

(ii) Find the equation of the straight line N which is the **normal** to this curve at the point (1,0).

Examiner Only	
Marks	Remark

Answer ______________________________ [2]

(iii) **Hence** find the coordinates of the point where the lines T and N meet.

Answer __________________ [2]

[Turn over

14 A wholesaler provides bags of coal, bags of logs and bags of peat briquettes, priced at £x, £y and £z per bag respectively, to three local garages.

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Marks	Remark

Garage A purchases 150 bags of coal, 100 bags of logs and 50 bags of peat briquettes at a total cost of £2250

(i) Show that x, y and z satisfy the equation

$$3x + 2y + z = 45$$

[1]

Garage B purchases 195 bags of coal, 170 bags of logs and 75 bags of peat briquettes at a total cost of £3195

(ii) Show that x, y and z also satisfy the equation

$$39x + 34y + 15z = 639$$

[1]

Garage C purchases 150 bags of coal, 75 bags of logs and 60 bags of peat briquettes at a total cost of £2130

(iii) Show that x, y and z also satisfy the equation

$$10x + 5y + 4z = 142$$

[1]

(iv) Solve these equations, showing clearly each stage of your solution, to find the cost of a bag of coal, a bag of logs and a bag of peat briquettes.

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Answer ____ [8]

[Turn over

15 The sketch below shows the curve with equation $y = x^2 - 2x - 3$

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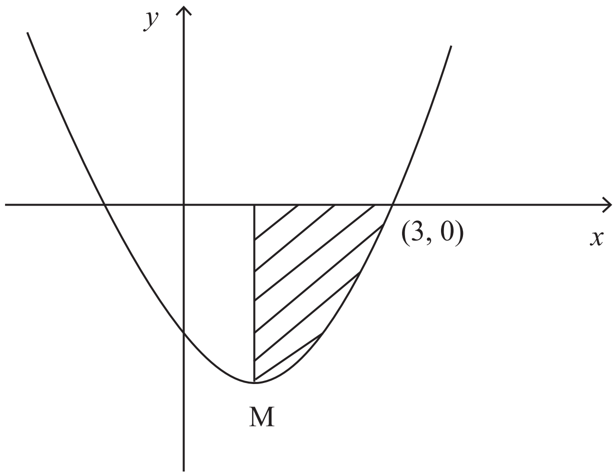


(i) Find the x-coordinate of the minimum point M.

Answer ___________________ [2]

(ii) **Hence** find the area of the shaded region shown.

Examiner Only	
Marks	Remark

Answer ____________________ [4]

[Turn over

16 A curve is defined by the equation $y = x^3 - 5x^2 + 8x$

(i) Find the coordinates of the turning points of the curve.

Answer ____ [5]

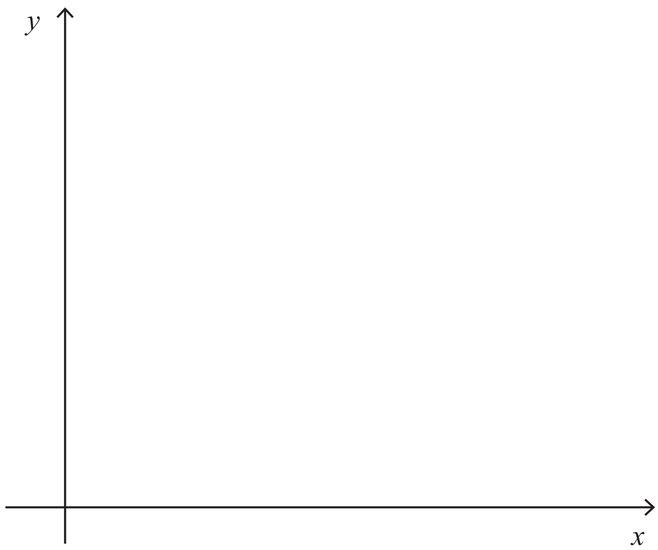
Examiner Only	
Marks	Remark

Examiner Only	
Marks	Remark

(ii) Identify each turning point as either a maximum or a minimum point. You **must** show working to justify your answers.

Answer ____ [2]

(iii) Given that the curve passes through the point (0,0), use your answers from parts **(i)** and **(ii)** to sketch the curve $y = x^3 - 5x^2 + 8x$ on the axes below.



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For Examiner's use only	
Question Number	Marks
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