MATHEMATICS

Paper 9280/11
Paper 11

General comments

While presentation of work for many candidates was satisfactory there were significant numbers of candidates whose work was untidy and difficult to follow. The rubric on the front of the Question Paper says 'You are reminded of the need for clear presentation in your answers'. It is generally true that marks cannot be awarded if methods used are not clear.

It is worth noting here a very common error: forgetting the ± sign. For example, in **Question 8(ii)**, $p^2 = 4 \Rightarrow p = \pm 2$. Mistakes of this kind were also seen in **Questions 2(ii)**, **7**, **9(i)**.

Comments on specific questions

Question 1

This question was rather poorly attempted. Significant numbers of candidates made no attempt at it and some of the attempts that were made seem to have been the result of guesswork. It was expected that candidates would note that the curve has been translated in the positive y direction by 1 unit. Hence $a = 1$. Also that the difference between maximum and minimum values is 4 units compared with the standard sine function in which the difference is 2 units. Hence $b = 2$.

Answers: $a = 1$, $b = 2$.

Question 2

In part **(i),** candidates experienced difficulty in dealing with the $4x^2$ term and with the required form of the answer. A significant number of candidates kept 4 outside the bracket. Very few candidates thought of putting the two expressions identically equal to each other and equating coefficients, which is an alternative way of dealing with such questions. There were two ways of approaching part **(ii)**. Candidates could have used their answer to part **(i)**, taking their value of b to the other side of the equation to obtain $(2x - 3)^2 > 16$. Taking the square root of both sides gives $2x - 3 > 4$ or $2x - 3 < -4$, from which the solutions come very easily. Alternatively, candidates could have made the given inequality into a quadratic equation, solved the equation giving two critical values and deciding whether to take the region between the two critical values or the region outside the critical values. Candidates who chose the first method often made a mistake when taking the square root and wrote $2x - 3 > \pm 4$ giving one correct solution and one incorrect solution. Candidates who employed the other method often obtained the correct critical values but did not always manage to proceed correctly to obtain the correct solution to the inequality.

Answers: **(i)** $(2x - 3)^2 - 9$; **(ii)** $x < -\frac{1}{2}$, $x > 3\frac{1}{2}$.

Question 3

Although most candidates showed some understanding of the binomial expansion, relatively few reached the answer without making at least one error. The majority of candidates wrote down all of the terms of the expansion and amongst these terms, the term that could simplify to a term independent of x could usually be found. This term was ${}^8C_6(4x^3)^2(2x)^{-6}$. However, 4 was often not raised to the power of 2, and 2 was often not raised to the power –6. In fact, 2 was sometimes raised to the power +6. The outcome was that this was a low-scoring question.

Answer: 7.

Question 4

Where candidates were able to differentiate correctly full marks were often scored. Some ca attempted to treat the function as a fraction and use the formula for differentiating a fraction. While th valid method it is much easier to write the function as $4(3x + 1)^{-2}$ and use the chain rule. It is most impor that candidates are confident about using the chain rule and that they can do so accurately.

Answer: $y = 3x + 4$.

Question 5

In part **(i),** most candidates were able to write down a correct expression for at least one of S_{100} or S_{200}. However, candidates seemed to find difficulty in writing down a correct equation linking the two expressions. Often the 4 was on the wrong side of the equation. Even more difficulty was experienced in simplifying the equation and finally solving it. Many candidates did not spot that they could divide throughout by 100 at an early stage and got embroiled in an equation involving large numbers, increasing the likelihood of arithmetic errors. In part **(ii)**, most candidates recognised the need to substitute their answer for d into the expression $a + 99d$.

Answers: **(i)** $d = 2a$; **(ii)** $199a$.

Question 6

In part **(i)**, most candidates were able to find the area of the sector, but often made errors in finding the area of the triangle. In part **(ii)**, the length of the arc DE was usually found correctly, but once again it was trigonometry which defeated many who were unable to obtain a correct expression for AC. Some candidates also employed an incorrect strategy for finding the perimeter of the shaded region, thinking that this was equal to the perimeter of the triangle – the perimeter of the sector. Those candidates who had successfully obtained all the correct terms for the perimeter often did not notice that the resulting expression could be simplified as it included a +2 term and a –2 term.

Answers: **(i)** $8\tan\alpha - 2\alpha$; **(ii)** $4\tan\alpha + 2\alpha + \dfrac{4}{\cos\alpha}$.

Question 7

The majority of candidates were able to obtain two correct equations and to realise that they needed to substitute from the linear equation into the quadratic equation. Faulty algebra was often to blame for failure to reach the correct quadratic equation in a single variable. Very few candidates spotted that they could, for example, substitute for $2 - b$ instead of the more usual b and this led very simply to $5(a - 3)^2 = 125$ and then to $a - 3 = \pm 5$. Nevertheless, there was a pleasing number of correct solutions.

Answer: $a = -2$ or 8, $b = 12$ or -8.

Question 8

In part **(i)**, candidates were generally familiar with the conditions needed for vectors to be perpendicular and were able to set up a quadratic equation in p^2. Many candidates solved the equation for p^2 correctly, discarding the solution $p^2 = -1$ and obtaining $p^2 = 4$. Unfortunately many candidates then gave the single solution $p = 2$. In part **(ii)**, most candidates found the vector **BA** (or in a significant number of cases, **AB**) but were then unable to find the unit vector in the direction of **BA**.

Answers: **(i)** $p = \pm 2$; **(ii)** $\dfrac{1}{13}\begin{pmatrix} 12 \\ 5 \\ 0 \end{pmatrix}$.

Question 9

Part **(i)** was not very well done. Because this is a 'proof', candidates are expected to show cle intermediate steps. The standard method involves putting both terms over a common denominator, the identity $\cos^2\theta + \sin^2\theta = 1$, factorising the numerator and cancelling $(1-\cos\theta)$, and finally using t identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. Algebraic errors were often responsible for disappointing outcomes for this part. In part **(ii)**, the majority of candidates reached $\tan^2\theta = \dfrac{1}{4}$ but many then wrote '$\tan\theta = \dfrac{1}{2}$', omitting the negative root and therefore missing the second solution for *θ*.

Answer: **(ii)** 26.6°, 153.4°.

Question 10

Part **(i)** was not really understood by many candidates. Many did not appreciate that the combined range of the two functions was required, whilst yet more candidates combined the two domains. In part **(ii)**, it would have been of great assistance to candidates if they had drawn the line $y = x$ (a line joining the two end-points of the curve and the origin) before reflecting the given graph in this line. Very few candidates drew the line and hence the graph of $y = \mathrm{f}^{-1}(x)$ was usually incorrectly drawn. In part **(iii)**, reasonable attempts were made at the two inverse expressions, but their domains were often missing or incorrect.

Answers: **(i)** $-5 \le \mathrm{f}(x) \le 4$; **(iii)** LINE: $\mathrm{f}^{-1}(x) = \dfrac{1}{3}(x+2)$ for $-5 \le x \le 1$,

CURVE: $\mathrm{f}^{-1}(x) = 5 - \dfrac{4}{x}$ for $1 < x \le 4$.

Question 11

Many candidates were unsure how to proceed with part **(i)**. The expected method (but not the only one) is to eliminate *y* between the two equations to obtain a quadratic equation in *x* and then to apply the condition for equal roots ($b^2 = 4ac$). While the marks scored for part **(i)** were disappointingly low, part **(ii)** was far more successful. Most candidates found the points of intersection correctly and then found the area under the curve by integration and subtracted from this the area under the straight line.

Answers: **(i)** $c = 12$; **(ii)** $1\frac{1}{3}$.

Question 12

In part **(i)**, most candidates attempted to integrate although there were errors seen, particularly with attempts to simplify division by the fractions ⅓ and ½. Some candidates forgot the constant of integration and therefore made no further progress. Part **(ii)** was very well done with many candidates scoring full marks. In part **(iii)**, candidates generally understood the need to solve the equation d*y*/d*x* = 0, although there were some dubious methods used in finding a solution. The *y* coordinate of the stationary point was often incorrect but attempts to determine its nature usually met with success. Overall, candidates scored well on this question.

Answers: **(i)** $y = \dfrac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - \dfrac{2}{3}$; **(ii)** $\dfrac{1}{2}x^{-\frac{1}{2}} + \dfrac{1}{2}x^{-\frac{3}{2}}$; **(iii)** (1, –2) minimum.

MATHEMATICS

Paper 9280/21
Paper 21

Key Messages

Candidates are advised to read the questions carefully and make sure they are actually answering the question. Candidates should also ensure that they are working to the required level of accuracy.

General Comments

There were many scripts of a high standard submitted showing that candidates had a good understanding of the syllabus objectives and were able to apply techniques learned both appropriately and correctly.

Comments on Specific Questions

Question 1

Most candidates chose to square both sides of the inequality and solve the resulting quadratic equation, obtaining the correct critical values in most cases. Many candidates, however, were unable to determine the correct range of values required. It should be noted that it is good practice for discontinuous ranges to be given as 2 discrete expressions (see solution below) rather than one expression (e.g. $-\frac{1}{2} \geq x \geq 3$).

Solution: $x \leq -\frac{1}{2}$, $x \geq 3$

Question 2

(i) Most candidates were able to differentiate 3sin$x$ correctly. Problems arose when differentiating tan2x, with many candidates offering the response $\sec^2 x$, rather than the correct response of $2\sec^2 2x$. Unfortunately the former gave a fortuitous answer of 5. Use of the double angle formula for tan2x followed by differentiation of a quotient was another approach.

(ii) Different approaches were used with varying degrees of success. If candidates realised that the derivative of 6 with respect to x was zero, then use of the product or quotient rule was often successful. For those candidates who used the chain rule, there was often a missing exponential term of $2e^{2x}$.

Solution: **(i)** 5 **(ii)** -3

Question 3

(i) Most candidates adopted the process of algebraic long division, usually with great success. Most candidates that used this approach were able to obtain correct quotients and thus show that there was a remainder of 7. Similarly those candidates that chose to use a method involving an identity were in general, equally successful. Problems arose when candidates attempted to use synthetic division or the remainder theorem with substitutions of $x = \pm 2$. The synthetic division approach was acceptable, provided candidates realised that they needed to obtain a cubic expression after division by either $x - 2$ or $x + 2$, and then repeat the process with a further division, using the unused factor, to obtain the quotient and remainder. Use of the remainder theorem with $x = \pm 2$ yielded a remainder of 7 for both divisions, but this is insufficient and does not result in a quotient being obtained.

(ii) It was realised by most candidates that they were able to make use of the quotient o... **(i)** to solve the given quartic equation. Some lost a mark by forgetting to give the solu... in addition to those obtained by solving the quadratic quotient equation.

Solution: **(i)** $6x^2 - x - 2$ **(ii)** $\pm 2,\ -\frac{1}{2},\ \frac{2}{3}$.

Question 4

(i) The standard of curve sketching was generally poor. Most candidates have a good idea of what the graphs should look like, but present them badly. A sketch does not need to be done on graph paper. It is sufficient for it to appear within the body of the rest of the question. Axes are meant to be straight lines. It would be useful and good practice to mark in the coordinates of the points where the curves cross the coordinate axes. The question asked the candidates to show that there is exactly one real root. So a conclusion was expected to be seen stating that, as there is only one point of intersection between the two curves, there is only one real solution to the given equation.

(ii) Most candidates used the correct approach of considering the value of $3\ln x - 15 + x^3$ or equivalent when $x = 2$ and when $x = 2.5$. It is necessary to give actual numeric values, rather than just state that the result is either positive or negative. Again, many candidates were not giving a conclusion. A statement was expected to be seen to the effect that, as there was a change of sign, the root was between the two given values.

(iii) As usual, most candidates were able to obtain full marks for this part of the question, showing a good understanding of the process of iteration. It was surprising to note that there are still candidates who do not make full use of their calculators and the 'Answer' function to make quick work of the iteration process. Ensuring that the final answer is given to the required level of accuracy is also of importance.

Solution: **(iii)** 2.319

Question 5

(i) Provided candidates were able to express the left hand side of the expression as a single fractions, most were able to get full marks. There were also many correct solutions starting with the right hand side of the expression with candidates making use of $1 = \sin^2\theta + \cos^2\theta$.

(ii) It was realised by most candidates that they needed to use the result from part **(i)** and most were able to write the given expressions correctly using part **(i)**. Some candidates are still unaware of the implications of the word 'Hence' and the phrase 'exact value'.

Solution: **(ii) (a)** $2\sqrt{2}$ **(b)** 3

Question 6

(a) There was a pleasing number of correct solutions for this part of the question. The fact that candidates were told to show that the integral was equal to ln125 prompted most to use a form of ln(2*x* – 7). Limits were usually substituted in correctly and the laws governing the use of logarithms were usually correct with the exception of the all too frequent incorrect statement $3\ln 125 - 3\ln 5 = 3\frac{\ln 125}{\ln 5}$.

(b) Many completely correct solutions were seen showing a good understanding of the trapezium rule. There were occasional errors in the *x*-coordinates used, leading on to incorrect *y*-values being used. Candidates should be reminded to make sure they are giving their answers to the correct level of accuracy.

Solution: **(b)** 13.5

Question 7

(i) Most candidates realised that they had to differentiate implicitly with respect to *x*. An usually occurred when trying to deal with the product 3*xy* or 3. Correct numerical gradients usually obtained, together with a correct equation of the tangent. Many candidates did not g their answer in the required form, however, leaving their equation with fractional coefficients.

(ii) Very few correct solutions to this part of the question were seen. Most candidates were able to reach the conclusion that $4x+3y=0$, but were unsure of what to do with this result. Very few candidates realised that they needed to use this result together with the original equation and thus obtain a result of either $-\frac{1}{8}y^2=3$ or $-\frac{2}{9}x^2=3$, neither of which has any real solutions. Again a conclusion to this effect was expected.

Solution: **(i)** $5x+4y-6=0$

MATHEMATICS

Paper 9280/41
Paper 41

General Comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen. The presentation of the work was good in most cases.

Some candidates lost marks due to not giving answers to 3 significant figures as requested and also due to prematurely approximating within their calculations leading to the final answer, particularly in **Question 4**. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

One of the rubrics on this paper is to take g = 10 and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct answer unless this value is used.

Comments on Specific Questions

Question 1

Most candidates performed very well on this question. They applied the relationship $P = Fv$ and used the fact that the driving force was equal to the resistive force in this case. A large number of well presented correct answers was seen.

Answer: $V = 47.5$

Question 2

(i) Many candidates solved this problem correctly by resolving forces perpendicular to the plane and equating this to the given normal reaction. An error that was occasionally seen was to use sin α instead of cos α when resolving forces.

Answer: $\alpha = 16.3$

(ii) Most candidates realised that this part of the question involved the application of $F = \mu R$. The majority of candidates found R, substituted into the equation $F = \mu R$ and solved for μ. Others realised that the value of μ was simply the tangent of the angle found in part **(i)**.

Answer: $\mu = \frac{7}{24}$ or 0.292

Question 3

This question was well done with most candidates resolving the given forces horizontally and vertically. There were some errors in signs and also sin 30 and cos 30 or sin 60 and cos 60 were sometimes mixed up when resolving. Often very little detail of the calculations involving the components of the forces was shown, with candidates simply quoting a value for the net horizontal and vertical forces. This did not matter provided the forces were found correctly, but for those who made errors in their calculations it could lead to a loss of method marks. It is an illustration of why candidates should be advised to show all of their working. The first 4 marks were often earned and although the resultant force was found by most candidates, a large number did not find the direction of the resultant which lost the final two marks.

Answer: Resultant = 1.23 N, Direction = 152.9° anticlockwise from the positive x-axis (or equivalent)

Question 4

Most candidates attempted this question by finding the height travelled by the particle until it came t and this was generally found correctly. It was then possible without further calculation to write down the t distance travelled although a few candidates not only doubled the height reached but also the given heigh above ground. Overall this question was well done by most candidates. A variety of methods was used to find the total time. The most obvious was to determine the time taken to reach the highest point and then to determine the time taken to reach the ground and add these times together. A few candidates realised that, with the upward direction taken as positive, the net displacement of the particle was -3.15 and by using the constant acceleration equation $s = ut + ½ at^2$ the total time taken could be evaluated in one step.

Answer: Total distance travelled = 11.25 m. Total time taken = 2.1 s

Question 5

(i) This question was not generally well done by candidates. Many determined the total potential energy at the top of the incline when in fact the question referred to finding kinetic and potential energies at a distance x along the incline. This meant that very few candidates achieved the correct value of k. Although it was possible to answer the question using Newton's second law, the question used the word "hence" which was directing candidates towards using the work/energy method.

Answer: $k = 5.5$

(ii) There were similar problems in this part of the question and a common error was that as x was given as the distance travelled then the distance of the particle from A as it moved along AB was not x but $x – 1760$. It was possible to solve this problem by using Newton's second law or by using energy principles as no method was referred to in this part.

Answer: Given

Question 6

(i) Most candidates made good attempts at this part of the question with the majority writing down Newton's second law for each particle and using it to determine the acceleration. Once candidates had found the acceleration, most simply used the constant acceleration equations to determine correctly the distance travelled.

Answer: Acceleration = 5 ms^{-2}. Distance travelled = 0.9 m

(ii) V was found correctly by most candidates but the value of T was often wrongly given as 0.3 as candidates forgot that they had to add on the extra 0.6 seen on the graph in the question paper.

Answer: $V = 3$, $T = 0.9$

(iii) Only a few candidates managed to score full marks on this part. Often the distance travelled upwards was found correctly but errors were made when considering the downward motion.

Answer: Distance travelled upwards = 1.35 m, distance travelled downwards = 2.45 m, $h = 2.45 – 1.35 = 1.1$

Question 7

(i) This part was well done by almost all candidates. Valid confirmation that the distance travelled was 1600 m was completed by most and the speed of the cyclist P was also found correctly by almost all candidates.

Answer: Speed of P = 5 ms^{-1}

(ii) This part was not generally well done. Candidates often continued to assume that … also travelling with constant acceleration although the form of velocity given showed th… not the case. Only a few candidates correctly integrated the given velocity expression and … this to the distance travelled. Most were able to show where the maximum speed occ… although the wrong value of *k* had already been found.

Answer: $k = \frac{4}{3}$, Maximum speed of Q = $\frac{16}{3}$ ms^{-1} = 5.33 ms^{-1}

(iii) This part was well done by most candidates with the time taken found correctly by the majority of candidates. Most candidates then used the value of the time taken to travel from *B* to *C* along with the given information to attempt to find the acceleration *a*, although in many cases the initial velocity had not been found correctly. However, candidates were still able to earn the method marks.

Answer: Time taken to travel from *B* to *C* = 280 s, $a = \frac{11}{420}$ = 0.0262

MATHEMATICS

Paper 9280/61
Paper 61

General comments

A considerable number of candidates found the paper too challenging. Many were unaware of both the Binomial and Normal Distributions and lacked the basic concepts of Statistics. There was no evidence of candidates having insufficient time. Many scripts lost marks through careless and untidy presentation; examples included reading off 0.516 instead of 0.5106 in tables, misreading their own writing, not using a ruler to draw lines and not working to at least 4 significant figures prior to an answer corrected to 3 significant figures.

Comments on specific questions

Question 1

Those candidates who were familiar with the Normal Distribution were comfortable with this standard type of question. It was pleasing to find very few instances of confusion between standard deviation/variance and the non-use of a continuity correction. The common errors were in the reading off of z = –0.5106 for the probability of 0.6952 and in the establishment of the required probability. The latter is made easier with the help of a simple diagram.

Answer: 0.537

Question 2

Again, the comments for the previous question are valid here. A diagram would clearly demonstrate the need to obtain the z-values corresponding to 0.96 and 0.68 as +1.751 and +0.468 respectively. The many good attempts at solving the simultaneous equations were all too often marred by substituting the first value obtained, corrected to 3 significant figures, and thus producing an error in the second value. It cannot be stressed enough that working should be at least to 4 significant figures if answers are required to 3 significant figures.

Answers: $\mu = 7.91$ and $\sigma = 2.34$

Question 3

Many candidates were unfamiliar with the Binomial Distribution. The response from those who were familiar with it revealed inconsistencies and errors.

(i) There was confusion, with the requirements for approximating the Binomial Distribution to the Normal Distribution often being cited. There were many lengthy presentations but all that was required were 3 of the answers stated below.

(ii) The most common errors were interpreting ‘probability at least 3’ as ‘probability of 3 exactly’, adding the value of P(3), omitting the value P(0) and truncating the values of the 3 constituent probabilities before adding.

Answers: **(i)** Constant / given probability,
Trials are independent,
Fixed / given number of trials,
Only two outcomes.

(ii) 0.520

Question 4

(i) The given answer of 3/14 was arrived at correctly in several ways with no particular prefe using combinations, multiplying a series of fractions or a tree diagram. Many of the latter w spoilt by untidy presentation and careless enumerating of probabilities. When an answer is given, the solution must be fully correct with all the explanation detailed. It is not sufficient to just simply to add 'x 6' without either $^4C_2$ being quoted, the 6 relevant branches being identified on the tree diagram, or the 6 possible outcomes stated.

(ii) A large minority of candidates did not realise that there were only 3 possible values of X. A few used $E(X) = 3$ to realise symmetry and thus easily found the probabilities.

(iii) Despite the reminder in the question that $E(X) = 3$, a surprising number of scripts omitted $\{E(X)\}^2$ in the calculation.

Answers: **(i)** 3/14 **AG**

(ii)

x	2	3	4
Probability	3/14	8/14	3/14

(iii) 3/7 (0.429)

Question 5

It was pleasing to see the many correct responses to this question and that conditional probability is becoming increasingly understood. The tree diagram approach was popular, but again some responses were marred by untidy presentation and careless enumerations.

(i) The final mark was often lost by converting fractions into decimals and approximating prematurely, e.g. 1/9 as 0.11 instead of at least 0.1111

(ii) A common error was in simplifying the denominator of $\frac{1}{4}\times\frac{1}{12}+\frac{1}{2}\times\frac{4}{16}$

Those who resorted to a calculator invariably obtained the wrong answer.

Answers: **(i)** 53/288 (0.184) **(ii)** 1/7 (0.143)

Question 6

Parts **(i)** and **(ii)** were well understood by candidates but very few grasped that in the remainder of the question the constraints placed on N and A reduced the choices available.

(i) The presence of the two 'N's was often overlooked and consequently division by 2! was not seen.

(ii) Again the two 'N's and the three 'A's were overlooked and 4! × 4! was given as the final answer.

(iii) Only a handful of candidates recognised the importance of 'exactly 1 N and 1 A' meant that there were only 3 letters remaining to fill 2 places. Hence $^3C_2$ was the required number of ways.

(iv) Similarly, 'exactly 1 N' required the 4 separate cases of 0, 1, 2 or 3 'A's to be considered, i.e. filling 3, 2, 1 or 0 places with the 3 letters (T, Z and I).

Answers: **(i)** 360 **(ii)** 48 **(iii)** 3 **(iv)** 8

Question 7

Surprisingly, the majority of candidates scored less than half marks on this basic question on groupe

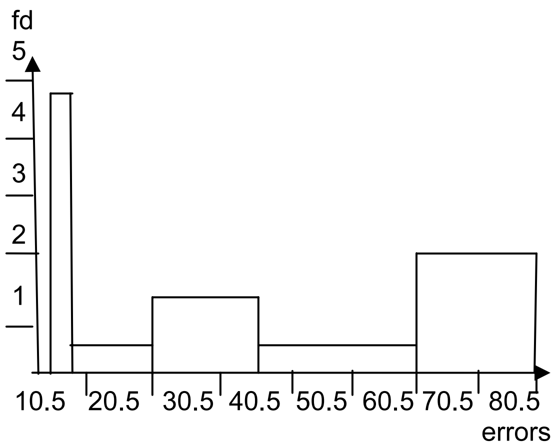
(i) Very few Centres were familiar with 'frequency density' and most candidates produced a freque histogram or a cumulative frequency curve instead. The class boundaries of 0.5, 5.5, 20.5, etc were often not realised and marks were lost on the graph through untidy presentation. A line across the top of a rectangle indicating the height (value) gets no credit if it is not horizontal or several mm thick.

(ii) A very large minority of candidates did not use the 'mid class values', preferring to use wrongly the class widths or various class boundaries instead.

(iii) Frequently 27.75 and 83.25 (or 28 and 84) were given as the quartile values and the question of which class was ignored.

Answers:

<u>**(i)**</u> **(ii)** 40.2



(iii) (6-20) Class, (61-80) Class, 41.