Past Year: Chapter 7 Series

May/June 2002

4 A progression has a first term of 12 and a fifth term of 18.

(i) Find the sum of the first 25 terms if the progression is arithmetic. [3]

(ii) Find the 13th term if the progression is geometric. [4]

Nov/Dec 2002

1 Find the value of the term which is independent of x in the expansion of $\left(x+\frac{3}{x}\right)^4$. [3]

2 A geometric progression, for which the common ratio is positive, has a second term of 18 and a fourth term of 8. Find

(i) the first term and the common ratio of the progression, [3]

(ii) the sum to infinity of the progression. [2]

May/June 03

1 Find the value of the coefficient of $\frac{1}{x}$ in the expansion of $\left(2x-\frac{1}{x}\right)^5$. [3]

4 In an arithmetic progression, the 1st term is -10, the 15th term is 11 and the last term is 41. Find the sum of all the terms in the progression. [5]

May/June 2004

1 A geometric progression has first term 64 and sum to infinity 256. Find

(i) the common ratio, [2]

(ii) the sum of the first ten terms. [2]

4 Find the coefficient of x^3 in the expansion of

(i) $(1+2x)^6$, [3]

(ii) $(1-3x)(1+2x)^6$. [3]

May/June 2005

4 **(i)** Find the first 3 terms in the expansion of $(2-x)^6$ in ascending powers of x. [3]

(ii) Find the value of k for which there is no term in x^2 in the expansion of $(1+kx)(2-x)^6$. [2]

6 A geometric progression has 6 terms. The first term is 192 and the common ratio is 1.5. An arithmetic progression has 21 terms and common difference 1.5. Given that the sum of all the terms in the geometric progression is equal to the sum of all the terms in the arithmetic progression, find the first term and the last term of the arithmetic progression. [6]

May/June 2006

3 Each year a company gives a grant to a charity. The amount given each year increases by 5% of its value in the preceding year. The grant in 2001 was \$5000. Find

(i) the grant given in 2011, [3]

(ii) the total amount of money given to the charity during the years 2001 to 2011 inclusive. [2]

4 The first three terms in the expansion of $(2 + ax)^n$, in ascending powers of x, are $32 - 40x + bx^2$. Find the values of the constants n, a and b. [5]

May/June 2007

7 The second term of a geometric progression is 3 and the sum to infinity is 12.

(i) Find the first term of the progression. [4]

An arithmetic progression has the same first and second terms as the geometric progression.

(ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

May/June 2008

7 The first term of a geometric progression is 81 and the fourth term is 24. Find

(i) the common ratio of the progression, [2]

(ii) the sum to infinity of the progression. [2]

The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression.

(iii) Find the sum of the first ten terms of the arithmetic progression. [3]

May/June 2009

3 **(i)** Find the first 3 terms in the expansion of $(2 + 3x)^5$ in ascending powers of x. [3]

(ii) Hence find the value of the constant a for which there is no term in x^2 in the expansion of $(1 + ax)(2 + 3x)^5$. [2]

7 **(a)** Find the sum to infinity of the geometric progression with first three terms 0.5, 0.5^3 and 0.5^5. [3]

(b) The first two terms in an arithmetic progression are 5 and 9. The last term in the progression is the only term which is greater than 200. Find the sum of all the terms in the progression. [4]

Oct/Nov 2001

8 A precious metal is extracted from a mine. In the first year of operation, 2000 kg of the metal was extracted. In each succeeding year, the amount extracted was 90% of the previous year's amount. Find

(i) the amount of metal extracted in the 10th year of operation, [2]

(ii) the total amount of metal extracted in the first 20 years of operation, [2]

(iii) the total amount of metal that would be extracted over a very long period of time. [3]

Oct/Nov 2002

1 Find the value of the term which is independent of x in the expansion of $\left(x + \frac{3}{x}\right)^4$. [3]

2 A geometric progression, for which the common ratio is positive, has a second term of 18 and a fourth term of 8. Find

(i) the first term and the common ratio of the progression, [3]

(ii) the sum to infinity of the progression. [2]

Oct/Nov 2003

3 **(a)** A debt of \$3726 is repaid by weekly payments which are in arithmetic progression. The first payment is \$60 and the debt is fully repaid after 48 weeks. Find the third payment. [3]

(b) Find the sum to infinity of the geometric progression whose first term is 6 and whose second term is 4. [3]

Oct/Nov 2004

1 Find the coefficient of x in the expansion of $\left(3x - \frac{2}{x}\right)^5$. [4]

2 Find

(i) the sum of the first ten terms of the geometric progression 81, 54, 36, ... , [3]

(ii) the sum of all the terms in the arithmetic progression 180, 175, 170, ... , 25. [3]

Oct/Nov 2005

6 A small trading company made a profit of \$250 000 in the year 2000. The company considered two different plans, plan A and plan B, for increasing its profits.

Under plan A, the annual profit would increase each year by 5% of its value in the preceding year. Find, for plan A,

(i) the profit for the year 2008, [3]

(ii) the total profit for the 10 years 2000 to 2009 inclusive. [2]

Under plan B, the annual profit would increase each year by a constant amount \$$D$.

(iii) Find the value of D for which the total profit for the 10 years 2000 to 2009 inclusive would be the same for both plans. [3]

Oct/Nov 2006

1 Find the coefficient of x^2 in the expansion of $\left(x + \frac{2}{x}\right)^6$. [3]

6 **(a)** Find the sum of all the integers between 100 and 400 that are divisible by 7. [4]

(b) The first three terms in a geometric progression are 144, x and 64 respectively, where x is positive. Find

(i) the value of x,

(ii) the sum to infinity of the progression.

[5]

Oct/Nov 2007

3 **(i)** Find the first three terms in the expansion of $(2 + u)^5$ in ascending powers of u. [3]

(ii) Use the substitution $u = x + x^2$ in your answer to part **(i)** to find the coefficient of x^2 in the expansion of $\left(2 + x + x^2\right)^5$. [2]

4 The 1st term of an arithmetic progression is a and the common difference is d, where $d \neq 0$.

(i) Write down expressions, in terms of a and d, for the 5th term and the 15th term. [1]

The 1st term, the 5th term and the 15th term of the arithmetic progression are the first three terms of a geometric progression.

(ii) Show that $3a = 8d$. [3]

(iii) Find the common ratio of the geometric progression. [2]

Oct/Nov 2008

1 Find the value of the coefficient of x^2 in the expansion of $\left(\frac{x}{2}+\frac{2}{x}\right)^6$. [3]

3 The first term of an arithmetic progression is 6 and the fifth term is 12. The progression has n terms and the sum of all the terms is 90. Find the value of n. [4]

Oct/Nov 2009/11

3 **(i)** Find the first 3 terms in the expansion of $(2-x)^6$ in ascending powers of x. [3]

(ii) Given that the coefficient of x^2 in the expansion of $(1+2x+ax^2)(2-x)^6$ is 48, find the value of the constant a. [3]

8 The first term of an arithmetic progression is 8 and the common difference is d, where $d \neq 0$. The first term, the fifth term and the eighth term of this arithmetic progression are the first term, the second term and the third term, respectively, of a geometric progression whose common ratio is r.

(i) Write down two equations connecting d and r. Hence show that $r = \frac{3}{4}$ and find the value of d. [6]

(ii) Find the sum to infinity of the geometric progression. [2]

(iii) Find the sum of the first 8 terms of the arithmetic progression. [2]

Oct/Nov 2009/12

2 **(i)** Find, in terms of the non-zero constant k, the first 4 terms in the expansion of $(k+x)^8$ in ascending powers of x. [3]

(ii) Given that the coefficients of x^2 and x^3 in this expansion are equal, find the value of k. [2]

3 A progression has a second term of 96 and a fourth term of 54. Find the first term of the progression in each of the following cases:

(i) the progression is arithmetic, [3]

(ii) the progression is geometric with a positive common ratio. [3]

May/June 2010/11

2 **(i)** Find the first 3 terms in the expansion of $\left(2x-\frac{3}{x}\right)^5$ in descending powers of x. [3]

(ii) Hence find the coefficient of x in the expansion of $\left(1+\frac{2}{x^2}\right)\left(2x-\frac{3}{x}\right)^5$. [2]

3 The ninth term of an arithmetic progression is 22 and the sum of the first 4 terms is 49.

(i) Find the first term of the progression and the common difference. [4]

The nth term of the progression is 46.

(ii) Find the value of n. [2]

May/June 2010/12

6 **(i)** Find the first 3 terms in the expansion of $(1+ax)^5$ in ascending powers of x. [2]

(ii) Given that there is no term in x in the expansion of $(1-2x)(1+ax)^5$, find the value of the constant a. [2]

(iii) For this value of a, find the coefficient of x^2 in the expansion of $(1-2x)(1+ax)^5$. [3]

7 **(a)** Find the sum of all the multiples of 5 between 100 and 300 inclusive. [3]

(b) A geometric progression has a common ratio of $-\frac{2}{3}$ and the sum of the first 3 terms is 35. Find

(i) the first term of the progression, [3]

(ii) the sum to infinity. [2]

May/June 2010/13

1 The first term of a geometric progression is 12 and the second term is −6. Find

(i) the tenth term of the progression, [3]

(ii) the sum to infinity. [2]

2 **(i)** Find the first three terms, in descending powers of x, in the expansion of $\left(x - \frac{2}{x}\right)^6$. [3]

(ii) Find the coefficient of x^4 in the expansion of $(1 + x^2)\left(x - \frac{2}{x}\right)^6$. [2]

Oct/Nov 2010/11

2 In the expansion of $(1 + ax)^6$, where a is a constant, the coefficient of x is −30. Find the coefficient of x^3. [4]

6 **(a)** The fifth term of an arithmetic progression is 18 and the sum of the first 5 terms is 75. Find the first term and the common difference. [4]

(b) The first term of a geometric progression is 16 and the fourth term is $\frac{27}{4}$. Find the sum to infinity of the progression. [3]

Oct/Nov 2010/12

1 **(i)** Find the first 3 terms in the expansion, in ascending powers of x, of $(1 - 2x^2)^8$. [2]

(ii) Find the coefficient of x^4 in the expansion of $(2 - x^2)(1 - 2x^2)^8$. [2]

5 **(a)** The first and second terms of an arithmetic progression are 161 and 154 respectively. The sum of the first m terms is zero. Find the value of m. [3]

(b) A geometric progression, in which all the terms are positive, has common ratio r. The sum of the first n terms is less than 90% of the sum to infinity. Show that $r^n > 0.1$. [3]

Oct/Nov 2010/13

1 Find the term independent of x in the expansion of $\left(x - \frac{1}{x^2}\right)^9$. [3]

9 **(a)** A geometric progression has first term 100 and sum to infinity 2000. Find the second term. [3]

(b) An arithmetic progression has third term 90 and fifth term 80.

(i) Find the first term and the common difference. [2]

(ii) Find the value of m given that the sum of the first m terms is equal to the sum of the first $(m + 1)$ terms. [2]

(iii) Find the value of n given that the sum of the first n terms is zero. [2]