Past Year: Chapter 8 Differentiation

May/June 2002

8 A hollow circular cylinder, open at one end, is constructed of thin sheet metal. The total external surface area of the cylinder is $192\pi\,\text{cm}^2$. The cylinder has a radius of r cm and a height of h cm.

(i) Express h in terms of r and show that the volume, $V\,\text{cm}^3$, of the cylinder is given by

$$V = \tfrac{1}{2}\pi(192r - r^3).$$ [4]

Given that r can vary,

(ii) find the value of r for which V has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

9 A curve is such that $\dfrac{dy}{dx} = \dfrac{12}{(2x+1)^2}$ and $P(1, 5)$ is a point on the curve.

(i) The normal to the curve at P crosses the x-axis at Q. Find the coordinates of Q. [4]

(ii) Find the equation of the curve. [4]

(iii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of increase of the y-coordinate when $x = 1$. [3]

Nov/Dec 2002

4 The gradient at any point (x, y) on a curve is $\sqrt{(1+2x)}$. The curve passes through the point $(4, 11)$. Find

(i) the equation of the curve, [4]

(ii) the point at which the curve intersects the y-axis. [2]

8 A curve has equation $y = x^3 + 3x^2 - 9x + k$, where k is a constant.

(i) Write down an expression for $\dfrac{dy}{dx}$. [2]

(ii) Find the x-coordinates of the two stationary points on the curve. [2]

(iii) Hence find the two values of k for which the curve has a stationary point on the x-axis. [3]

May/June 03

10 The equation of a curve is $y = \sqrt{(5x+4)}$.

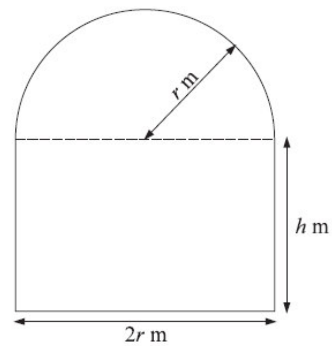
(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of x has the constant value 0.03 units per second. Find the rate of increase of y at the instant when $x = 1$. [2]

(iii) Find the area enclosed by the curve, the x-axis, the y-axis and the line $x = 1$. [5]

May/June 2004

8



The diagram shows a glass window consisting of a rectangle of height h m and width $2r$ m and a semicircle of radius r m. The perimeter of the window is 8 m.

(i) Express h in terms of r. [2]

(ii) Show that the area of the window, A m^2, is given by

$$A = 8r - 2r^2 - \tfrac{1}{2}\pi r^2.$$ [2]

Given that r can vary,

(iii) find the value of r for which A has a stationary value, [4]

(iv) determine whether this stationary value is a maximum or a minimum. [2]

May/June 2005

2 Find the gradient of the curve $y = \dfrac{12}{x^2 - 4x}$ at the point where $x = 3$. [4]

9 A curve has equation $y = \dfrac{4}{\sqrt{x}}$.

(i) The normal to the curve at the point (4, 2) meets the x-axis at P and the y-axis at Q. Find the length of PQ, correct to 3 significant figures. [6]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$. [4]

May/June 2006

9 A curve is such that $\dfrac{dy}{dx} = \dfrac{4}{\sqrt{(6 - 2x)}}$, and $P\,(1, 8)$ is a point on the curve.

(i) The normal to the curve at the point P meets the coordinate axes at Q and at R. Find the coordinates of the mid-point of QR. [5]

(ii) Find the equation of the curve. [4]

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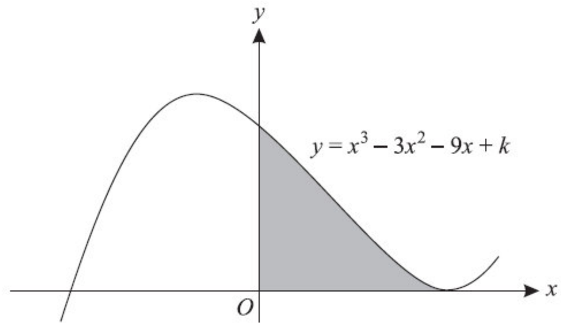


The diagram shows the curve $y = x^3 - 3x^2 - 9x + k$, where k is a constant. The curve has a minimum point on the x-axis.

(i) Find the value of k. [4]

(ii) Find the coordinates of the maximum point of the curve. [1]

(iii) State the set of values of x for which $x^3 - 3x^2 - 9x + k$ is a decreasing function of x. [1]

(iv) Find the area of the shaded region. [4]

May/June 2007

10 The equation of a curve is $y = 2x + \frac{8}{x^2}$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve and determine the nature of the stationary point. [3]

(iii) Show that the normal to the curve at the point $(-2, -2)$ intersects the x-axis at the point $(-10, 0)$. [3]

(iv) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$. [3]

May/June 2008

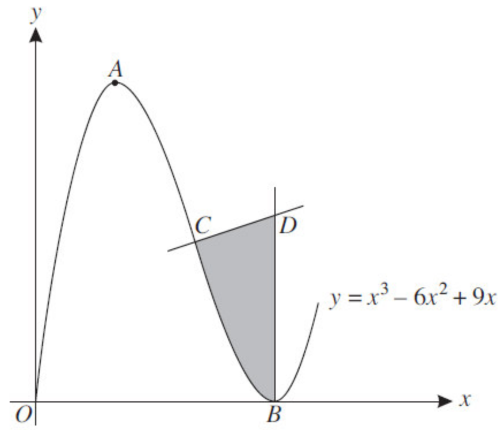
6 The function f is such that $f(x) = (3x + 2)^3 - 5$ for $x \geqslant 0$.

(i) Obtain an expression for $f'(x)$ and hence explain why f is an increasing function. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

May/June 2009

11



The diagram shows the curve $y = x^3 - 6x^2 + 9x$ for $x \geqslant 0$. The curve has a maximum point at A and a minimum point on the x-axis at B. The normal to the curve at C (2, 2) meets the normal to the curve at B at the point D.

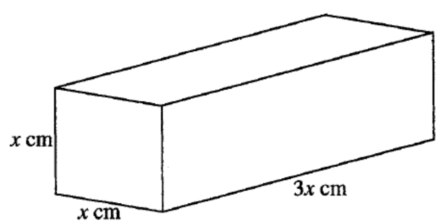
(i) Find the coordinates of A and B. [3]

(ii) Find the equation of the normal to the curve at C. [3]

(iii) Find the area of the shaded region. [5]

Oct/Nov 2001

5



The diagram shows a rectangular block of ice, x cm by x cm by $3x$ cm.

(i) Obtain an expression, in terms of x, for the total surface area, A cm^2, of the block and write down an expression for $\frac{dA}{dx}$. [3]

(ii) Given that the ice is melting in such a way that A is decreasing at a constant rate of 0.14 cm^2 s^{-1}, calculate the rate of decrease of x at the instant when $x = 2$. [3]

9 A curve is such that $\frac{dy}{dx} = \frac{24}{x^3} - 3$.

(i) Given that the curve passes through the point (1, 16), find the equation of the curve. [4]

(ii) Find the coordinates of the stationary point on the curve. [4]

Oct/Nov 2002

8 A curve has equation $y = x^3 + 3x^2 - 9x + k$, where k is a constant.

(i) Write down an expression for $\frac{dy}{dx}$. [2]

(ii) Find the x-coordinates of the two stationary points on the curve. [2]

(iii) Hence find the two values of k for which the curve has a stationary point on the x-axis. [3]

Oct/Nov 2003

8 A solid rectangular block has a base which measures $2x$ cm by x cm. The height of the block is y cm and the volume of the block is 72 cm^3.

(i) Express y in terms of x and show that the total surface area, $A\text{ cm}^2$, of the block is given by

$$A = 4x^2 + \frac{216}{x}.$$ [3]

Given that x can vary,

(ii) find the value of x for which A has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

Oct/Nov 2004

10 A curve has equation $y = x^2 + \frac{2}{x}$.

(i) Write down expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(iii) Find the volume of the solid formed when the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated completely about the x-axis. [6]

Oct/Nov 2005

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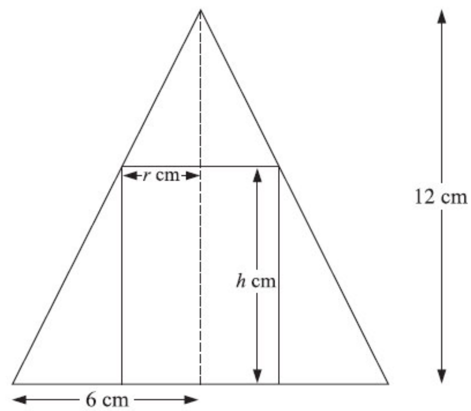

The diagram shows the cross-section of a hollow cone and a circular cylinder. The cone has radius 6 cm and height 12 cm, and the cylinder has radius r cm and height h cm. The cylinder just fits inside the cone with all of its upper edge touching the surface of the cone.

(i) Express h in terms of r and hence show that the volume, V cm^3, of the cylinder is given by

$$V = 12\pi r^2 - 2\pi r^3.$$ [3]

(ii) Given that r varies, find the stationary value of V. [4]

8 A function f is defined by $\mathrm{f} : x \mapsto (2x - 3)^3 - 8$, for $2 \leqslant x \leqslant 4$.

(i) Find an expression, in terms of x, for $\mathrm{f}'(x)$ and show that f is an increasing function. [4]

(ii) Find an expression, in terms of x, for $\mathrm{f}^{-1}(x)$ and find the domain of f^{-1}. [4]

Oct/Nov 2006

8 The equation of a curve is $y = \dfrac{6}{5 - 2x}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of y has a constant value of 0.02 units per second. Find the rate of increase of x when $x = 1$. [2]

(iii) The region between the curve, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through 360° about the x-axis. Show that the volume obtained is $\frac{12}{5}\pi$. [5]

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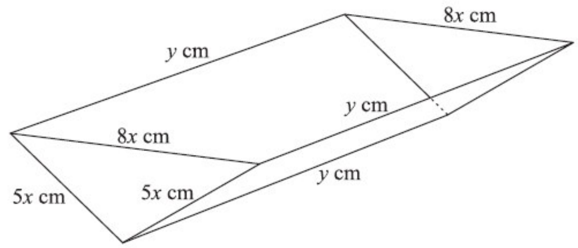

The diagram shows an open container constructed out of $200\,\text{cm}^2$ of cardboard. The two vertical end pieces are isosceles triangles with sides $5x$ cm, $5x$ cm and $8x$ cm, and the two side pieces are rectangles of length y cm and width $5x$ cm, as shown. The open top is a horizontal rectangle.

(i) Show that $y = \dfrac{200 - 24x^2}{10x}$. [3]

(ii) Show that the volume, $V\,\text{cm}^3$, of the container is given by $V = 240x - 28.8x^3$. [2]

Given that x can vary,

(iii) find the value of x for which V has a stationary value, [3]

(iv) determine whether it is a maximum or a minimum stationary value. [2]

Oct/Nov 2007

8 The equation of a curve is $y = (2x - 3)^3 - 6x$.

(i) Express $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ in terms of x. [3]

(ii) Find the x-coordinates of the two stationary points and determine the nature of each stationary point. [5]

Oct/Nov 2008

8 The equation of a curve is $y = 5 - \dfrac{8}{x}$.

(i) Show that the equation of the normal to the curve at the point $P\,(2, 1)$ is $2y + x = 4$. [4]

This normal meets the curve again at the point Q.

(ii) Find the coordinates of Q. [3]

(iii) Find the length of PQ. [2]

Oct/Nov 2009/11

4 The equation of a curve is $y = x^4 + 4x + 9$.

(i) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 1$. [3]

7 The equation of a curve is $y = \dfrac{12}{x^2 + 3}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

(ii) Find the equation of the normal to the curve at the point $P\,(1, 3)$. [3]

(iii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.012 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

Oct/Nov 2009/12

8 The function f is such that $f(x) = \dfrac{3}{2x+5}$ for $x \in \mathbb{R}$, $x \neq -2.5$.

(i) Obtain an expression for $f'(x)$ and explain why f is a decreasing function. [3]

(ii) Obtain an expression for $f^{-1}(x)$. [2]

(iii) A curve has the equation $y = f(x)$. Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 2$ is rotated through 360° about the x-axis. [4]

May/June 2010/11

6 A curve is such that $\dfrac{dy}{dx} = 3x^{\frac{1}{2}} - 6$ and the point (9, 2) lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the x-coordinate of the stationary point on the curve and determine the nature of the stationary point. [3]

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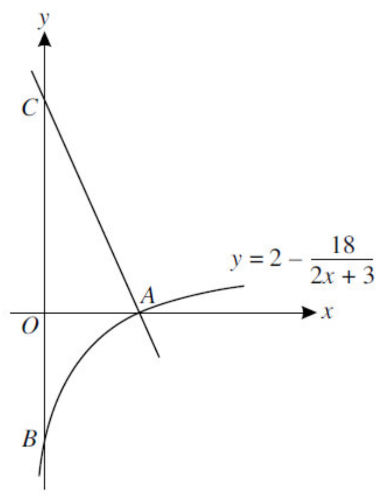

The diagram shows part of the curve $y = 2 - \dfrac{18}{2x+3}$, which crosses the x-axis at A and the y-axis at B. The normal to the curve at A crosses the y-axis at C.

(i) Show that the equation of the line AC is $9x + 4y = 27$. [6]

(ii) Find the length of BC. [2]

May/June 2010/12

8 A solid rectangular block has a square base of side x cm. The height of the block is h cm and the total surface area of the block is 96 cm^2.

(i) Express h in terms of x and show that the volume, V cm^3, of the block is given by

$$V = 24x - \tfrac{1}{2}x^3.$$ [3]

Given that x can vary,

(ii) find the stationary value of V, [3]

(iii) determine whether this stationary value is a maximum or a minimum. [2]

10 The equation of a curve is $y = \frac{1}{6}(2x-3)^3 - 4x$.

(i) Find $\frac{dy}{dx}$. [3]

(ii) Find the equation of the tangent to the curve at the point where the curve intersects the y-axis. [3]

(iii) Find the set of values of x for which $\frac{1}{6}(2x-3)^3 - 4x$ is an increasing function of x. [3]

May/June 2010/13

5 The equation of a curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{(3x-2)}}$. Given that the curve passes through the point $P(2, 11)$, find

(i) the equation of the normal to the curve at P, [3]

(ii) the equation of the curve. [4]

Oct/Nov 2010/11

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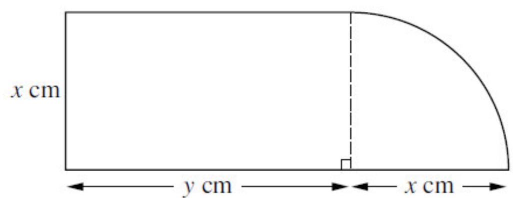

The diagram shows a metal plate consisting of a rectangle with sides x cm and y cm and a quarter-circle of radius x cm. The perimeter of the plate is 60 cm.

(i) Express y in terms of x. [2]

(ii) Show that the area of the plate, A cm^2, is given by $A = 30x - x^2$. [2]

Given that x can vary,

(iii) find the value of x at which A is stationary, [2]

(iv) find this stationary value of A, and determine whether it is a maximum or a minimum value. [2]

10 The equation of a curve is $y = 3 + 4x - x^2$.

(i) Show that the equation of the normal to the curve at the point (3, 6) is $2y = x + 9$. [4]

(ii) Given that the normal meets the coordinate axes at points A and B, find the coordinates of the mid-point of AB. [2]

(iii) Find the coordinates of the point at which the normal meets the curve again. [4]

11 The equation of a curve is $y = \frac{9}{2-x}$.

(i) Find an expression for $\frac{dy}{dx}$ and determine, with a reason, whether the curve has any stationary points. [3]

(ii) Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 1$ is rotated through 360° about the x-axis. [4]

(iii) Find the set of values of k for which the line $y = x + k$ intersects the curve at two distinct points. [4]

Oct/Nov 2010/12

3 The length, x metres, of a Green Anaconda snake which is t years old is given approximately by the formula

$$x = 0.7\sqrt{(2t - 1)},$$

where $1 \leqslant t \leqslant 10$. Using this formula, find

(i) $\frac{dx}{dt}$, [2]

(ii) the rate of growth of a Green Anaconda snake which is 5 years old. [2]

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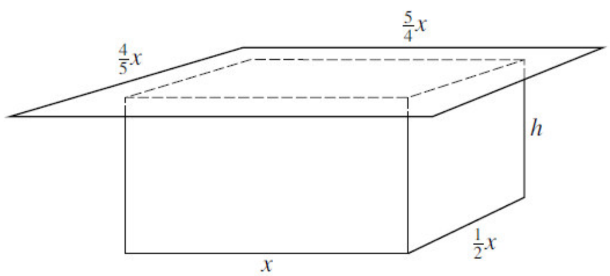

The diagram shows an open rectangular tank of height h metres covered with a lid. The base of the tank has sides of length x metres and $\frac{1}{2}x$ metres and the lid is a rectangle with sides of length $\frac{5}{4}x$ metres and $\frac{4}{5}x$ metres. When full the tank holds $4\,\text{m}^3$ of water. The material from which the tank is made is of negligible thickness. The external surface area of the tank together with the area of the top of the lid is $A\,\text{m}^2$.

(i) Express h in terms of x and hence show that $A = \frac{3}{2}x^2 + \frac{24}{x}$. [5]

(ii) Given that x can vary, find the value of x for which A is a minimum, showing clearly that A is a minimum and not a maximum. [5]

Oct/Nov 2010/13

5 A curve has equation $y = \frac{1}{x-3} + x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [2]

(ii) Find the coordinates of the maximum point A and the minimum point B on the curve. [5]

6 A curve has equation $y = f(x)$. It is given that $f'(x) = 3x^2 + 2x - 5$.

(i) Find the set of values of x for which f is an increasing function. [3]

(ii) Given that the curve passes through $(1, 3)$, find $f(x)$. [4]