Q1.

6	$\frac{dy}{dx} = 3\sqrt{x} - 6 \quad (9, 2)$		
	(i) $y - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 6x\ (+c)$	B2,1	Loses 1 for each error – ignore $+c$
	$(9, 2)\ \ 2 = 54 - 54 + c$ $\rightarrow c = 2.$	M1 A1 [4]	Uses (9, 2) with integration to find c. co.
	(ii) $\frac{dy}{dx} = 0 \rightarrow x = 4$	B1	Ignore any y-value
	$\frac{d^2y}{d^2x} - \frac{3x^{-\frac{1}{2}}}{2}$ $\rightarrow$ +ve (or ¾) Minimum	M1 A1 [3]	Any valid method. co.

Q2.

7	$y - 2 - \frac{18}{2x+3}$		
	(i) A is (3, 0)	B1	Anywhere – but not from given answer
	$\frac{dy}{dx} = 18(2x+3)^{-2} \times 2$	B1 B1	B1 for $18(2x+3)^{-2}$, B1 for $\times 2$
	If $x = 3$, $m = \frac{4}{9}$.		
	m of normal $= -\frac{9}{4}$	M1	Use of $m_1m_2 = -1$ with m from dy/dx
	Equation of normal $y - -\frac{9}{4}(x-3)$	M1	Correct method for normal
	$\rightarrow\ 4y + 9x - 27$	A1 [6]	co (answer was given)
	(ii) Normal meets y-axis at (0, 6¾) Curve meets y-axis at (0, −4) $\rightarrow BC = 10¾$	M1 A1 [2]	Needs to put $x = 0$ in both normal and curve. co

Q3.

2	$\left(\frac{dv}{dr} -\right) 4\pi r^2$	M1	
	$-4\pi \times 10^2$	A1	SOI at any point
	$\frac{dr}{dt} - \frac{\frac{dv}{dt}}{\frac{dv}{dr}}$ **OE** used	M1	Correct link between differentials with $\frac{dr}{dt}$ finally as subject
	$\frac{50}{4\pi \times 10^2} - \frac{1}{8\pi}$ or 0.0398	A1 [4]	Allow $\frac{50}{400\pi}$. Non-calculus methods $\frac{0}{4}$

Q4.

6 (i) $z-3x+2\left(\frac{600}{x}\right)$ or $x\frac{(z-3x)}{2}-600$ OE $\rightarrow$ **AG**	B1 [1]	
(ii) $\frac{dz}{dx}-3-\frac{1200}{x^2}$ or $\frac{dz}{dy}-2-\frac{1800}{y^2}$	B1	
$=0\rightarrow x=20$ or $=0\rightarrow y=30$	M1A1	Set to 0 & attempt to solve. Allow ±20
$z-60+\frac{120}{20}-120$	A1√	Ft from *their* x provided positive Or other valid method
$\frac{d^2z}{dx^2}-\frac{2400}{x^3}$	B1√	Dep. on $\frac{d^2z}{dx^2}-\frac{k}{x^3}$ $(k>0)$ or other
$>0\Rightarrow$ minimum	B1 [6]	valid method.

Q5.

4 (a) $y-\frac{2x^3+5}{x}-2x^2+\frac{5}{x}$	M1	Knows to divide numerator by x
$d/dx=4x-\frac{5}{x^2}$ or $4x-5x^{-2}$	A1 + A1 [3]	co
(b) $\int(3x-2)^5dx-\frac{(3x-2)^6}{6}\div 3\ (+c)$	B1 B1	B1 without "÷ 3". B1 for "÷ 3". (ignore $(+c)$)
$\int_0^1(3x-2)^5dx-\left[\frac{(3x-2)^6}{18}\right]$	M1	Uses limits after integration.
Limits used correctly $\rightarrow -3½$	A1 [4]	co

Q6.

4 $1000k=3.2\Rightarrow k-\frac{3.2}{1000}$ or $\frac{2}{625}$ or 0.0032 oe	**M1A1**	
$\left(\frac{dM}{dr}\right)-3kr^2$	**B1**	
$\frac{dM}{dt}-\frac{dM}{dr}\times\frac{dr}{dt}$ **used** e.g. $3\times k\times 10^2\times 0.1$	**M1**	Must eventually make dM/dt subject
0.096	**A1** **[5]**	cao. Non-calculus methods (e.g. $\rightarrow$ 0.09696) can score only 1st 2 marks

Q7.

10 (i) $3x^2-4x+1(<)5$ $(3x+2)(x-2)<0$ $-\frac{2}{3}<x<2$ or $\left[-\frac{2}{3},2\right]$ or $\left(-\frac{2}{3},2\right)$. Allow ‹	**M1** **M1** **A2** **[4]**	Attempt differentiate & put 5 on RHS Attempt to factorise or solve SC Allow A1 for $-\frac{2}{3}$ and 2 seen
(ii) $3x^2-4x+1-0\Rightarrow(3x-1)(x-1)-0$	**M1**	Derivative = 0 & **any** attempt to solve
$x-\frac{1}{3}$ or 1	**A1**	Both
$y-\frac{4}{27}$ or 0	**A1**	Both
$f''(x)=6x-4\rightarrow f'\left(\frac{1}{3}\right)=-2\ (<0)$; $f''(1)=2\ (>0)$	**M1**	Or other valid method
max at $\left(\frac{1}{3},\frac{4}{27}\right)$; min at (1, 0) cao	**A1** **[5]**	Allow just x values or just y values given for identification

Q8.

7 $y-\frac{10}{2x+1}-2$ **(i)** $\frac{dy}{dx}-\frac{-10}{(2x+1)^2}\times 2$	B1 B1	Without the "×2". For the "×2".
At A, $y=0$, $\rightarrow$ $x=2$ m at $x=2$, is $-\frac{4}{5}$	B1	For $x=2$
Eqn of tangent is $y--\frac{4}{5}(x-2)$ $\rightarrow$ $5y+4x=8$	M1 A1 [5]	Must be using differential as m co – answer given.
(ii) C (0, 1.6) $d=\sqrt{(1.6^2+2^2)}=2.56$	M1 A1 [2]	Correct method – needs √. co

Q9.

1 $f'(x)-(2x-5)^2\times 2+1$ or $24\left(x-\frac{5}{2}\right)^2+1$ >0 (allow≥)	**B1B1** **B1** √ **[3]**	**B1** for $3(2x-5)^2$, **B1** for $(\times 2+1)$ SC B1 for $24x^2-120x+151$ Dep on $k(2x-5)^2+c$ $(k>0)$, $(c\geq 0)$ Subst of particular values is B0

Q10.

9 **(i)**	$3u+\frac{3}{u}-10-0$	**B1**	Or $3x-10\sqrt{x}+3-0$ Or $(3\sqrt{x}-1)(\sqrt{x}-3)$ or apply formula etc.
	$3u^2-10u+3-0 \Rightarrow (3u-1)(u-3)-0$	**M1**	
	$\sqrt{x}-\frac{1}{3}$ or 3	**A1**	
	$\sqrt{x}-\frac{1}{9}$ or 9	**A1** [4]	
(ii)	$f''(x)=\frac{3}{2}x^{-\frac{1}{2}}-\frac{3}{2}x^{-\frac{3}{2}}$	**B1**	Allow anywhere
	At $x=\frac{1}{9}$		
	$f''(x)=\frac{3}{2}(3)-\frac{3}{2}(27)(=-36)<0 \rightarrow$ Max	**M1**	Valid method. Allow innac subs, even 3, $\frac{1}{3}$
	At $x=9$		
	$f''(x)=\frac{3}{2}\times\frac{1}{3}-\frac{3}{2}\times\frac{1}{27}(-\frac{4}{9})>0 \rightarrow$ Min	**A1** [3]	Fully correct. No working, no marks.
(iii)	$f(x)=2x^{\frac{3}{2}}+6x^{\frac{1}{2}}-10x\ (+c)$	**B2**	B1 for 2/3 terms correct. Allow in **(i)**
	$-7=16+12-40+c$	**M1**	Sub (4, –7). c must be present.
	$c=5$	**A1** [4]	

Q11.

6	$u=x^2y \quad y+3x-9$ $u=x^2(9-3x)$ or $\left(\frac{9-y}{3}\right)^2 y$	M1		Expressing u in terms of 1 variable
	$\frac{du}{dx}=18x-9x^2$ or $\frac{du}{dy}=27-12y+y^2$	DM1A1		Knowing to differentiate.
	$=0$ when $x=2$ or $y=3 \rightarrow u=12$	DM1 A1		Setting differential to 0.
	$\frac{d^2u}{dx^2}=18-18x$ –ve	DM1 A1	[7]	Any valid method

Q12.

4 $y = x^4 + 4x + 9$		
(i) Differential $= 4x^3 + 4$ Sets to 0 + solution $\rightarrow (-1, 6)$ 2^{nd} differential $= 12x^2$ Positive, $\rightarrow$ Minimum	B1 M1 A1 B1 [4]	co Differentiates and sets to 0. co. Statement only.
(ii) $A = \left[\frac{x^5}{5} + 2x^2 + 9x\right]$ Limits from 0 to 1 $\rightarrow$ 11.2	B1 M1 A1 [3]	co. Value at "1" − value at "0" in integral of y.

Q13.

7 $y = \frac{12}{x^2 + 3}$		
(i) $\frac{dy}{dx} = -12(x^2 + 3)^{-2} \times 2x$	B1 B1 [2]	Without the "×2x". For "×2x". Accept unsimplified answer
(ii) At $x = 1$, $m = -\frac{3}{2}$ m of normal $= \frac{2}{3}$ Eqn of normal $y - 3 = \frac{2}{3}(x - 1)$	M1 M1 A1 [3]	Uses $m_1 m_2 = -1$ ....algebraic ok. Correct form of equation. co unsimplified
(iii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = -\frac{3}{2} \times 0.012$ $\rightarrow -0.018$	M1 A1√ [2]	Correct link between differentials co to his $\frac{dy}{dx}$. (Omission of x in part **(i)** causes fortuitous results in **(ii)** and **(iii)**.)

Q14.

8	**(i)** $2x+2y+\frac{\pi x}{2}-60$	M1		Linking 60 with sum of at least 4 sides and use of radians
	$\rightarrow y-30-x-\frac{\pi x}{4}$	A1	[2]	co
	(ii) $A-xy+\frac{\pi x^2}{4}$			
	$= x\,(30-x-\frac{\pi x}{4})+\frac{\pi x^2}{4}$	M1		Subs "y" into area eqn and use $\frac{1}{2}r^2\theta$
	$=30x-x^2$	A1	[2]	co.
	(iii) $\frac{dA}{dx}=30-2x$			Knowing to differentiate
	$=0$ when $x=15$ cm	M1 A1	[2]	Sets differential to 0 + solution. co.
	(iv) Max.	M1 A1	[2]	Any valid method. co.

Q15.

10	$y=4x-x^2+3$			
	(i) $\frac{dy}{dx}=4-2x$	B1		co
	At $x=3$, $m=-2$			
	Gradient of normal $=\frac{1}{2}$	M1		Use of $m_1m_2=-1$
	Eqn of normal $y-6-\frac{1}{2}(x-3)$	M1 A1		Use of $y-k=m(x-h)$ or $y=mx+c$
	$\rightarrow 2y=x+9$		[4]	(where m is gradient of normal)
	(ii) Meets axes at $(0, \frac{9}{2})$ and $(-9, 0)$	M1		Sets x and y to 0 + midpoint formula.
	Mid-point is $\left(\frac{-9}{2}, \frac{9}{4}\right)$	A1	[2]	co.
	(iii) $2y=x+9$, $y=4x-x^2+3$			
	$\rightarrow 2x^2-7x+3=0$ oe	M1 A1		Eliminates x completely. Correct eqn.
	$\rightarrow$ (½, 4¾)	M1 A1	[4]	Solution of quadratic. co

Q16.

5	(i) $\frac{dy}{dx} = \frac{-1}{(x-3)^2} + 1$	B1		oe
	$\frac{d^2y}{dx^2} = \frac{2}{(x-3)^3}$	B1	[2]	oe
	(ii) $(x-3)^2 = 1 \Rightarrow x-3 = \pm 1$	M1		Set $\frac{dy}{dx} = 0$ & reasonable attempt to solve
	$x = 4, 2$	A1		
	$y = 5, 1$	A1		
	When $x = 4$ $\frac{d^2y}{dx^2} > 0 \ (= 2) \Rightarrow$ min	M1		Investigate signs of f'' at a point or other method
	When $x = 2$ $\frac{d^2y}{dx^2} < 0 \ (= -2) \Rightarrow$ max	A1	[5]	

Q17.

4	(i) 3	B1	[1]	
	(ii) $f(x) = x^2 - 6x(+c)$	M1A1		Dependent on c present
	Subst $(3,-4)$	M1		cao
	$c = 5 \rightarrow f(x) = x^2 - 6x + 5$	A1	[4]	

Q18.

7	(i) $y = \frac{1}{6(48-8x)}$ oe	B1	[1]	
	(ii) $A = 4xy + 2xy$ or $3xy + 3xy = 6xy$	M1		
	$A = x(48-8x) = 48x - 8x^2$	A1	[2]	AG
	(iii) $\frac{\delta A}{\delta x} - 48 - 16x$	B1		
	$A = 72$ cao	M1A1		Attempt to solve derivative $= 0$ Expect $x = 3$
	$\frac{\delta^2 A}{\delta x^2} = -16 \ (< 0) \Rightarrow$ Maximum	B1	[4]	www Accept other complete methods

Q19.

8	**(i)** $f'(3) = 0 \Rightarrow 18 + 3k - 12 = 0$ $k = -2$ $(x - 3)(x + 2) = 0$ $x = -2$, (Allow also = 3) **(ii)** $f''(x) = 4x - 2$ $f''(3) > 0$ hence min at P $f''(-2) < 0$ hence max at Q **(iii)** $f(x) -\frac{2}{3}x^3 - x^2 - 12x$ $(+c)$ Sub $(3,-10) \rightarrow -10 = 18 - 9 - 36 + c$ $c = 17$	M1 A1 M1 A1 [4] B1 B1 [2] B2,1,0 M1 A1 [4]	 AG 3 min, –2 max independent of f''(x) Accept anywhere in question Dependent on c present Condone y = , or equation =

Q20.

3	$A = \pi r^2 \rightarrow \left(\frac{dA}{dr}\right) = 2\pi r$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ **used** $\frac{dr}{dt} = 3$ soi 300π (or 942)	**B1** **M1** **B1** **A1** **[4]**	

Q21.

5 2	$\frac{dy}{dx} = 2 - 2(x-1)^{-2}$ Sub $x = 2 \rightarrow \frac{dy}{dx} = 2 - 2 = 0 \Rightarrow$ stat value at $x =$ $\frac{d^2y}{dx^2} = 6(x-1)^{-4}$ (and sub $x = 2$) (At $x = 2$, $\frac{d^2y}{dx^2} = 6$) $> 0 \Rightarrow$ Minimum	**B2,1,0** **B1** **M1** **A1** **[5]**	–1 each error in 2, –2, $(x-1)^{-2}$ **AG** Reasonable attempt to diff form $(x-1)^{-n}$ Correct $\frac{d^2y}{dx^2}$ and 'minimum' is required **Or** other valid method for last 2 marks

Q22.

11 **(i)** $\frac{dy}{dx} = [6] \times \left[\frac{1}{3}(6x+2)^{-\frac{2}{3}}\right]$ Equation of tangent is $y-2=m(x-1)$ Equation of normal is $y-2=-\frac{1}{m(x-1)}$ Both eqns correct with $m=\frac{1}{2}$ cao **(ii)** $B=(0, 1\frac{1}{2})$; $C=(2, 0)$ $BC=\sqrt{2^2+\left(1\frac{1}{2}\right)^2}$ $= 2\frac{1}{2}$ **(iii)** BC: $y-1\frac{1}{2}=-\frac{3}{4(x-0)}$ or $y=-\frac{3}{4(x-2)}$ Intersection (E): $-\frac{3}{4}x+1\frac{1}{2}=2x$ $x=\frac{6}{11}$; $y=\frac{12}{11}$ Mid-point of $OA = (\frac{1}{2}, 1) \rightarrow E$ not mid-point	**B1B1** **M1** **M1** **A1** **[5]** **B1** **M1A1**√ **[3]** **M1** **M1** **A1** **B1** **[4]**	Independent Where m = numerical $\frac{dy}{dx}$ Including use of $m_1 m_2 = -1$ SC 1/3 Blatant tangent/normal reversal Both cao ft from *their B* and *C* or $y=-\frac{3}{4}x+1\frac{1}{2}$ cao Dependent on correct x values or y values for both E and the mid-point of OA

Q23.

2 $f'(x) =$ -4 2 $< 0 \Rightarrow$ decreasing function	B1 B1 B1 [3]	Dependent upon minus signs & even powers

Q24.

8 **(i)** **(ii)** $f'(-1) = 2+6-8=0$ hence stat value at $x=$ $f''(-1) = 9-6=3$ > 0 hence minimum **(iii)** y $+(c)$ Sub $(-1, 5) \rightarrow$ $+c=5$ $\rightarrow$	B2,1,0 [2] B1 B1 [2] B1 B1 B1 M1 A1 [5]	1 each error AG allow unsimplified $\frac{4}{15}$ Dependent on c present cao

Q25.

8 (i) $A - 2xr + \pi r^2$ $2x + 2\pi r - 400 \ (\Rightarrow x - 200 - \pi r)$ $A - 400r - \pi r^2$	**B1** **B1** **M1A1** **[4]**	Subst & simplify to **AG** (www)
(ii) $\frac{dA}{dr} - 400 - 2\pi r$ $= 0$ $r - \frac{200}{\pi}$ oe $x = 0 \Rightarrow$ no straight sections **AG** $\frac{d^2A}{dr^2} - -2\pi \ (< 0)$ Max	**B1** **M1** **A1** **A1** **B1** **[5]**	Differentiate Set to zero and attempt to find r Dep on -2π, or use of other valid reason

Q26.

9 $\frac{dy}{dx} - -k^2(x+2)^{-2} + 1 - 0$	M1A1	Attempt differentiation & set to zero
$x + 2 = \pm k$	DM1	Attempt to solve
$x = -2 \pm k$	A1	cao
$\frac{d^2y}{dx^2} - 2k^2(x+2)^{-3}$	M1	Attempt to differentiate again
	M1	Sub their x value with k in it into $\frac{d^2y}{dx^2}$
When $x = -2 = k$, $\frac{d^2y}{dx^2} - \left(\frac{2}{k}\right)$ which is (> 0) min	A1	Only 1 of bracketed items needed for each
When $x = -2 - k$, $\frac{d^2y}{dx^2} - \left(\frac{2}{-k}\right)$ which is (< 0)	A1	but $\frac{d^2y}{dx^2}$ and x need to be correct.
max	[8]	