**Q1.**

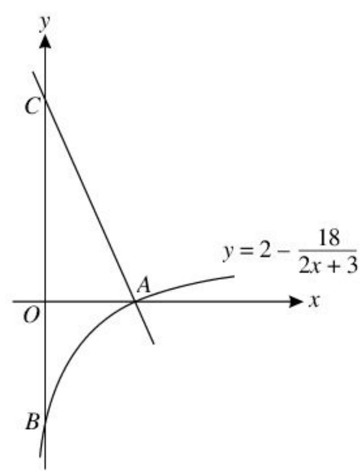
6 A curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$ and the point (9, 2) lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the $x$-coordinate of the stationary point on the curve and determine the nature of the stationary point. [3]

**Q2.**

7




The diagram shows part of the curve $y = 2 - \frac{18}{2x+3}$, which crosses the $x$-axis at $A$ and the $y$-axis at $B$. The normal to the curve at $A$ crosses the $y$-axis at $C$.

(i) Show that the equation of the line $AC$ is $9x + 4y = 27$. [6]

(ii) Find the length of $BC$. [2]

**Q3.**

2 The volume of a spherical balloon is increasing at a constant rate of 50 $cm^3$ per second. Find the rate of increase of the radius when the radius is 10 cm. [Volume of a sphere = $\frac{4}{3}\pi r^3$.] [4]

**Q4.**

6 The variables $x$, $y$ and $z$ can take only positive values and are such that

$$z = 3x + 2y \quad \text{and} \quad xy = 600.$$

(i) Show that $z = 3x + \frac{1200}{x}$. [1]

(ii) Find the stationary value of $z$ and determine its nature. [6]

**Q5.**

4 **(a)** Differentiate $\dfrac{2x^3+5}{x}$ with respect to $x$. [3]

**(b)** Find $\int (3x-2)^5\,dx$ and hence find the value of $\int_0^1 (3x-2)^5\,dx$. [4]

**Q6.**

4 A watermelon is assumed to be spherical in shape while it is growing. Its mass, $M$ kg, and radius, $r$ cm, are related by the formula $M = kr^3$, where $k$ is a constant. It is also assumed that the radius is increasing at a constant rate of 0.1 centimetres per day. On a particular day the radius is 10 cm and the mass is 3.2 kg. Find the value of $k$ and the rate at which the mass is increasing on this day. [5]

**Q7.**

10 It is given that a curve has equation $y = f(x)$, where $f(x) = x^3 - 2x^2 + x$.

**(i)** Find the set of values of $x$ for which the gradient of the curve is less than 5. [4]

**(ii)** Find the values of $f(x)$ at the two stationary points on the curve and determine the nature of each stationary point. [5]

**Q8.**

7 The curve $y = \dfrac{10}{2x+1} - 2$ intersects the $x$-axis at $A$. The tangent to the curve at $A$ intersects the $y$-axis at $C$.

**(i)** Show that the equation of $AC$ is $5y + 4x = 8$. [5]

**(ii)** Find the distance $AC$. [2]

**Q9.**

1 It is given that $f(x) = (2x-5)^3 + x$, for $x \in \mathbb{R}$. Show that f is an increasing function. [3]

**Q10.**

9 A curve has equation $y = f(x)$ and is such that $f'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$.

**(i)** By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find the values of $x$ for which the curve $y = f(x)$ has stationary points. [4]

**(ii)** Find $f''(x)$ and hence, or otherwise, determine the nature of each stationary point. [3]

**(iii)** It is given that the curve $y = f(x)$ passes through the point $(4, -7)$. Find $f(x)$. [4]

**Q11.**

6 The non-zero variables $x$, $y$ and $u$ are such that $u = x^2y$. Given that $y + 3x = 9$, find the stationary value of $u$ and determine whether this is a maximum or a minimum value. [7]

**Q12.**

4 The equation of a curve is $y = x^4 + 4x + 9$.

**(i)** Find the coordinates of the stationary point on the curve and determine its nature. [4]

**(ii)** Find the area of the region enclosed by the curve, the $x$-axis and the lines $x = 0$ and $x = 1$. [3]

**Q13.**

7 The equation of a curve is $y = \frac{12}{x^2 + 3}$.

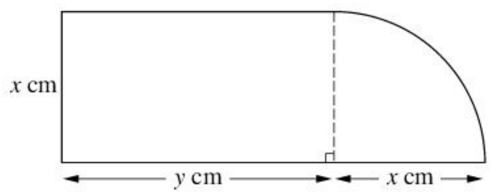
**(i)** Obtain an expression for $\frac{dy}{dx}$. [2]

**(ii)** Find the equation of the normal to the curve at the point $P(1, 3)$. [3]

**(iii)** A point is moving along the curve in such a way that the $x$-coordinate is increasing at a constant rate of 0.012 units per second. Find the rate of change of the $y$-coordinate as the point passes through $P$. [2]

**Q14.**

8




The diagram shows a metal plate consisting of a rectangle with sides $x$ cm and $y$ cm and a quarter-circle of radius $x$ cm. The perimeter of the plate is 60 cm.

**(i)** Express $y$ in terms of $x$. [2]

**(ii)** Show that the area of the plate, $A\text{ cm}^2$, is given by $A = 30x - x^2$. [2]

Given that $x$ can vary,

**(iii)** find the value of $x$ at which $A$ is stationary, [2]

**(iv)** find this stationary value of $A$, and determine whether it is a maximum or a minimum value. [2]

**Q15.**

**10** The equation of a curve is $y = 3 + 4x - x^2$.

**(i)** Show that the equation of the normal to the curve at the point $(3, 6)$ is $2y = x + 9$. [4]

**(ii)** Given that the normal meets the coordinate axes at points $A$ and $B$, find the coordinates of the mid-point of $AB$. [2]

**(iii)** Find the coordinates of the point at which the normal meets the curve again. [4]

**Q16.**

**5** A curve has equation $y = \frac{1}{x-3} + x$.

**(i)** Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [2]

**(ii)** Find the coordinates of the maximum point $A$ and the minimum point $B$ on the curve. [5]

**Q17.**

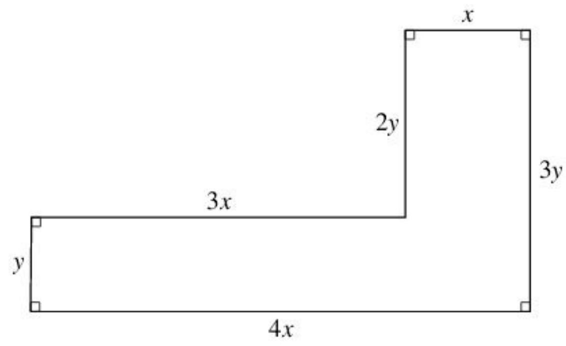
**4** A function f is defined for $x \in \mathbb{R}$ and is such that $f'(x) = 2x - 6$. The range of the function is given by $f(x) \geqslant -4$.

**(i)** State the value of $x$ for which $f(x)$ has a stationary value. [1]

**(ii)** Find an expression for $f(x)$ in terms of $x$. [4]

**Q18.**

**7**




The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48 m.

**(i)** Find an expression for $y$ in terms of $x$. [1]

**(ii)** Given that the area of the garden is $A\ \text{m}^2$, show that $A = 48x - 8x^2$. [2]

**(iii)** Given that $x$ can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

**Q19.**

**8** A curve $y = f(x)$ has a stationary point at $P\ (3, -10)$. It is given that $f'(x) = 2x^2 + kx - 12$, where $k$ is a constant.

**(i)** Show that $k = -2$ and hence find the $x$-coordinate of the other stationary point, $Q$. [4]

**(ii)** Find $f''(x)$ and determine the nature of each of the stationary points $P$ and $Q$. [2]

**(iii)** Find $f(x)$. [4]

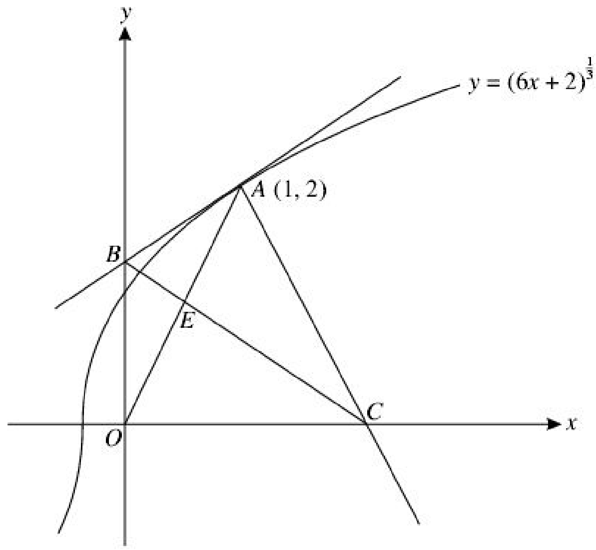
**Q20.**

**Q21.**

**5** A curve has equation $y = 2x + \frac{1}{(x-1)^2}$. Verify that the curve has a stationary point at $x = 2$ and determine its nature. [5]

**Q22.**

**11**




The diagram shows the curve $y = (6x + 2)^{\frac{1}{3}}$ and the point $A\ (1,\ 2)$ which lies on the curve. The tangent to the curve at $A$ cuts the $y$-axis at $B$ and the normal to the curve at $A$ cuts the $x$-axis at $C$.

**(i)** Find the equation of the tangent $AB$ and the equation of the normal $AC$. [5]

**(ii)** Find the distance $BC$. [3]

**(iii)** Find the coordinates of the point of intersection, $E$, of $OA$ and $BC$, and determine whether $E$ is the mid-point of $OA$. [4]

**Q23.**

**2** It is given that $\mathrm{f}(x) = \dfrac{1}{x^3} - x^3$, for $x > 0$. Show that f is a decreasing function. [3]

**Q24.**

**8** A curve is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2(3x + 4)^{\frac{3}{2}} - 6x - 8.$$

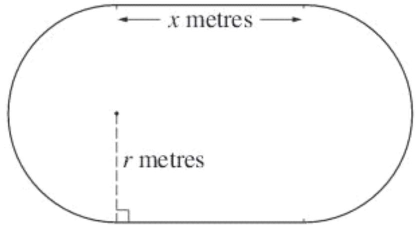
**(i)** Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [2]

**(ii)** Verify that the curve has a stationary point when $x = -1$ and determine its nature. [2]

**(iii)** It is now given that the stationary point on the curve has coordinates $(-1,\ 5)$. Find the equation of the curve. [5]

**Q25.**

**8**




The inside lane of a school running track consists of two straight sections each of length $x$ metres, and two semicircular sections each of radius $r$ metres, as shown in the diagram. The straight sections are perpendicular to the diameters of the semicircular sections. The perimeter of the inside lane is 400 metres.

**(i)** Show that the area, $A$ m$^2$, of the region enclosed by the inside lane is given by $A = 400r - \pi r^2$. [4]

**(ii)** Given that $x$ and $r$ can vary, show that, when $A$ has a stationary value, there are no straight sections in the track. Determine whether the stationary value is a maximum or a minimum. [5]

## Q26.

**9** A curve has equation $y = \frac{k^2}{x+2} + x$, where $k$ is a positive constant. Find, in terms of $k$, the values of $x$ for which the curve has stationary points and determine the nature of each stationary point. [8]