

Q1.

<p>5 $x \mapsto 2\sin^2 x - 3\cos^2 x$</p> <p>(i) $2(1 - \cos^2 x) - 3\cos^2 x$ $\rightarrow 2 - 5\cos^2 x$ ($a=2, b=-5$)</p> <p>(ii) Values are -3 and 2</p> <p>(iii) $2 - 5\cos^2 x = -1$ $\rightarrow \cos^2 x = 0.6$ $x = 0.685, 2.46$ (accept 0.684)</p>	<p>M1</p> <p>A1</p> <p>[2]</p> <p>B1√ B1√</p> <p>[2]</p> <p>B1√</p> <p>B1 B1√</p> <p>[3]</p>	<p>Uses $s^2 + c^2 = 1$</p> <p>co</p> <p></p> <p></p> <p>co √ for π - (first answer)</p> <p>SC B1 for both 39.2 and 140.8</p>
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Q2.

<p>9 (i) $2x^2 - 12x + 7 = 2(x-3)^2 - 11$</p> <p>(ii) Range of $f \geq -11$</p> <p>(iii) $2x^2 - 12x + 7 < 21$ $\rightarrow 2x^2 - 12x - 14$ or $2(x-3)^2 < 32$ \rightarrow end-values of 7 or -1 $\rightarrow -1 < x < 7$</p> <p>(iv) $gf(x) = 2(2x^2 - 12x + 7) + k = 0$ Use of $b^2 - 4ac$ $\rightarrow 24^2 - 16(14 + k)$ $\rightarrow k = 22$</p>	<p>$3 \times$ B1</p> <p>[3]</p> <p>B1√</p> <p>[1]</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>B1 for each value – accept if a, b, c not specifically quoted.</p> <p>√ to his “c”. allow $>$ or \geq.</p> <p>3-term quadratic to 0 or $2(x-3)^2 < 32$</p> <p>Correct end-values</p> <p>co</p> <p>Puts f into g. co.</p> <p>Used correctly with quadratic</p> <p>co.</p>
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Q3.

<p>3 $f: x \mapsto a + b\cos x$</p> <p>(i) $f(0) = 10, a + b = 10$ $f(\frac{2}{3}\pi) = 1, a - \frac{b}{2} = 1$ $\rightarrow a = 4, b = 6$</p> <p>(ii) Range is -2 to 10.</p> <p>(iii) $\cos\left(\frac{5}{6}\pi\right) = -\cos\left(\frac{1}{6}\pi\right) = -\frac{\sqrt{3}}{2}$ $\rightarrow 4 - 3\sqrt{3}$</p>	<p>B1</p> <p>B1</p> <p>B1√</p> <p>[2]</p> <p>[1]</p> <p>B1</p> <p>B1</p> <p>[2]</p>	<p>EITHER OF THESE</p> <p>both co</p> <p>√ for his “$a - b$” to “$a + b$”</p> <p>For $\cos 30^\circ = \frac{1}{2}\sqrt{3}$ used somewhere.</p> <p>co</p>
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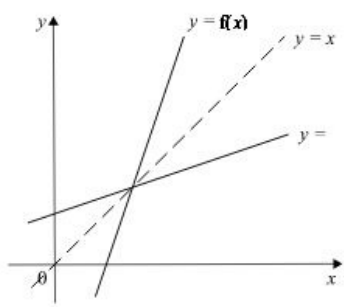
Q4.

<p>10 $f: x \mapsto 2x^2 - 8x + 14$</p> <p>(i) $y + kx = 12$, Sim Eqns. $\rightarrow 2x^2 - 8x + kx + 2 = 0$ Use of $b^2 - 4ac$ $\rightarrow (k - 8)^2 = 16 \rightarrow k = 12$ or 4.</p> <p>(ii) $2x^2 - 8x + 14 = 2(x - 2)^2 + 6$</p> <p>(iii) Range of $f \geq 6$.</p> <p>(iv) Smallest $A = 2$</p> <p>(v) Makes x the subject Order of operations correct.</p> $g^{-1}(x) = \sqrt{\frac{x - 6}{2}} + 2$	<p>M1 A1 M1 A1</p> <p>[4]</p> <p>B1×3 [3]</p> <p>B1√ [1] B1√ [1] [1]</p> <p>M1 M1</p> <p>A1 [3]</p>	<p>Complete elimination of y (or x)</p> <p>Uses $b^2 - 4ac$ on eqn = 0, no “x” in a, b, c. co.co</p> <p>√ for c or from calculus.</p> <p>√ to answer to (ii).</p> <p>Could interchange x, y first. Order must be correct.</p> <p>co</p>
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Q5.

<p>11 (i) $fg(x) = 2x^2 - 3, \quad gf(x) = 4x^2 + 4x - 1$</p> <p>(ii) $2a^2 - 3 - 4a^2 + 4a - 1 \Rightarrow 2a^2 + 4a + 2 = 0$ $(a + 1)^2 = 0$ $a = -1$</p> <p>(iii) $b^2 - b - 2 = 0 \rightarrow (b + 1)(b - 2) = 0$ $b = 2$ Allow $b = -1$ in addition</p> <p>(iv) $f^{-1}(x) = \frac{1}{2}(x - 1)$ $f^{-1}g(x) = \frac{1}{2}(x^2 - 3)$</p> <p>(v) $x = (\pm)\sqrt{y + 2}$ $h^{-1}(x) = -\sqrt{x + 2}$</p>	<p>B1, B1 [2]</p> <p>M1 M1 A1</p> <p>[3]</p> <p>B1</p> <p>B1√ [2]</p> <p>M1 A1 [2]</p>	<p>fg & gf clearly transposed gets B0B0</p> <p>Dep. quadratic. Allow x for all 3 marks Allow marks in (ii) if transposed in (i)</p> <p>Allow in terms of x for M1 only Correct answer without working B2</p> <p>Must be simplified. Ft from <i>their</i> f^{-1}</p>
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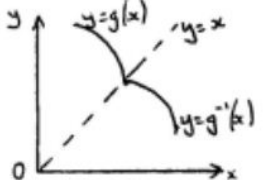
Q6.

10 $f: x \mapsto 3x - 4$ $g: x \mapsto 2(x-1)^2 + 8$		
(i) $fg(2) = f(10) = 26$	M1 A1 [2]	Must use g first, then f . ∞
(ii) 	B1 B1 B1 [3]	$y = f(x)$ correct in 1 st , 4 th quadrants. $y = f^{-1}(x)$ correct in 1 st , 2 nd quadrants. $y = x$ marked, or quoted.
(iii) $g'(x) = 6(x-1)^2$ $g'(x) > 0 \rightarrow$ no turning points $\rightarrow g$ is 1 : 1, g has an inverse.	B1 B1 ✓ B1 ✓ [3]	∞ allow only for incorrect "6" following from incorrect "6"
(iv) $f^{-1}(x) = \frac{x+4}{3}$ Attempt at making x Order correct. $-8, \div 2, \sqrt[3]{\quad}, +1$ $g^{-1}(x) = \sqrt[3]{\frac{x-8}{2}} + 1$	B1 M1 M1 A1 [4]	∞ May change x and y first. Must all be correct, but allow for $+8, -1$ ∞ as function of x , not y .

Q7.

8 (i) $(x-2)^2 - 4 + k$	B1B1 [2]	$a = -2, b = -4$
(ii) $f(x) > k - 4$ or $[k-4, \infty)$ or $(k-4, \infty)$ oe	B1 ✓ [1]	fit <i>their</i> $k-4$. Accept $>$
(iii) smallest value of $p = 2$	B1 ✓ [1]	fit <i>their</i> 2
(iv) $x - 2 - (\pm)\sqrt{y+4-k}$ $x - 2 + \sqrt{y+4-k}$ $f^{-1}(x) - 2 + \sqrt{x+4-k}$ Domain is $x > k-4$ or $[k-4, \infty)$ or $(k-4, \infty)$ oe	M1 A1 ✓ A1 B1 ✓ [4]	fit from <i>their</i> part (i) ∞ fit from <i>their</i> part (ii). Accept $>$

Q8.

<p>11 $f(x) = 8 - (x-2)^2$,</p> <p>(i) Stationary point at $x = 2$ y-coordinate = 8 Nature Maximum (or $y = -x^2 + 4x + 4$ $-2x + 4 = 0 \rightarrow (2, 8)$ Max)</p> <p>(ii) $k = 2$</p> <p>(iii) $y = 8 - (x-2)^2$ $\rightarrow (x-2)^2 + y - 8$ $\rightarrow (x-2) = \pm\sqrt{8-y}$ $\rightarrow x^{-1} = 2 + \sqrt{8-x}$</p> <p>(iv)</p> 	<p>B1 B1 B1</p> <p>[3]</p> <p>B1✓ [1]</p> <p>M1 M1 A1</p> <p>[3]</p> <p>B1 B1 B1</p> <p>[3]</p>	<p>co co co independent of first two marks</p> <p>✓ on "x-value"</p> <p>Attempt to make x the subject Order of operations correct Must be $f(x)$.</p> <p>B1 arc 1st quad (no tp, no axes) B1 Evidence of symmetry about $y = x$ B1 all correct as shown left</p>
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Q9.

<p>8 (i) $2(x-3)^2 - 5$ or $a = 2, b = -3, c = -5$</p> <p>(ii) 3</p>	<p>B1B1B1 [3]</p> <p>B1 ✓ [1]</p>	<p>ft on <i>their</i> b. Allow $k \geq 3$ or $x \geq 3$</p>
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<p>(iii) $(y) \geq 27$</p> <p>(iv) $2(x-3)^2 - (y+5)$ $x - 3 - (\pm)\sqrt{\frac{1}{2}(y+5)}$ $x - 3 + / \pm \sqrt{\frac{1}{2}(y+5)}$ $(f^{-1}(x)) - 3 + \sqrt{\frac{1}{2}(x+5)}$ for $x \geq 27$</p>	<p>B1 [1]</p> <p>M1</p> <p>M1</p> <p>A1 ✓</p> <p>A1B1 ✓ [5]</p>	<p>Allow $>$. Allow $27 \leq y \leq \infty$ etc. OR (x/y interchange as 1st operation)</p> <p>$x - 2(y-3)^2 - 5$ $(y-3)^2 - \frac{1}{2}(x+5)$ $y - 3 - (\pm)\sqrt{\frac{1}{2}(x+5)}$</p> <p>ft on <i>their</i> 27 from (iii)</p>
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Q10.

10	$f: x \mapsto 2x + k, g: x \mapsto x^2 - 6x + 8,$			
(i)	$2(2x + 3) + 3 = 25$ $\rightarrow x = 4$ or $\{f(11) = 25, f(4) = 11\}$	M1 A1	[2]	$ff(x)$ needs to be correctly formed
(ii)	$x^2 - 6x + 8 = 2x + k$ $x^2 - 8x + 8 - k = 0$ Uses $b^2 - 4ac < 0$ $\rightarrow k < -8$	M1 M1 A1	[3]	Eliminates y to form eqn in x . Uses the discriminant – even if $=0, >0$
(iii)	$x^2 - 6x + 8 = (x - 3)^2 - 1$ $y = (x - 3)^2 - 1$ Makes x the subject $\rightarrow \pm\sqrt{(x + 1) + 3}$ Needs specifically to lose the “-”.	B1 B1 M1 A1√	[4]	For “-3” and “-1” Makes x the subject, in terms of x and without – or \pm .

Q11.

10	$f: x \mapsto 2x + 1, x \in \mathbb{R}, x > 0$ $g: x \mapsto \frac{2x - 1}{x + 3}, x \in \mathbb{R}, x \neq -3.$			
(i)	$gf(x) = \frac{2(2x + 1) - 1}{2x + 1 + 3} = \frac{4x + 1}{2x + 4}$ Equates to $x \rightarrow 2x^2 = 1$ $\rightarrow x = \frac{1}{2}\sqrt{2}$	M1 M1 A1	[3]	Must be gf , needs x replacing twice. Forms quadratic + solution Co. condone \pm .
(ii)	$f^{-1}(x) = \frac{1}{2}(x - 1)$ To find $g^{-1}(x)$, make x the subject Order must be correct $\rightarrow g^{-1}(x) = \frac{-1 - 3x}{x - 2}$ or $\frac{1 + 3x}{2 - x}$	B1 M1 M1 A1	[4]	Co Attempt at x as the subject. Order correct. Allow for sign errors. Co – must be $f(x)$.
(iii)	$\frac{1 + 3x}{2 - x} = x \rightarrow x^2 + x + 1 = 0$ Looks at $b^2 - 4ac$ \rightarrow negative \rightarrow no roots.	M1 M1 A1	[3]	Forms quadratic equation. Looks at discriminant or attempts to solve and finds $\sqrt{(\text{negative})}$. Co
(iv)		B1 B1 B1	[3]	Correct $y = 2x + 1$ on graph from $(0, 1)$ Correct $y = \frac{1}{2}(x - 1)$ on graph from $(1, 0)$ (if -ve x plotted, B1 s.c. for both) Shows or states or implies that f, f^{-1} are reflections in $y = x$.

Q12.

3	$f: x \mapsto 2x + 3,$ $g: x \mapsto x^2 - 2x,$ $gf(x) = (2x + 3)^2 - 2(2x + 3)$ $= 4x^2 + 8x + 3$ $= 4(x + 1)^2 - 1$	M1 A1 $3 \times B1\sqrt{}$	[5]	Must be f into g , not g into f . co Allow all these as $\sqrt{}$ for either fg or gf .
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Q13.

6	(i) $(3x + 5)(x - 1) > 0$ -5/3, 1 $x < -5/3, x > 1$	M1 A1 A1	[3]	Attempt at factorisation Both required Ignore any words between answers Condone < >
	(ii) $f(x) = x^3 + x^2 - 5x + c$ $3 = 1 + 1 - 5 + c$ $f(x) = x^3 + x^2 - 5x + 6$	M1 A1 M1 A1		[4]

Q14.

7	(i) Range is $0 < f(x) < 4$, 0 to 4	B1	[1]	Accept in two parts. Condone <
	(ii) $y = x$ drawn or implied Correct sketch of f^{-1}	B1 B1		SC if f missing, (2, 2) (4, 6) must be shown
	(iii) $(x \mapsto) \sqrt{2x}$ for $0 < x < 2$ $(x \mapsto) 2x - 2$ for $2 < x < 4$	B1B1 B1B1	[2]	Condone < <
			[4]	

Q15.

11	(i) $2(x-2)^2 + 2$	B 1, B1, B1	[3]	For 2, -2, 2
	(ii) $2 \leq f(x) \leq 10$ oe	B1	[1]	Allow < etc. Ignore notation
	(iii) $2 \leq x \leq 10$	B1√	[1]	Ft from part (ii). Ignore notation
	(iv) $f(x) \approx$ half parabola from (0,10) to (2,2) $g(x)$: line through 0 at $\approx 45^\circ$ $f^{-1}(x)$: reflection of <i>their</i> $f(x)$ in $g(x)$ Everything totally correct	B1 B1 B1√ B1	[4]	Or from int with y axis to int with <i>their</i> $y - x$

(v) $(x-2)^2 - \frac{1}{2}(y-2)$ $x = 2 \pm \sqrt{\frac{1}{2}(y-2)}$ $f^{-1}(x) = 2 - \sqrt{\frac{1}{2}(x-2)}$	M1 M1 A1	[3]	Allow $+\sqrt{\quad}$ or $-\sqrt{\quad}$. Dep on final ans as f^n of x cao
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Q16.

<p>9 (i) $f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$</p> $2x + 3 - \frac{1}{2}x - \frac{3}{2} \Rightarrow x - -3$ <p>(ii) 2 lines approximately correct, reflected in $y=x$ & meeting at $(-3, -3)$</p> <p>(iii) $gf(x) = (2x + 3)^2 - 6(2x + 3)$ $4x^2 - 9$ $4x^2 - 9 < 16 \Rightarrow x^2 < \frac{25}{4}$ $-\frac{5}{2} \leq x \leq 0$</p>	<p>B1</p> <p>M1A1 [3]</p> <p>B3,2,1,0 [3]</p> <p>M1 A1 M1</p> <p>A1A1 [5]</p>	<p>Can be implied by graph or in writing. Ignore lines extended</p> <p>Solving any quadratic to do with f and g <16, to x =</p> <p>Condone < and ></p>
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Q17.

<p>10 (i) $4(x - 3)^2 - 25$ Vertex is $(3, -25)$</p> <p>(ii) range is $(g(x)) > -9$ Allow ></p> <p>(iii) $(x - 3)^2 = \frac{1}{4(y + 25)}$ $x - 3 = \frac{(\pm)1}{2}\sqrt{y + 25}$ $g^{-1}(x) = 3 - \frac{1}{2}\sqrt{x + 25}$ Domain is $x > -9$</p>	<p>B1B1B1 B1✓ [4]</p> <p>B1B1 [2]</p> <p>M1 DM1 A1 B1✓ [4]</p>	<p>Or $a = 4, b = 3, c = -25$ fit to <i>their</i> (b, c). Accept if not 'hence'</p> <p>B1 for >, B1 for -9 Accept e.g. $[-9, \infty]$</p> <p>Attempt to square root both sides cao fit from <i>their</i> (ii)</p>
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Q18.

<p>6 (i) $\cos\left(\frac{1}{2}x + \frac{\pi}{6}\right) (= 1)$ $x = \frac{\pi}{3}$</p> <p>(ii) $\frac{1}{2}\cos x + \frac{\pi}{6} = 1$ $\cos x = 0.9528$ $x = \pm 0.31$</p>	<p>B1 B1 [2]</p> <p>B1 B1 B1 B1✓ [4]</p>	<p>cao</p> <p>✓ for negative of first answer</p>
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Q19.

<p>7 (i) $11 x^2 = 5 x \Rightarrow x^2 x 6(=0)$</p> <p>$p = 3; q = 2$</p> <p>(ii)</p>	<p>B1</p> <p>B1 B1 [3]</p> <p>B1 B1 B1</p> <p>B1 B1 [5]</p>	<p>oe</p> <p>cao</p>
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Q20.

<p>5 (i) $x - (\pm)\sqrt{y-1}$ $f^{-1}: x \mapsto \sqrt{x-1}$ for $x > 1$</p> <p>(ii) $f(x) - (x^2 + 1)^2 + 1$ $x^2 + 1 - (\pm)3/4$ $x = 3/2$</p> <p>Alt. (ii) $f(x) - f^{-1}(185/16) - 13/4$ M1 $x - f^{-1}(13/4)$ M1 $x - 3/2$ A1</p>	<p>B1</p> <p>B1B1 [3]</p> <p>B1</p> <p>M1</p> <p>A1 [3]</p>	<p>OR $y^2 - x - 1$ (x/y interchange 1st)</p> <p>Or $x^4 + 2x^2 - (153/16) - 0$</p> <p>Or $x^2 - 9/4, (-17/4)$</p> <p>www. Condone $\pm 3/2$</p> <p>Alt. (ii) $f(3/2) = 13/4$ B1 $f(13/4) = 185/16$ B1 $x = 3/2$ B1 SC.B2 answer 1.5 with no working</p>
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Q21.

<p>10 (i) Range is $(y) > c^2 + 4c$ $x^2 + 4x = (x + 2)^2 - 4$ (Smallest value of c is) -2</p> <p>(ii) $5a + b = 11$ $(a + b)^2 + 4(a + b) = 21$ $(11 - 5a + a)^2 + 4(11 - 5a + a) = 21$ $(8)(2a^2 - 13a + 18) = (8)(2a - 9)(a - 2) = 0$ $a = \frac{9}{2}, 2$ OR $b = \left(-\frac{23}{2}\right), 1$</p> <p>Alt. (ii) Last 5 marks $f^{-1}(x) = \sqrt{x + 4} - 2$ B1 $g(1) = f^{-1}(21)$ used M1 $a + b = \sqrt{25} - 2 = 3$ A1 Solve $a + b = 3, 5a + b = 11$ M1 $a = 2, b = 1$ A1</p>	<p>B1 M1 A1 [3] B1 B1 M1 M1 A1 A1 [6]</p>	<p>Allow $>$ OR $\frac{dy}{dx} = 2x + 4 = 0$ -2 with no (wrong) working gets B2</p> <p>OR corresponding equation in b OR $(8)(2b + 23)(b - 1) = 0$</p> <p>A1 for either a or b correct. Condone 2nd value. Spotted solution scores only B marks.</p> <p>Alt. (ii) Last 4 marks $(a + b + 7)(a + b - 3) = 0$ M1A1 (Ignore solution involving $a + b = -7$) Solve $a + b = 3, 5a + b = 11$ M1 $a = 2, b = 1$ A1</p>
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