Q1.

5 (i)	Make recognizable sketch over the given range of two suitable graphs, e.g. $y = \ln x$ and $y = 2 - x^2$	B1+B1
	State or imply link between intersections and roots and justify given answer	B1
		[3]
(ii)	Consider sign of $\ln x - (2 - x^2)$ at $x = 1$ and $x = 1.4$, or equivalent	M1
	Complete the argument correctly with appropriate calculation	A1
		[2]
(iii)	Use the given iterative formula correctly with $1 \leq x_n \leq 1.4$	M1
	Obtain final answer 1.31	A1
	Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (1.305, 1.315)	A1
		[3]

Q2.

2	(i)	Use the given iterative formula correctly at least ONCE with $x_1 = 3$	M1	
		Obtain final answer 3.142	A1	
		Show sufficient iterations to justify its accuracy to 3 d.p.	A1	3
	(ii)	State any suitable equation e.g. $x = \frac{1}{5}\left(4x + \frac{306}{x^4}\right)$	B1	
		Derive the given answer α (or x) = $\sqrt[5]{306}$	B1	2

Q3.

3	(i)	Use the given iterative formula correctly at least once	M1	
		Obtain final answer $\alpha = 1.68$	A1	
		Show sufficient iterations to justify the answer to 2 dp	B1	3
	(ii)	State equation, e.g. $x = \frac{3}{4}x + \frac{2}{x^3}$, in any correct form	B1	
		Derive the exact answer α (or x) = $\sqrt[4]{8}$, or equivalent	B1	2

Q4.

6	(i) Make recognizable sketch of an appropriate exponential curve, e.g. $y = 9e^{-2x}$	B1	
	Sketch the appropriate second curve, e.g. $y = x$ correctly and justify the given statement	B1	2
	(ii) Consider sign of $x - 9e^{-2x}$ at $x = 1$ and $x = 2$, or equivalent	M1	
	Complete the argument correctly with appropriate calculations	A1	2
	(iii) State or imply the equation $x = \frac{1}{2}(\ln 9 - \ln x)$	B1	
	Rearrange this in the form given in part (i), or work *vice versa*	B1	2
	(iv) Use the iterative formula correctly at least once	M1	
	Obtain final answer $x = 1.07$	A1	
	Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.065, 1.075)	A1	3

Q5.

5	(i)	Make recognisable sketch of a relevant graph, e.g. $y = \sec x$	B1	
		Sketch an appropriate second graph, e.g. $y = 3 - x$, correctly and justify the given statement	B1	[2]
	(ii)	Consider sign of $\sec x - (3 - x)$ at $x = 1$ and $x = 1.2$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]
	(iii)	Show that the given equation is equivalent to $\sec x = 3 - x$, or *vice versa*	B1	[1]
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.04	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.035, 1.045)	B1	[3]

Q6.

7	(i)	Use product rule	M1*	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve for x	M1(dep*)	
		Obtain answer $x = -\frac{1}{2}$ correctly	A1	
		Obtain $y = -1/(2e)$ or exact equivalent	A1	[5]
	(ii)	Show that $20 = xe^{2x}$ is equivalent to $x = \frac{1}{2}\ln(20 / x)$ or *vice versa*	B1	[1]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.35	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p.	A1	[3]

Q7.

7	**(i)**	Use product rule	M1*	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve for x	M1(dep*)	
		Obtain answer $x = -\frac{1}{2}$ correctly	A1	
		Obtain $y = -1/(2e)$ or exact equivalent	A1	[5]
	(ii)	Show that $20 = xe^{2x}$ is equivalent to $x = \frac{1}{2}\ln(20 / x)$ or *vice versa*	B1	[1]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.35	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p.	A1	[3]

Q8.

7	**(i)**	Make a recognisable sketch of a relevant graph, e.g. $y = 2 - x$	B1	
		Sketch an appropriate second graph, e.g. $y = e^{2x}$, and justify the given statement	B1	[2]
	(ii)	Consider sign of $e^{2x} - (2 - x)$ at $x = 0$ and $x = 0.5$, or equivalent	M1	
		Complete the argument correctly with correct calculations	A1	[2]
	(iii)	Show that $e^{2x} = 2 - x$ is equivalent to $x = \frac{1}{2}\ln(2 - x)$, or *vice versa*	B1	[1]
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.27	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.265, 0.275)	A1	[3]

Q9.

6	**(i)**	Make a recognisable sketch of a relevant graph, e.g. $y = \ln x$ or $y = 2 - x^2$	B1	
		Sketch a second relevant graph and justify the given statement	B1	[2]
	(ii)	Consider sign of $\ln x - (2 - x^2)$ at $x = 1.3$ and $x = 1.4$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]

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	(iii)	Show that given equation is equivalent to $x = \sqrt{(2 - \ln x)}$ or *vice versa*	B1	[1]
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.31	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.305, 1.315)	B1	[3]

Q10.

7	**(i)**	Draw correct sketch of $y = e^{2x}$	B1	
		Draw correct sketch of $y = 14 - x^2$	B1	
		Indicate two real roots only from correct sketches	B1	[3]
	(ii)	Consider sign of $e^{2x} + x^2 - 14$ for 1.2 and 1.3 or equivalent	M1	
		Justify conclusion with correct calculations (f(1.2) = –1.54, f(1.3) = 1.15)	A1	[2]
	(iii)	Confirm given answer $x = \frac{1}{2}\ln\left(14 - x^2\right)$	B1	[1]
	(iv)	Use the iteration process correctly at least once	M1	
		Obtain final answer 1.26	A1	
		Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval (1.255, 1.256) [1.2 → 1.2653 → 1.2588 → 1.2595 ; 1.25 → 1.2604 → 1.2593 → 1.2594 ; 1.3 → 1.2522 → 1.2598 → 1.2594]	A1	[3]

Q11.

3	**(i)**	Use the iteration process correctly at least once	M1	
		Obtain at least two correct iterates to 5 decimal places	A1	
		Conclude $\alpha = 0.952$ [1 → 0.95647 → 0.95257 → 0.95223 → 0.95220]	A1	[3]
	(ii)	State or imply equation is $x = \frac{1}{2}\sqrt[3]{x^2 + 6}$	B1	
		Obtain $8x^3 - x^2 - 6 = 0$	B1	[2]

Q12.

6	**(i)**	Obtain derivative of form $k(2t + 1)^{-3}$	M1	
		Obtain $-4(2t + 1)^{-3}$ or equivalent as derivative of x	A1	
		Obtain $\frac{1}{2}(t + 2)^{-\frac{1}{2}}$ or equivalent as derivative of y	B1	
		Equate attempt at $\frac{dy}{dx}$ to –1	M1	
		Obtain $(2p + 1)^3 = 8(p + 2)^{\frac{1}{2}}$ or equivalent	A1	
		Confirm given answer $p = (p + 2)^{\frac{1}{6}} - \frac{1}{2}$	A1	[6]
	(ii)	Use iteration process correctly at least once	M1	
		Obtain final answer 0.678	A1	
		Show sufficient iterations to 5 decimal places to justify answer or show a sign change in the interval (0.6775, 0.6785) [0.7 → 0.68003 → 0.67857 → 0.67847 → 0.67846]	A1	[3]

Q13.

6	(i)	Attempt use of quotient rule or equivalent	M1	
		Obtain $\frac{2(x+2)\cos 2x - \sin 2x}{(x+2)^2}$ or equivalent	A1	
		Equate numerator to zero and attempt rearrangement	M1	
		Confirm given result $\tan 2x = 2x + 4$	A1	[4]
	(ii)	Consider sign of $\tan 2x - 2x - 4$ for 0.6 and 0.7 or equivalent	M1	
		Obtain –2.63 and 0.40 or equivalents and justify conclusion	A1	[2]
	(iii)	Use iteration process correctly at least once	M1	
		Obtain final answer 0.694	A1	
		Show sufficient iterations to 5 decimal places to justify answer or show a sign change in the interval (0.6935, 0.6945)	A1	[3]
		[0.6 → 0.69040 → 0.69352 → 0.69363		
		0.65 → 0.69215 → 0.69358 → 0.69363		
		0.7 → 0.69384 → 0.69364 → 0.69363]		

Q14.

6	(i)	Make a recognisable sketch of a relevant graph, e.g. $y = \cot x$ or $y = 4x - 2$	B1	
		Sketch a second relevant graph and justify the given statement	B1	[2]
	(ii)	Consider sign of $4x - 2 - \cot x$ at $x = 0.7$ and $x = 0.9$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]
	(iii)	Show that given equation is equivalent to $x = \frac{1 + 2\tan x}{4\tan x}$, or vice versa	B1	[1]
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.76	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (0.755, 0.765)	B1	[3]

Q15.

6	(i)	Make a recognisable sketch of a relevant graph, e.g. $y = 3e^x$ or $y = 8 - 2x$	B1	
		Sketch a second relevant graph and justify the given statement	B1	[2]

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(ii) Consider sign of $3e^x - 8 + 2x$ at $x = 0.7$ and $x = 0.8$, or equivalent — M1
Complete the argument correctly with appropriate calculations — A1 [2]
($f(0.7) = -0.559$, $f(0.8) = 0.277$ or equivalent)

(iii) Show that given equation is equivalent to $x = \ln\left(\frac{8-2x}{3}\right)$, or vice versa — B1 [1]

(iv) Use the iterative formula correctly at least once — M1
Obtain final answer 0.768 — A1
Show sufficient iterations to justify its accuracy to 3 d.p.

$x_0 = 0.7$	$x_0 = 0.75$	$x_0 = 0.8$
0.78846	0.77319	0.75769
0.76129	0.76603	0.77082
0.76971	0.76825	0.76676
0.76711	0.76756	0.76802
0.76791		0.76763
0.76766		

or show there is a sign change in the interval (0.7675, 0.7685) — B1 [3]

Q16.

4	(i)	Make recognisable sketches over the given range of a suitable pair of graphs e.g. $y = \sin x$ and $y = \frac{1}{x^2}$	B1	
		State or imply connection between intersections and roots and justify given statement	B1	2
	(ii)	Calculate values (or signs) of $\sin x - \frac{1}{x^2}$ at $x = 1$ and $x = 1.5$	M1	
		Derive given result correctly	A1	2
	(iii)	Rearrange $\sin x = \frac{1}{x^2}$ and obtain given answer	B1	1
	(iv)	Use the iterative formula correctly with $1 \leq x_n \leq 1.5$	M1	
		Obtain final answer 1.07	A1	
		Show sufficient iterations to justify its accuracy to 3d.p., or show there is a sign change in the interval (1.065, 1.075)	A1	3

Q17.

5 (i)	Make recognisable sketch of $y = 2^x$ or $y = x^2$, for $x < 0$	B1
	Sketch the other graph correctly	B1
		[2]
(ii)	Consider sign of $2^x - x^2$ at $x = -1$ and $x = -0.5$, or equivalent	M1
	Complete the argument correctly with appropriate calculations	A1
		[2]
(iii)	Use the iterative form correctly	M1
	Obtain final answer -0.77	A1
	Show sufficient iterations to justify its accuracy to 2 s.f., or show there is a sign change in the interval $(-0.775, -0.765)$	A1
		[3]

Q18.

6 (i)	Make recognisable sketch of an appropriate trig curve, e.g. $y = \cot x$, for $0 < x < \frac{1}{2}\pi$	B1	
	Sketch the appropriate second curve e.g. $y = x$ correctly and justify the given statement	B1	**2**

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(ii)	Consider sign of $\cot x - x$ at $x = 0.8$ and $x = 0.9$, or equivalent	M1	
	Complete the argument correctly with appropriate calculations	A1	**2**
(iii)	Show, using $\cot x \equiv \frac{1}{\tan x}$, that $\cot x = x$ is equivalent to $x = \arctan\left(\frac{1}{x}\right)$ (or vice versa)	B1	**1**
(iv)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.86	A1	
	Show sufficient iterations to justify its accuracy to 2 decimal places, or show that there is a sign change in (0.855, 0.865)	B1	**3**

Q19.

5	(i)	Make recognizable sketch of a relevant graph, e.g. $y = 1/x$	B1	
		Sketch an appropriate second graph, e.g. $y = \ln x$, correctly and justify the given statement	B1	2
	(ii)	Consider sign of $1/x - \ln x$ at $x = 1$ and $x = 2$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	2
	(iii)	Show that the given equation is equivalent to $1/x = \ln x$, or *vice versa*	B1	1
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.76	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in (1.755, 1.765)	B1	3

Q20.

5	(i)	Obtain area of shaded segment in terms of r and α, e.g. $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\sin\alpha$	B1	
		Equate area of shaded segment to $\frac{1}{6}\pi r^2$, or equivalent	M1	
		Obtain given answer correctly	A1	3
	(ii)	Consider sign of $x - \sin x - \frac{1}{3}\pi$ at $x = \frac{1}{2}\pi$ and $x = \frac{2}{3}\pi$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	2
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.97	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.965, 1.975)	B1	3

Q21.

2	(i)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 2.29	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. (must be working to 4 d.p.) – 3 iterations are sufficient	B1	[3]
	(ii)	State equation $x = \frac{2}{3}x + \frac{4}{x^2}$, or equivalent	B1	
		Derive the exact answer α (or x) $= \sqrt[3]{12}$, or equivalent	B1	[2]

Q22.

7	(i)	Make a recognizable sketch of a relevant graph, e.g. $y = \cos x$ or $y = 2 - 2x$	B1	
		Sketch a second relevant graph and justify the given statement	B1	[2]
	(ii)	Consider sign of $\cos x - (2 - 2x)$ at $x = 0.5$ and $x = 1$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]
	(iii)	Show that the given equation is equivalent to $x = 1 - \frac{1}{2}\cos x$, or *vice versa*	B1	[1]
	(iv)	Use the iterative formula correctly at least once	Ml	
		Obtain final answer 0.58	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (0.575, 0.585)	B1	[3]

Q23.

7	(i)	*EITHER*:	Integrate $1 - e^{-x}$ obtaining $x \pm e^{-x}$	M1	
			Obtain indefinite integral $x - e^{-x}$	A1	
			Substitute limits $x = 0$, $x = p$ correctly	M1	
			Obtain answer $p + e^{-p} - 1$, or equivalent	A1	
		OR:	Integrate e^{-x} obtaining $\pm e^{-x}$	M1	
			Substitute limits $x = 0$, $x = p$ correctly	M1	
			Obtain area below curve is $1 - e^{-p}$	A1	
			Obtain answer $p + e^{-p} - 1$, or equivalent	A1	[4]
	(ii)		Show that $p + e^{-p} - 1 = 1$ is equivalent to $p = 2 - e^{-p}$ or *vice versa*	B1	[1]
	(iii)		Use the iterative formula correctly at least once	M1	
			Obtain final answer 1.84	A1	
			Show sufficient iterations to justify its accuracy to 2 d.p.	A1	[3]

Q24.

7	(i)	Use product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and express tan x in terms of x	M1	
		Obtain given answer	A1	[4]
	(ii)	Consider sign of $\tan x - \frac{2}{x}$ at $x = 1$ and $x = 1.2$, or equivalent	M1	
		Complete the argument with correct calcuations	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.08	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.075, 1.085)	A1	[3]

Q25.

6	(i)	Consider sign of $\frac{6}{x^2} - x - 1$ at $x = 1.4$ and $x = 1.6$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]
	(ii)	State $\frac{6}{x^2} = x + 1$	B1	
		Rearrange equation to given equation or *vice versa*	B1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.54	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.535, 1.545)	B1	[3]

Q26.

2	**(i)**	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.82	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.815, 1.825)	B1	[3]
	(ii)	State equation $x = \frac{7x}{8} + \frac{5}{2x^4}$, or equivalent	B1	
		Derive the exact answer α (or x) = $\sqrt[5]{20}$	B1	[2]

Q27.

6	**(i)**	Consider sign of $x^3 - 2x^2 + 5x - 3$ at $x = 0.7$ and $x = 0.8$	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]
	(ii)	Rearrange equation to given equation or *vice versa*	B1	
		State $a = 2$ and $b = 5$	B1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.74	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (0.735, 0.745)	B1	[3]

Q28.

5	**(i)**	Make a recognisable sketch of a relevant graph, e.g. $y = \sin x$ or $y = \frac{1}{x}$	B1	
		Sketch a second relevant graph and justify the given statement	B1	[2]
	(ii)	Consider sign of $\frac{1}{x} - \sin x$ at $x = 1.1$ and $x = 1.2$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.11	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.105, 1.115)	B1	[3]

Q29.

7	(i)	At any stage, state the correct derivative of $e^{\frac{1}{2}x}$	B1	
		Use product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to 3 and obtain given equation correctly	A1	[4]
	(ii)	Consider sign of $2 + 6e^{-\frac{1}{2}x} - x$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.21	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (3.205, 3.215)	B1	[3]

Q30.

5	(i)	Attempt to integrate and use limits θ and π	M1	
		Obtain $1 - \sin\theta$	A1	[2]
	(ii)	State that area of rectangle $= \theta\cos\theta$, equate area of rectangle to area of R and rearrange to given equation	B1	[1]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.56	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (0.555, 0.565)	B1	[3]

Q31.

4	(i)	State or imply correct ordinates 1.4142…, 1.1370…, 1	B1	
		Use correct formula, or equivalent, correctly with $h = \frac{\pi}{4}$ and three ordinates	M1	
		Obtain answer 1.84 with no errors seen	A1	[3]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.06	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.055, 1.065)	B1	[3]

Q32.

2 **(i)** Consider sign of $x^4 + 2x - 9$ at $x = 1.5$ and $x = 1.6$ M1
Complete the argument correctly with appropriate calculations A1 [2]
$(f(1.5) = -0.9375, f(1.6) = 0.7536)$

(ii) Rearrange $x^4 + 2x - 9 = 0$ to given equation or *vice versa* B1 [1]

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 1.56 A1
Show sufficient iterations to justify its accuracy to 2 d.p. B1 [3]

$x_0 = 1.5$	$x_0 = 1.55$	$x_0 = 1.6$
1.5874	1.5614	1.5362
1.5424	1.5556	1.5685
1.5653		1.5520
1.5536		1.5604
1 5595		1.5561
1.5565		

or show there is a sign change in the interval (1.555, 1.565)

Q33.

7 **(i)** Integrate to obtain terms $4x^2$ and $\frac{1}{2}e^x$ B1 + B1
Substitute limits correctly M1
Obtain correct equation in any form $4a^2 + \frac{1}{2}e^a - \frac{1}{2} = \frac{1}{2}$ A1
Rearrange to given answer correctly A1 [5]

(ii) Consider sign of $\sqrt{\frac{2 - e^a}{8}} - a$, or equivalent M1
Complete the argument correctly with appropriate calculations A1 [2]
$(f(0.2) = 0.112,\ f(0.3) = -0.015)$

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 0.29 A1
Show sufficient iterations to justify its accuracy to 2 d.p. B1

$x_0 = 0.2$	$x_0 = 0.25$	$x_0 = 0.3$
0.3120	0.2992	0.2851
0.2815	0.2853	0.2894
0.2905	0.2894	
0.2879		

or show there is a sign change in the interval (0.285, 0.295) [3]

Q34.

4	(i)	Sketch, showing the correct shape of each, $y = 3\ln x$ and $y = 15 - x^3$	B1	
		Indicate the correct intercepts (1,0) and (0,15)	B1	
		Indicate one real root from two correct sketches	B1	[3]
	(ii)	Consider sign of $3\ln x + x^3 - 15$ for 2.0 and 2.5 or equivalent	M1	
		Justify conclusion with correct calculations (–4.9 and 3.4 or equivalents)	A1	[2]
	(iii)	Use the iteration process correctly at least once	M1	
		Obtain final answer 2.319	A1	
		Show sufficient iterations to 5 decimal places to justify answer or show a sign change in the interval (2.3185, 2.3195)	A1	[3]

Q35.

1	(i)	Either	Square both sides to obtain linear equation	M1	
			Obtain $x = \frac{165}{30}$ or $\frac{33}{6}$ or $\frac{11}{2}$	A1	[2]
		Or	Solve linear equation in which, initially, signs of x are different	M1	
			Obtain $x + 2 = -x + 13$ or equivalent and hence $\frac{11}{2}$ or equivalent	A1	[2]
	(ii)		Apply logarithms and use power law	M1	
			Obtain $y\log 3 = \log\frac{11}{2}$ and hence $y = 1.55$	A1	[2]

Q36.

7	(i)	Integrate to obtain $ke^{3x} + mx^3$	M1	
		Apply both limits to obtain $\frac{1}{6}e^{3a} + \frac{1}{3}a^3 - \frac{1}{6} = 10$ or equivalent	A1	
		Rearrange to form involving natural logarithm	DM1	
		Obtain $a = \frac{1}{3}\ln(61 - 2a^3)$ with no errors seen (AG)	A1	[4]
	(ii)	Consider sign of $a - \frac{1}{3}\ln(61 - 2a^3)$ for 1.0 and 1.5 or equivalent	M1	
		Obtain –0.36 and 0.17 or equivalent and justify conclusion	A1	[2]
	(iii)	Use iteration process correctly at least once	M1	
		Obtain final answer 1.343	A1	
		Show sufficient iterations to 5 decimal places to justify answer or show a sign change in the interval (1.3425, 1.3435)	A1	[3]

Q37.

6	**(i)**	Use quotient rule or equivalent	M1	
		Obtain $\frac{2x\left(1+e^{3x}\right)-3x^2e^{3x}}{\left(1+e^{3x}\right)^2}$ or equivalent	A1	
		Equate first derivative to zero and attempt rearrangement to $x = \ldots$	DM1	
		Obtain $x=\frac{2}{3}\left(1+e^{-3x}\right)$ with sufficient detail and no errors seen (AG)	A1	[4]
	(ii)	Consider sign of $x-\frac{2}{3}\left(1+e^{-3x}\right)$ at 0.7 and 0.8 or equivalent	M1	
		Obtain correct values (–0.05 and 0.07 or equivalents) and conclude appropriately	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.739	A1	
		Show sufficient iterations to 5 decimal places to justify result or show a sign change in the interval (0.7385, 0.7395)	A1	[3]

Q38.

1	Either	Square both sides obtaining 3 terms on each side	M1	
		Solve 3-term quadratic equation	M1	
		Obtain $-\frac{4}{5}$ and 6	A1	[3]
	Or	Obtain value 6 from graphical method, inspection, linear equation, …	B1	
		Obtain value $-\frac{4}{5}$ similarly	B2	[3]

Q39.

6	**(i)**	Identify $x-3$ as divisor	B1	
		Divide by linear expression at least as far as x term	M1	
		Obtain quotient $x^3+3x-16$	A1	
		Obtain zero remainder with no errors in the division	A1	
		Equate quotient to zero and confirm $x=\sqrt[3]{16-3x}$ (AG)	A1	[5]
	(ii)	Use iteration process correctly at least once	M1	
		Obtain final answer 2.13	A1	
		Show sufficient iterations to 4 decimal places or show a sign change in the interval (2.125, 2.135)	A1	[3]

P3 (variant1 and 3)

Q1.

6	**(i)**	Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}r^2\sin\theta$, or equivalent, form an equation	M1	
		Obtain a correct equation in r and x and/or $x/2$ in any form	A1	
		Obtain the given equation correctly	A1	[3]
	(ii)	Consider the sign of $x-(\frac{3}{4}\pi-\sin x)$ at $x = 1.3$ and $x = 1.5$, or equivalent	M1	
		Complete the argument with correct calculations	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.38	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.375, 1.385)	A1	[3]

Q2.

6	**(i)**	Use correct quotient or product rule	M1	
		Obtain correct derivative in any form, e.g. $\frac{1}{x(x+1)}-\frac{\ln x}{(x+1)^2}$	A1	
		Equate derivative to zero and obtain the given equation correctly	A1	
		Consider the sign of $x-\frac{(x+1)}{\ln x}$ at $x = 3$ and $x = 4$, or equivalent	M1	
		Complete the argument with correct calculated values	A1	[5]
	(ii)	Use the iterative formula correctly at least once, using or reaching a value in the interval (3, 4)	M1	
		Obtain final answer 3.59	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (3.585, 3.595)	A1	[3]

Q3.

6	**(i)**	State or imply area of segment is $\frac{1}{2}r^2\theta-\frac{1}{2}r^2\sin\theta$ or $50\theta-50\sin\theta$	B1	
		Attempt to form equation from area of segment = $\frac{1}{5}$ of area of circle, or equivalent	M1	
		Confirm given result $\theta = \frac{2}{5}\pi + \sin\theta$	A1	[3]
	(ii)	Use iterative formula correctly at least once	M1	
		Obtain value for θ of 2.11	A1	
		Show sufficient iterations to justify value of θ or show sign change in interval (2.105, 2.115)	A1	
		Use correct trigonometry to find an expression for the length of AB e.g. $20\sin 1.055$ or $\sqrt{200-200\cos 2.11}$	M1	
		Hence 17.4	A1	[5]
		[2.1 → 2.1198 → 2.1097 → 2.1149 → 2.1122]		

Q4.

6	**(i)**	Make recognisable sketch of a relevant graph over the given range	B1	
		Sketch the other relevant graph and justify the given statement	B1	[2]
	(ii)	Consider the sign of $\cot x - \left(1 + x^2\right)$ at $x = 0.5$ and $x = 0.8$, or equivalent	M1	
		Complete the argument with correct calculated values	A1	[2]
	(iii)	Use the iterative formula correctly at least once with $0.5 \leq x_n \leq 0.8$	M1	
		Obtain final answer 0.62	A1	
		Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.615, 0.625)	A1	[3]

Q5.

1	State or imply $4 - 2^x = -10$ and 10	B1	
	Use correct method for solving equation of form $2^x = a$	M1	
	Obtain 3.81	A1	[3]

Q6.

10	**(i)**	Use correct identity for $\tan 2x$ and obtains $at^4 + bt^3 + ct^2 + dt = 0$, where b may be zero	M1	
		Obtain correct horizontal equation, e.g. $4t + 5t^2 - 5t^4 = 0$	A1	
		Obtain $kt(t^3 + et + f) = 0$ or equivalent	M1	
		Confirm given results $t = 0$ and $t = \sqrt[3]{t + 0.8}$	A1	[4]
	(ii)	Consider sign of $t - \sqrt[3]{t + 0.8}$ at 1.2 and 1.3 or equivalent	M1	
		Justify the given statement with correct calculations (–0.06 and 0.02)	A1	[2]
	(iii)	Use the iterative formula correctly at least once with $1.2 < t_n < 1.3$	M1	
		Obtain final answer 1.276	A1	
		Show sufficient iterations to justify answer or show there is a change of sign in interval (1.2755, 1.2765)	A1	[3]
	(iv)	Evaluate $\tan^{-1}$ (answer from part **(iii)**) to obtain at least one value	M1	
		Obtain –2.24 and 0.906	A1	
		State $-\pi$, 0 and π	B1	[3]
		[SR If A0, B0, allow B1 for any 3 roots]		

Q7.

7	(i)	Substitute for x and dx throughout the integral	M1	
		Obtain $\int 2u\cos u\,\mathrm{d}u$	A1	
		Integrate by parts and obtain answer of the form $au\sin u + b\cos u$, where $ab \neq 0$	M1	
		Obtain $2u\sin u + 2\cos u$	A1	
		Use limits $u = 0$, $u = p$ correctly and equate result to 1	M1	
		Obtain the given answer	A1	[6]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer $p = 1.25$	A1	
		Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.245, 1.255)	A1	[3]

Q8.

4	(i)	Either	State or imply non-modular equation $(4x-1)^2 = (x-3)^2$ or pair of linear equations $4x-1 = \pm(x-3)$	B1	
			Solve a three-term quadratic equation or two linear equations	M1	
			Obtain $-\frac{2}{3}$ and $\frac{4}{5}$	A1	
		Or	Obtain value $-\frac{2}{3}$ from inspection or solving linear equation	B1	
			Obtain value $\frac{4}{5}$ similarly	B2	[3]
	(ii)		State or imply at least $4^y = \frac{4}{5}$, following a positive answer from part (i)	B1√	
			Apply logarithms and use $\log a^b = b\log a$ property	M1	
			Obtain –0.161 and no other answer	A1	[3]

Q9.

6	(i)	State the correct derivatives $2e^{2x-3}$ and $2/x$	B1	
		Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$	M1	
		Obtain the given result correctly (or work *vice versa*)	A1	[3]
	(ii)	Consider the sign of $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent	M1	
		Complete the argument with correct calculated values	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.35	A1	
		Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355)	A1	[3]

Q10.

3	(i)	Use the iterative formula correctly at least once	M1	
		State final answer 2.78	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in an appropriate function in (2.775, 2.785)	A1	[3]
	(ii)	State a suitable equation, e.g. $x = \frac{3}{4}x + \frac{15}{x^3}$	B1	
		State that the exact value of a is $\sqrt[4]{60}$, or equivalent	B1	[2]

Q11.

4	(i)	Make recognisable sketch of a relevant graph over the given range	B1	
		Sketch the other relevant graph on the same diagram and justify the given statement	B1	[2]
	(ii)	Consider sign of $4x^2 - 1 - \cot x$ at $x = 0.6$ and $x = 1$, or equivalent	M1	
		Complete the argument correctly with correct calculated values	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.73	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.725, 0.735)	A1	[3]

Q12.

7	(i)	Attempt integration by parts	M1	
		Obtain $-x^{-1}\ln x + \int \frac{1}{x^2}\,dx$, $\frac{x\ln x - x}{x^2} + 2\int \frac{\ln x}{x^2}\,dx - 2\int \frac{1}{x^2}\,dx$ or equivalent	A1	
		Obtain $-x^{-1}\ln x - x^{-1}$ or equivalent	A1	
		Use limits correctly, equate to $\frac{2}{5}$ and attempt rearrangement to obtain a in terms of $\ln a$	M1	
		Obtain given answer $a = \frac{5}{3}(1 + \ln a)$ correctly	A1	[5]
	(ii)	Use valid iterative formula correctly at least once	M1	
		Obtain final answer 3.96	A1	
		Show sufficient iterations to › 4 dp to justify accuracy to 2 dp or show sign change in interval (3.955, 3.965)	A1	[3]
		[4 → 3.9772 → 3.9676 → 3.9636 → 3.9619]		
		SR: Use of $a_{n+1} = e^{\left(\frac{3}{5}a_n - 1\right)}$ to obtain 0.50 also earns 3/3.		

Q13.

5	(i)	Make recognisable sketch of a relevant graph over the given interval	B1	
		Sketch the other relevant graph and justify the given statement	B1	[2]
	(ii)	Consider the sign of sec $x - (3 - \frac{1}{2}x^2)$ at $x = 1$ and $x = 1.4$, or equivalent	M1	
		Complete the argument with correct calculated values	A1	[2]
	(iii)	Convert the given equation to sec $x = 3 - \frac{1}{2}x^2$ or work *vice versa*	B1	[1]
	(iv)	Use a correct iterative formula correctly at least once	M1	
		Obtain final answer 1.13	A1	
		Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change in the interval (1.125, 1.135)	A1	[3]
		[SR: Successive evaluation of the iterative function with $x = 1, 2, \ldots$ scores M0.]		

Q14.

5	(i)	**Either**		
		Use integration by parts and reach an expression $kx^2 \ln x \pm n\int x^2 . \frac{1}{x} dx$	M1	
		Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$ or equivalent	A1	
		Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
		Or		
		Use Integration by parts and reach an expression $kx(x\ln x - x) \pm m\int x\ln x - x dx$	M1	
		Obtain $I = (x^2 \ln x - x^2) - I + \int x dx$	A1	
		Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
		Substitute limits correctly and equate to 22, having integrated twice	DM1*	
		Rearrange and confirm given equation $a = \sqrt{\frac{87}{2\ln a - 1}}$	A1	[5]
	(ii)	Use iterative process correctly at least once	M1	
		Obtain final answer 5.86	A1	
		Show sufficient iterations to 4 d.p. to justify 5.86 or show a sign change in the interval (5.855, 5.865)	A1	
		(6 → 5.8030 → 5.8795 → 5.8491 → 5.8611 → 5.8564)		[3]

Q15.

8	(i)	Use correct product or quotient rule and use chain rule at least once	M1	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve an equation with at least two non-zero terms for real x	M1	
		Obtain answer $x = \frac{1}{\sqrt{2}}$, or exact equivalent	A1	[4]
	(ii)	State a suitable equation, e.g. $\alpha = \sqrt{(\ln(4 + 8^{\alpha}(2)))}$	B1	
		Rearrange to reach $e^{2\alpha}(e^{2\alpha} - 2) = 4 + 8\alpha^2$	B1	
		Obtain $\frac{1}{2} = e^{\alpha}(e^{\alpha} - \alpha(e^{2\alpha} - 2))\sqrt{(1 + 2(e^{2\alpha} - 2))}$, or work *vice versa*	B1	[3]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.86	A1	
		Show sufficient iterations to 4 d.p. to justify 1.86 to 2 d.p., or show there is a sign change in the interval (1.855, 1.865)	A1	[3]

Q16.

6	(i)		Find y for $x = -2$	M1	
			Obtain 0 and conclude that $\alpha = -2$	A1	[2]
	(ii)	Either	Find cubic factor by division or inspection or equivalent	M1	
			Obtain $x^3 + 2x - 8$	A1	
			Rearrange to confirm given equation $x = \sqrt[3]{8 - 2x}$	A1	
		Or	Derive cubic factor from given equation and form product with $(x - \alpha)$	M1	
			$(x+2)(x^3 + 2x - 8)$	A1	
			Obtain quartic $x^4 + 2x^3 + 2x^2 - 4x - 16\ (= 0)$	A1	
		Or	Derive cubic factor from given equation and divide the quartic by the cubic	M1	
			$(x^4 + 2x^3 + 2x^2 - 4x - 16) \div (x^3 + 2x - 8)$	A1	
			Obtain correct quotient and zero remainder	A1	[3]
	(iii)		Use the given iterative formula correctly at least once	M1	
			Obtain final answer 1.67	A1	
			Show sufficient iterations to at least 4 d.p. to justify answer 1.67 to 2 d.p. or show there is a change of sign in interval (1.665, 1.675)	A1	[3]

Q17.

6	(i)	State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$	B1	
		Use correct formula to express the area of sector ABC in terms of r and θ	M1	
		Use correct area formulae to express the area of a segment in terms of r and θ	M1	
		State a correct equation in r and θ in any form	A1	
		Obtain the given answer	A1	**[5]**
		[SR: If the complete equation is approached by adding two sectors to the shaded area above BO and OC give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle AOB or AOC, and a sector AOB or AOC.]		
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.95	A1	
		Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955)	A1	**[3]**

Q18.

5	(i)	Use integration by parts to obtain $\alpha x e^{-\frac{1}{2}x} + \int \beta e^{-\frac{1}{2}x}\,dx$	M1*	
		Obtain $-8xe^{-\frac{1}{2}x} + \int 8e^{-\frac{1}{2}x}\,dx$ or unsimplified equivalent	A1	
		Obtain $-8xe^{-\frac{1}{2}x} - 16e^{-\frac{1}{2}x}$	A1	
		Use limits correctly and equate to 9	M1(d*M)	
		Obtain given answer $p = 2\ln\left(\frac{8p+16}{7}\right)$ correctly	A1	[5]

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	(ii)	Use correct iteration formula correctly at least once	M1	
		Obtain final answer 3.77	A1	
		Show sufficient iterations to 5sf or better to justify accuracy 3.77 or show sign change in interval (3.765, 3.775)	A1	[3]
		$[3.5 \to 3.6766 \to 3.7398 \to 3.7619 \to 3.7696 \to 3.7723]$		

Q19.

8	(i)	Sketch $y = \text{cosec}\, x$ for at least $0, x, \pi$	B1	
		Sketch $y = x(\pi - x)$ for at least $0, x, \pi$	B1	
		Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y-values on graph via scales	B1	[3]
	(ii)	Use $\text{cosec}x = \frac{1}{\sin x}$ and commence rearrangement	M1	
		Obtain given equation correctly, showing sufficient detail	A1	[2]
	(iii)	(a) Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.66	A1	
		Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval $(0.655, 0.665)$	A1	[3]
		(b) Obtain 2.48	B1	[1]

Q20.

4	(i)	Consider sign of $x - 10/(e^{2x} - 1)$ at $x = 1$ and $x = 2$	M1	
		Complete the argument correctly with correct calculated values	A1	2
	(ii)	State or imply $\alpha = \frac{1}{2}\ln(1 + 10/\alpha)$	B1	
		Rearrange this as $\alpha = 10/(e^{2\alpha} - 1)$ or work *vice versa*	B1	2
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.14	A1	
		Show sufficient iterations to 4 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	A1	3

Q21.

6	(i)	Integrate and reach $bx\ln 2x - c\int x.\frac{1}{x}\,dx$, or equivalent	M1*	
		Obtain $x\ln 2x - \int x.\frac{1}{x}\,dx$, or equivalent	A1	
		Obtain integral $x\ln 2x - x$, or equivalent	A1	
		Substitute limits correctly and equate to 1, having integrated twice	M1(dep*)	
		Obtain a correct equation in any form, e.g. $a\ln 2a - a + 1 - \ln 2 = 1$	A1	
		Obtain the given answer	A1	[6]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.94	A1	
		Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign change in the interval (1.935, 1.945).	A1	[3]

Q22.

9	**(i)**	Sketch increasing curve with correct curvature passing through origin, for $x \geq 0$	B1	
		Recognisable sketch of $y = 40 - x^3$, with equation stated, for $x > 0$	B1	
		Indicate in some way the one intersection, dependent on both curves being roughly correct and both existing for some $x < 0$	B1	[3]
	(ii)	Consider signs of $x^3 + \ln(x+1) - 40$ at 3 and 4 or equivalent or compare values of relevant expressions for $x = 3$ and $x = 4$	M1	
		Complete argument correctly with correct calculations (–11.6 and 25.6)	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.377	A1	
		Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (3.3765, 3.3775)	A1	[3]
	(iv)	Attempt value of $\ln(x+1)$	M1	
		Obtain 1.48	A1	[2]