**These are P2 questions(all variants) as the syllabus is same as P3 :)**

## Q1.

6 The equation of a curve is $y = \dfrac{1}{1 + \tan x}$.

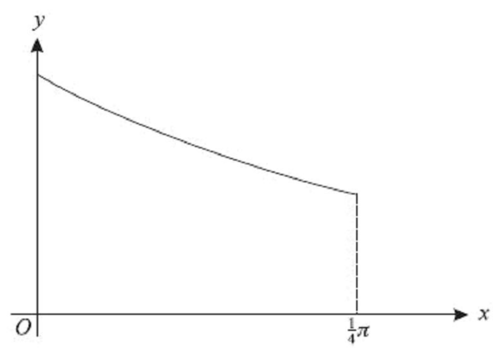
**(i)** Show, by differentiation, that the gradient of the curve is always negative. [4]

**(ii)** Use the trapezium rule with 2 intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 + \tan x}\, dx,$$

giving your answer correct to 2 significant figures. [3]

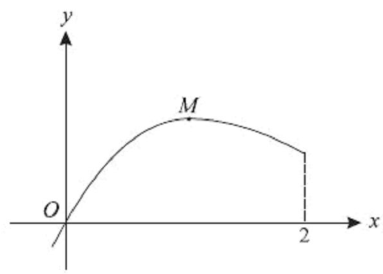
**(iii)**




The diagram shows a sketch of the curve for $0 \leqslant x \leqslant \frac{1}{4}\pi$. State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

## Q2.

5




The diagram shows the part of the curve $y = xe^{-x}$ for $0 \leqslant x \leqslant 2$, and its maximum point $M$.

**(i)** Find the $x$-coordinate of $M$. [4]

**(ii)** Use the trapezium rule with two intervals to estimate the value of

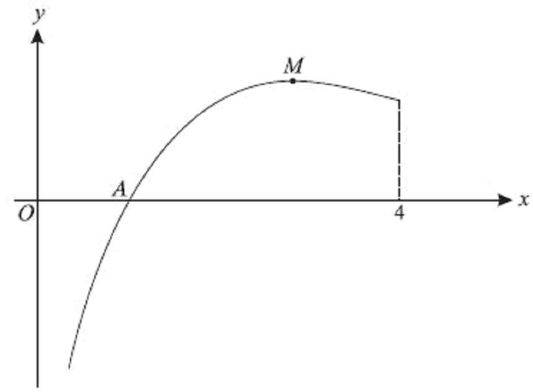
$$\int_0^2 xe^{-x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

**(iii)** State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

**Q3.**

6




The diagram shows the part of the curve $y = \frac{\ln x}{x}$ for $0 < x \leqslant 4$. The curve cuts the $x$-axis at $A$ and its maximum point is $M$.

**(i)** Write down the coordinates of $A$. [1]

**(ii)** Show that the $x$-coordinate of $M$ is e, and write down the $y$-coordinate of $M$ in terms of e. [5]

**(iii)** Use the trapezium rule with three intervals to estimate the value of

$$\int_1^4 \frac{\ln x}{x}\,dx,$$

correct to 2 decimal places. [3]

**(iv)** State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(iii)**. [1]

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## Q4.

7 **(i)** Differentiate $\ln(2x + 3)$. [2]

**(ii)** Hence, or otherwise, show that

$$\int_{-1}^{3} \frac{1}{2x + 3}\,dx = \ln 3.$$ [3]

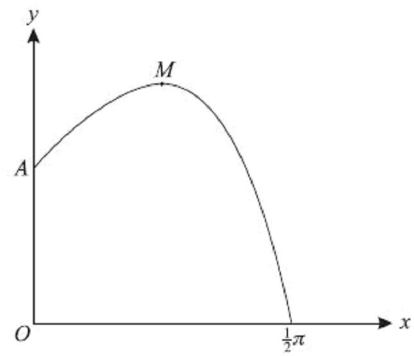
**(iii)** Find the quotient and remainder when $4x^2 + 8x$ is divided by $2x + 3$. [3]

**(iv)** Hence show that

$$\int_{-1}^{3} \frac{4x^2 + 8x}{2x + 3}\,dx = 12 - 3\ln 3.$$ [3]

## Q5.

7




The diagram shows the part of the curve $y = e^x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curve meets the $y$-axis at the point $A$. The point $M$ is a maximum point.

**(i)** Write down the coordinates of $A$. [1]

**(ii)** Find the $x$-coordinate of $M$. [4]

**(iii)** Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} e^x \cos x \, dx,$$

giving your answer correct to 2 decimal places. [3]

**(iv)** State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(iii)**. [1]

**Q6.**

**8** **(a)** Find the equation of the tangent to the curve $y = \ln(3x - 2)$ at the point where $x = 1$. [4]

**(b)** **(i)** Find the value of the constant $A$ such that

$$\frac{6x}{3x-2} \equiv 2 + \frac{A}{3x-2}.$$ [2]

**(ii)** Hence show that $\int_2^6 \frac{6x}{3x-2} \, dx = 8 + \frac{8}{3}\ln 2.$ [5]

**Q7.**

**8** (i) By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [3]

(ii) By expressing $\cot^2 x$ in terms of $\operatorname{cosec}^2 x$ and using the result of part (i), show that

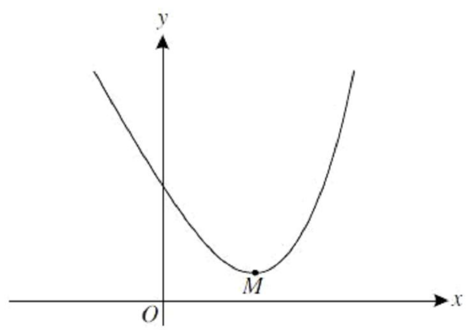
$$\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \cot^2 x \, dx = 1 - \tfrac{1}{4}\pi. \quad [4]$$

(iii) Express $\cos 2x$ in terms of $\sin^2 x$ and hence show that $\frac{1}{1 - \cos 2x}$ can be expressed as $\frac{1}{2}\operatorname{cosec}^2 x$. Hence, using the result of part (i), find

$$\int \frac{1}{1 - \cos 2x} \, dx. \quad [3]$$

**Q8.**

**5**




The diagram shows the curve $y = 4e^{\frac{1}{2}x} - 6x + 3$ and its minimum point $M$.

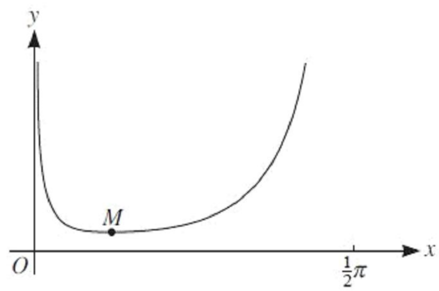
(i) Show that the $x$-coordinate of $M$ can be written in the form $\ln a$, where the value of $a$ is to be stated. [5]

(ii) Find the exact value of the area of the region enclosed by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. [4]

**Q9.**

7 **(a)** Find the exact area of the region bounded by the curve $y = 1 + e^{2x-1}$, the $x$-axis and the lines $x = \frac{1}{2}$ and $x = 2$. [4]

**(b)**




The diagram shows the curve $y = \dfrac{e^{2x}}{\sin 2x}$ for $0 < x < \frac{1}{2}\pi$, and its minimum point $M$. Find the exact $x$-coordinate of $M$. [5]

## Q10.

7 **(i)** By differentiating $\dfrac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\dfrac{dy}{dx} = -\operatorname{cosec}^2 x$. [3]

**(ii)** Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \operatorname{cosec}^2 x \, dx = \sqrt{3}$. [2]

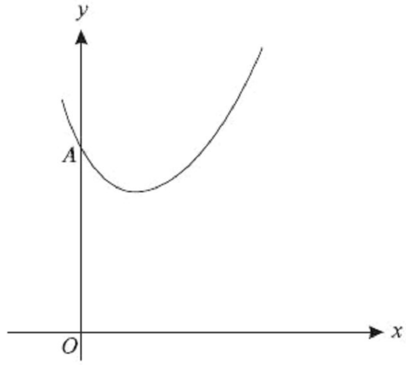
By using appropriate trigonometrical identities, find the exact value of

**(iii)** $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \cot^2 x \, dx$, [3]

**(iv)** $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{1 - \cos 2x} \, dx$. [3]

## Q11.

7




The diagram shows the curve $y = 2e^x + 3e^{-2x}$. The curve cuts the $y$-axis at $A$.

**(i)** Write down the coordinates of $A$. [1]

**(ii)** Find the equation of the tangent to the curve at $A$, and state the coordinates of the point where this tangent meets the $x$-axis. [6]

**(iii)** Calculate the area of the region bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = 1$, giving your answer correct to 2 significant figures. [4]

## Q12.

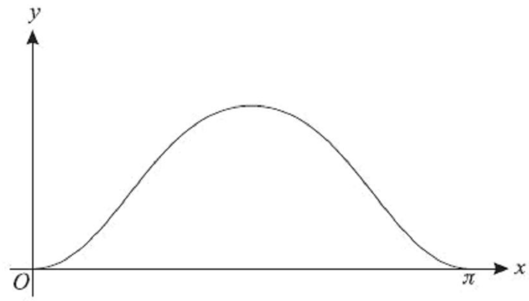
**6** A curve is such that $\frac{dy}{dx} = e^{2x} - 2e^{-x}$. The point (0, 1) lies on the curve.

**(i)** Find the equation of the curve. [4]

**(ii)** The curve has one stationary point. Find the $x$-coordinate of this point and determine whether it is a maximum or a minimum point. [5]

## Q13.

7




The diagram shows the part of the curve $y = \sin^2 x$ for $0 \leqslant x \leqslant \pi$.

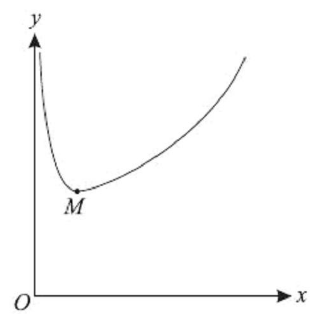
**(i)** Show that $\frac{dy}{dx} = \sin 2x$. [2]

**(ii)** Hence find the $x$-coordinates of the points on the curve at which the gradient of the curve is 0.5. [3]

**(iii)** By expressing $\sin^2 x$ in terms of $\cos 2x$, find the area of the region bounded by the curve and the $x$-axis between 0 and $\pi$. [5]

**Q14.**

6




The diagram shows the part of the curve $y = \frac{e^{2x}}{x}$ for $x > 0$, and its minimum point $M$.

**(i)** Find the coordinates of $M$. [5]

**(ii)** Use the trapezium rule with 2 intervals to estimate the value of

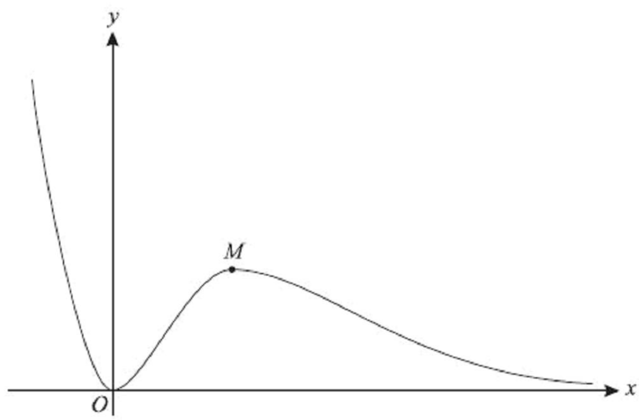
$$\int_1^2 \frac{e^{2x}}{x}\, dx,$$

giving your answer correct to 1 decimal place. [3]

**(iii)** State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

**Q15.**

**8**




The diagram shows the curve $y = x^2e^{-x}$ and its maximum point $M$.

**(i)** Find the $x$-coordinate of $M$. [4]

**(ii)** Show that the tangent to the curve at the point where $x = 1$ passes through the origin. [3]

**(iii)** Use the trapezium rule, with two intervals, to estimate the value of

$$\int_1^3 x^2e^{-x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

## Q16.

**8** **(i)** **(a)** Prove the identity

$$\sec^2 x + \sec x \tan x \equiv \frac{1 + \sin x}{\cos^2 x}.$$

**(b)** Hence prove that

$$\sec^2 x + \sec x \tan x \equiv \frac{1}{1 - \sin x}.$$

[3]

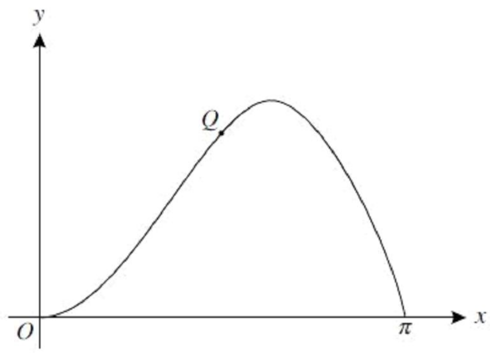
**(ii)** By differentiating $\frac{1}{\cos x}$, show that if $y = \sec x$ then $\frac{dy}{dx} = \sec x \tan x$. [3]

**(iii)** Using the results of parts **(i)** and **(ii)**, find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 - \sin x}\,dx.$$ [3]

## Q17.

8




The diagram shows the curve $y = x\sin x$, for $0 \leqslant x \leqslant \pi$. The point $Q\left(\frac{1}{2}\pi, \frac{1}{2}\pi\right)$ lies on the curve.

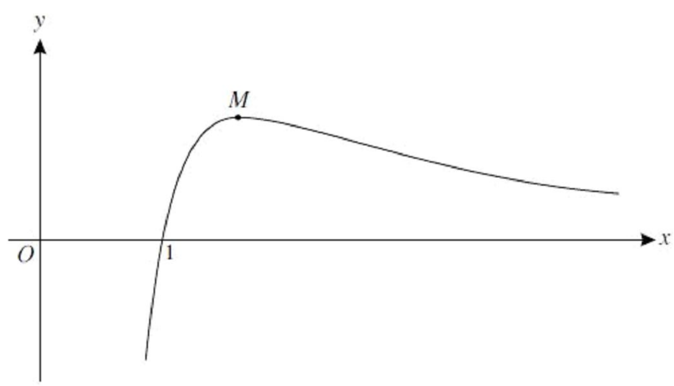
(i) Show that the normal to the curve at $Q$ passes through the point $(\pi, 0)$. [5]

(ii) Find $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x - x\cos x)$. [2]

(iii) Hence evaluate $\int_0^{\frac{1}{2}\pi} x\sin x\,\mathrm{d}x$. [3]

**Q18.**

7




The diagram shows the curve $y = \frac{\ln x}{x^2}$ and its maximum point $M$.

(i) Find the exact coordinates of $M$. [5]

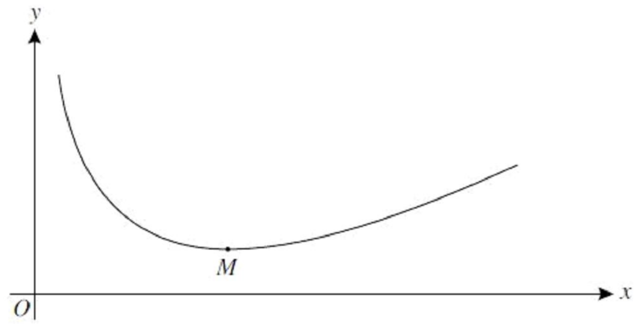
(ii) Use the trapezium rule with three intervals to estimate the value of

$$\int_1^4 \frac{\ln x}{x^2}\,\mathrm{d}x,$$

giving your answer correct to 2 decimal places. [3]

**Q19.**

3




The diagram shows the curve $y = x - 2\ln x$ and its minimum point $M$.

**(i)** Find the $x$-coordinate of $M$. [2]

**(ii)** Use the trapezium rule with three intervals to estimate the value of

$$\int_2^5 (x - 2\ln x)\,dx,$$

giving your answer correct to 2 decimal places. [3]

**(iii)** State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

## Q20.

8 **(i)** By differentiating $\frac{1}{\cos\theta}$, show that if $y = \sec\theta$ then $\frac{dy}{d\theta} = \tan\theta\sec\theta$. [3]

**(ii)** Hence show that

$$\frac{d^2y}{d\theta^2} = a\sec^3\theta + b\sec\theta,$$

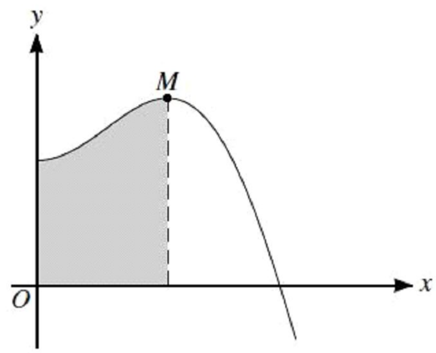
giving the values of $a$ and $b$. [4]

**(iii)** Find the exact value of

$$\int_0^{\frac{1}{4}\pi} (1 + \tan^2\theta - 3\sec\theta\tan\theta)\,d\theta.$$ [5]

## Q21.

5




The diagram shows part of the curve

$$y = 2\cos x - \cos 2x$$

and its maximum point $M$. The shaded region is bounded by the curve, the axes and the line through $M$ parallel to the $y$-axis.

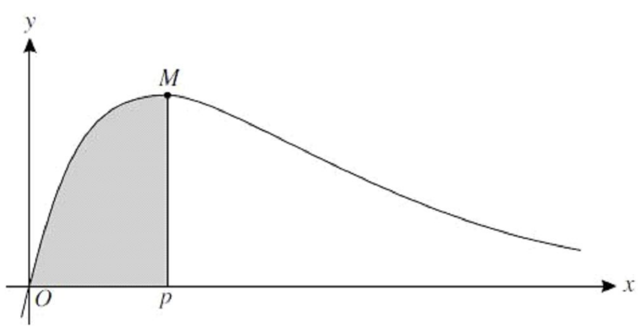
(i) Find the exact value of the $x$-coordinate of $M$. [4]

(ii) Find the exact value of the area of the shaded region. [4]

## P3 (variant1 and 3)

## Q1.

5



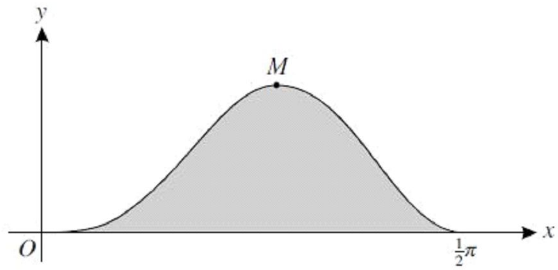

The diagram shows the curve $y = e^{-x} - e^{-2x}$ and its maximum point $M$. The $x$-coordinate of $M$ is denoted by $p$.

(i) Find the exact value of $p$. [4]

(ii) Show that the area of the shaded region bounded by the curve, the $x$-axis and the line $x = p$ is equal to $\frac{1}{8}$. [4]

## Q2.

8



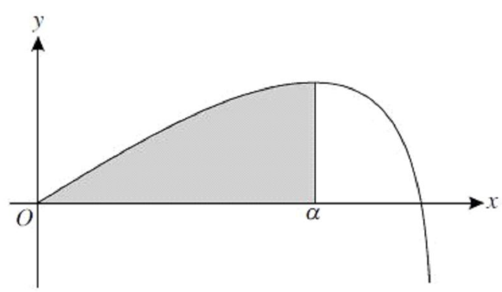

The diagram shows the curve $y = 5\sin^3 x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point $M$.

**(i)** Find the $x$-coordinate of $M$. [5]

**(ii)** Using the substitution $u = \cos x$, find by integration the area of the shaded region bounded by the curve and the $x$-axis. [5]

**Q3.**

5




The diagram shows the curve

$$y = 8\sin\tfrac{1}{2}x - \tan\tfrac{1}{2}x$$

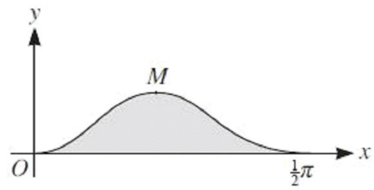
for $0 \leqslant x < \pi$. The $x$-coordinate of the maximum point is $\alpha$ and the shaded region is enclosed by the curve and the lines $x = \alpha$ and $y = 0$.

**(i)** Show that $\alpha = \frac{2}{3}\pi$. [3]

**(ii)** Find the exact value of the area of the shaded region. [4]

**Q4.**

9



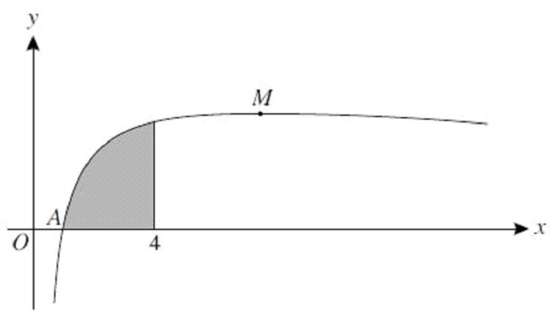

The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point $M$.

**(i)** Find the $x$-coordinate of $M$. [6]

**(ii)** Using the substitution $u = \sin x$, find by integration the area of the shaded region bounded by the curve and the $x$-axis. [4]

## Q5.

9




The diagram shows the curve $y = \frac{\ln x}{\sqrt{x}}$ and its maximum point $M$. The curve cuts the $x$-axis at the point $A$.

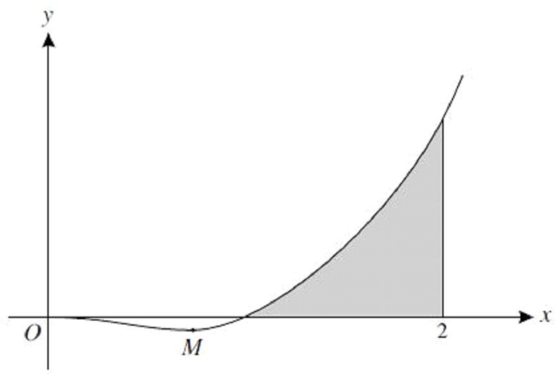
**(i)** State the coordinates of $A$. [1]

**(ii)** Find the exact value of the $x$-coordinate of $M$. [4]

**(iii)** Using integration by parts, show that the area of the shaded region bounded by the curve, the $x$-axis and the line $x = 4$ is equal to $8 \ln 2 - 4$. [5]

## Q6.

**9**




The diagram shows the curve $y = x^3 \ln x$ and its minimum point $M$.

**(i)** Find the exact coordinates of $M$. [5]

**(ii)** Find the exact area of the shaded region bounded by the curve, the $x$-axis and the line $x = 2$. [5]

## Q7.

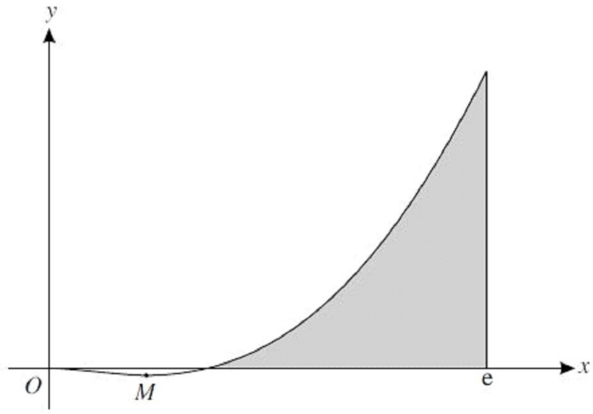
**4** It is given that $f(x) = 4\cos^2 3x$.

**(i)** Find the exact value of $f'(\frac{1}{9}\pi)$. [3]

**(ii)** Find $\int f(x)\,dx$. [3]

## Q8.

9




The diagram shows the curve $y = x^2 \ln x$ and its minimum point $M$.

**(i)** Find the exact values of the coordinates of $M$. [5]

**(ii)** Find the exact value of the area of the shaded region bounded by the curve, the $x$-axis and the line $x = \mathrm{e}$. [5]

**Q9.**

5 **(i)** By differentiating $\dfrac{1}{\cos x}$, show that if $y = \sec x$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sec x \tan x$. [2]

**(ii)** Show that $\dfrac{1}{\sec x - \tan x} \equiv \sec x + \tan x$. [1]

**(iii)** Deduce that $\dfrac{1}{(\sec x - \tan x)^2} \equiv 2\sec^2 x - 1 + 2\sec x \tan x$. [2]

**(iv)** Hence show that $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1}{(\sec x - \tan x)^2}\,\mathrm{d}x = \tfrac{1}{4}(8\sqrt{2} - \pi)$. [3]

**Q10.**

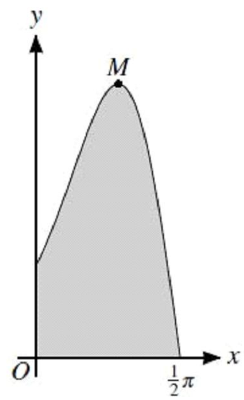
5 The expression $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 3x\mathrm{e}^{-2x}$.

**(i)** Find the exact value of $\mathrm{f}'\left(-\tfrac{1}{2}\right)$. [3]

**(ii)** Find the exact value of $\displaystyle\int_{-\frac{1}{2}}^{0} \mathrm{f}(x)\,\mathrm{d}x$. [5]

**Q11.**

**9**




The diagram shows the curve $y = e^{2\sin x}\cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point $M$.

**(i)** Using the substitution $u = \sin x$, find the exact value of the area of the shaded region bounded by the curve and the axes. [5]

**(ii)** Find the $x$-coordinate of $M$, giving your answer correct to 3 decimal places. [6]