Q1.

7 (i) State $\frac{dx}{d\theta} = 2 - 2\cos 2\theta$ or $\frac{dy}{d\theta} = 2\sin 2\theta$	B1

Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1

Obtain answer $\frac{dy}{dx} = \frac{2\sin 2\theta}{2 - 2\cos 2\theta}$ or equivalent	A1

Make relevant use of sin 2*A* and cos 2*A* formulae	(indep.) M1

Obtain given answer correctly	A1

[5]

(ii) Substitute $\theta = \frac{1}{4}\pi$ in $\frac{dy}{dx}$ and both parametric equations	M1

Obtain $\frac{dy}{dx} = 1$, $x = \frac{1}{2}\pi - 1$, $y = 2$	A1

Obtain equation $y = x + 1.43$, or any exact equivalent	A1√

[3]

(iii) State or imply that tangent is horizontal when $\theta = \frac{1}{2}\pi$ or $\frac{3}{2}\pi$	B1

Obtain a correct pair of *x*, *y* or *x*- or *y*-coordinates	B1

State correct answers $(\pi, 3)$ and $(3\pi, 3)$	B1

[3]

Q2.

6 (i) State that $\frac{dx}{dt} = 2 + \frac{1}{t}$ or $\frac{dy}{dt} = 1 - \frac{4}{t^2}$, or equivalent	B1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1

Obtain the given answer	A1	3

(ii) Substitute t = 1 in $\frac{dy}{dx}$ and both parametric equations	M1

Obtain $\frac{dy}{dx} = -1$ and coordinates (2, 5)	A1

State equation of tangent in any correct horizontal form e.g. x + y = 7	A1√	3

(iii) Equate $\frac{dy}{dx}$ to zero and solve for t	M1

Obtain answer t = 2	A1

Obtain answer y = 4	A1

Show by any method (but not via $\frac{d}{dt}(y')$) that this is a minimum point	A1	4

Q3.

5	(i)	Differentiate using chain or quotient rule	M1	
		Obtain derivative in any correct form	A1	
		Obtain given answer correctly	A1	3
	(ii)	State $\frac{dx}{d\theta} = \sec^2\theta$, or equivalent	B1	
		Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
		Obtain given answer correctly	A1	3

(iii) State that $\theta = \frac{\pi}{6}$	B1	
Obtain x-coordinate $1 + \frac{1}{\sqrt{3}}$, or equivalent	B1	
Obtain y-coordinate $\frac{2}{\sqrt{3}}$, or equivalent	B1	3

Q4.

3	State correct derivative $1 - 2\sin x$	B1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = \frac{1}{6}\pi$	A1	
	Carry out an appropriate method for determining the nature of a stationary point	M1	
	Show that $x = \frac{1}{6}\pi$ is a maximum with no errors seen	A1	
	Obtain second answer $x = \frac{5}{6}\pi$ in range	A1✓	
	Show this is a minimum point [f.t. is on the incorrect derivative $1 + 2\sin x$.]	A1✓	7

Q5.

5	(i)	State $2y\frac{dy}{dx}$ as the derivative of y^2	B1	
		State $2y + 2x\frac{dy}{dx}$ or equivalent, as derivative of $2xy$	B1	
		Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	M1	
		Obtain given relation $y = -3x$ correctly [The M1 is dependent on at least one B1 being earned earlier.]	A1	4
	(ii)	Carry out complete method for finding x^2 or y^2	M1	
		Obtain $x^2 = 1$ or $y^2 = 9$	A1	
		Obtain point $(1, -3)$	A1	
		Obtain second point $(-1, 3)$	A1	4

Q6.

3	(i)	State $\frac{dx}{dt} = 3 + \frac{1}{t-1}$ or $\frac{dy}{dt} = 2t$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain $\frac{dy}{dx}$ in any correct form, e.g. $\frac{2t(t-1)}{3t-2}$	A1	[3]
	(ii)	Equate derivative to 1 and solve for t	M1	
		Obtain roots 2 and $\frac{1}{2}$	A1	
		State or imply that only $t = 2$ is admissible c.w.o.	A1	
		Obtain coordinates (6, 5)	A1	[4]

Q7.

6	(i)	Use product rule	M1*	
		Obtain correct derivative in any form, e.g. $(x-1)e^x$	A1	
		Equate derivative to zero and solve for x	M1* (dep)	
		Obtain $x = 1$	A1	
		Obtain $y = -e$	A1	[5]
	(ii)	Carry out a method for determining the nature of a stationary point	M1	
		Show that the point is a minimum point, with no errors seen	A1	[2]

Q8.

7	(i)	State $2y\frac{dy}{dx}$ as derivative of y^2, or equivalent	B1	
		State $4y + 4x\frac{dy}{dx}$ as derivative of $4xy$, or equivalent	B1	
		Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	
		Obtain given answer correctly	A1	[4]
		[The M1 is dependent on at least one of the B marks being obtained.]		
	(ii)	State or imply that the coordinates satisfy $2y - x = 0$	B1	
		Obtain an equation in x^2 (or y^2)	M1	
		Solve and obtain $x^2 = 4$ (or $y^2 = 1$)	A1	
		State answer (2, 1)	A1	
		State answer (–2, –1)	A1	[5]

Q9.

4	State $\frac{dx}{d\theta} = 4\cos\theta$	B1	
	State $\frac{dy}{d\theta} = 4\sin 2\theta$, or equivalent	B1	
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
	Obtain $\frac{dy}{dx}$ in any correct form, e.g. $\frac{\sin 2\theta}{\cos\theta}$	A1	
	Simplify and obtain answer $2\sin\theta$	A1√	[5]
	[The f.t. is on gradients of the form $k\sin 2\theta / \cos\theta$, or equivalent.]		

Q10.

6	(i)	State $2xy + x^2\frac{dy}{dx}$ as derivative of x^2y	B1	
		State $2y\frac{dy}{dx}$ as derivative of y^2	B1	
		Equate derivatives of LHS and RHS, and solve for $\frac{dy}{dx}$	M1	
		Obtain given answer	A1	[4]
	(ii)	Substitute and obtain gradient $\frac{2}{5}$, or equivalent	B1	
		Form equation of tangent at the given point (1, 2)	M1	
		Obtain answer $2x - 5y + 8 = 0$, or equivalent	A1	[3]
		[The M1 is dependent on at least one of the B marks being obtained.]		

Q11.

5	(i)	Use product rule	M1	
		Obtain correct derivative in any form	A1	
		Show that derivative is equal to zero when $x = 3$	A1	[3]
	(ii)	Substitute $x = 1$ into gradient function, obtaining $2e^{-1}$ or equivalent	M1	
		State or imply required y-coordinate is e^{-1}	B1	
		Form equation of line through $(1, e^{-1})$ with gradient found (NOT the normal)	M1	
		Obtain equation in any correct form	A1	[4]

Q12.

2	State $\frac{dx}{dt} = 3 + 2\cos 2t$ or $\frac{dy}{dt} = -4\sin 2t$ (or both)	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain or imply $\frac{-4\sin 2t}{3 + 2\cos 2t}$	A1	
	Substitute $\frac{1}{6}\pi$ to obtain $-\frac{1}{2}\sqrt{3}$ or exact equivalent	A1	[4]

Q13.

5	(i)	Differentiate $\ln(x - 3)$ to obtain $\frac{1}{x-3}$	B1	
		Attempt to use product rule	M1	
		Obtain $\ln(x-3) + \frac{x}{x-3}$ or equivalent	A1	
		Substitute 4 to obtain 4	A1	[4]
	(ii)	Use correct quotient or product rule	M1	
		Obtain correct derivative in any form, e.g. $\frac{(x+1)-(x-1)}{(x+1)^2}$	A1	
		Substitute 4 to obtain $\frac{2}{25}$	A1	[3]

Q14.

5	Obtain $4y\frac{dy}{dx}$ as derivative of $2y^2$	B1	
	Differentiate LHS term by term to obtain expression including at least one $\frac{dy}{dx}$	M1	
	Obtain $2x + 4y\frac{dy}{dx} + 5 + 6\frac{dy}{dx}$	A1	
	Substitute 2 and -1 to attempt value of $\frac{dy}{dx}$	M1	
	Obtain $-\frac{9}{2}$	A1	
	Obtain equation $9x + 2y - 16 = 0$ or equivalent of required form	A1	[6]

Q15.

6	(i)	Attempt differentiation using product rule	M1	
		Obtain $8x\ln x + 4x$ (a.c.f.)	A1	
		Equate first derivative to zero and attempt solution	M1	
		Obtain 0.607	A1	
		Obtain –0.736 following their x-coordinate	A1√	[5]
	(ii)	Use an appropriate method for determining nature of stationary point	M1	
		Conclude point is a minimum (with no errors seen, second derivative = 8)	A1	[2]

Q16.

5	(i)	State $\frac{dx}{dt} = \frac{1}{t+1}$	B1	
		State $\frac{dy}{dt} = 2e^{2t} + 2$	B1	
		Attempt expression for $\frac{dy}{dx}$	M1	
		Obtain $\frac{dy}{dx} = (2e^{2t} + 2)(t+1)$ or equivalent	A1	[4]
	(ii)	Substitute $t = 0$ and attempt gradient of normal	M1	
		Obtain $-\frac{1}{4}$ following their expression for $\frac{dy}{dx}$	A1√	
		Attempt to find equation of normal through point (0, 1)	M1	
		Obtain $x + 4y - 4 = 0$	A1	[4]

Q17.

5	(i)	Use product rule to differentiate y	M1	
		Obtain correct derivative in any form	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain given answer correctly	A1	[4]
	(ii)	Substitute $t = 0$ in $\frac{dy}{dx}$ and both parametric equations	B1	
		Obtain $\frac{dy}{dx} = 2$ and coordinates (1, 0)	B1	
		Form equation of the normal at their point, using negative reciprocal of their $\frac{dy}{dx}$	M1	
		State correct equation of normal $y = -\frac{1}{2}x + \frac{1}{2}$ or equivalent	A1	[4]

Q18.

5 (i)	State $3\frac{dy}{dx}$ as derivative of $3y$, or equivalent	B1	
	State $4xy + 2x^2\frac{dy}{dx}$ as a derivative of $2x^2y$, or equivalent	B1	
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain given answer correctly	A1	[4]
(ii)	Substitute $x = 2$ into given equation and solve for y	M1	
	Obtain gradient $= \frac{12}{5}$ correctly	A1	
	Form equation of the normal at their point, using negative recip of their $\frac{dy}{dx}$	M1	
	State correct equation of normal $5x + 12y + 2 = 0$ or equivalent	A1	[4]

Q19.

7 (i)	State $6y\frac{dy}{dx}$ as the derivative of $3y^2$	B1	
	State $\pm 2x\frac{dy}{dx} \pm 2y$ as the derivative of $-2xy$ (allow any combination of signs here)	B1	
	Equate attempted derivative of LHS to 0 (or 10) and solve for $\frac{dy}{dx}$	M1	
	Obtain the given answer correctly [The M1 is dependent on at least one of the B marks being earned.]	A1	4
(ii)	State or imply the points lie on $y - 2x = 0$ or/ $(y-2x)/(3y-x) = 0$	B1	⊙
	Carry out complete method for finding one coordinate of a point of intersection of $y = kx$ with the given curve	M1	
	Obtain $10x^2 = 10$ or $2\frac{1}{2}y^2 = 10$ or 2-term equivalent	A1	
	Obtain one correct point e.g. $(1, 2)$ or/ 2 values of x (or y)	A1	⊙
	Obtain a second correct point e.g. $(-1, -2)$	A1 √	5 ⊙

Q20.

6 (i)	State A is (4, 0)	B1	
	State B is (0, 4)	B1	
			[2]
(ii)	Use the product rule to obtain the first derivative	M1(dep)	
	Obtain derivative $(4-x)e^x - e^x$, or equivalent	A1	
	Equate derivative to zero and solve for x	M1 (dep)	
	Obtain answer $x = 3$ only	A1	
			[4]
(iii)	Attempt to form an equation in p e.g. by equating gradients of OP and the tangent at P, or by substituting (0, 0) in the equation of the tangent at P	M1	
	Obtain equation in any correct form e.g. $\frac{4-p}{p} = 3-p$	A1	
	Obtain 3-term quadratic $p^2 - 4p + 4 = 0$, or equivalent	A1	
	Attempt to solve a quadratic equation in p	M1	
	Obtain answer $p = 2$ only	A1	
			[5]

Q21.

5 (i)	Use the product rule to obtain the first derivative (must involve 2 terms)	M1	
	Obtain derivative $2x \ln x + x^2 \frac{1}{x}$ or equivalent	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = e^{-0.5}$ or $\frac{1}{\sqrt{e}}$ or equivalent (e.g. 0.61)	A1	4
(ii)	Determine nature of stationary point using correct second derivative $(3 + 2\ln x)$ or correct first derivative or equation of the curve (3 y-values, central one $y(\exp(-0.5))$)	M1	
	Show point is a minimum completely correctly	A1	2

Q22.

4 (i)	State $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	
	State $9y + 9x\frac{dy}{dx}$ as derivative of $9xy$	B1	
	Express $\frac{dy}{dx}$ in terms of x and y	M1	
	Obtain given answer correctly	A1	4
	[The M1 is conditional on at least one B mark being obtained.]		
(ii)	Obtain gradient at (2, 4) in any correct unsimplified form	B1	
	Form the equation of the tangent at (2, 4)	M1	
	Obtain answer $5y - 4x = 12$, or equivalent	A1	3

Q23.

4	State derivative $2 - \sec^2 x$, or equivalent	B1	
	Equate derivative to zero and solve for x	M1	
	Obtain $x = \frac{1}{4}\pi$, or 0.785 (± 45° gains A1)	A1	
	Obtain $x = -\frac{1}{4}\pi$, (allow negative of first solution)	A1√	
	Obtain corresponding y-values $\frac{1}{2}\pi - 1$ and $-\frac{1}{2}\pi + 1$, ± 0.571	A1	[5]

Q24.

6	At any stage, state the correct derivative of $e^{-\frac{1}{2}x}$ or $e^{\frac{1}{2}x}$	B1	
	Use product or quotient rule	M1	
	Obtain correct first derivative in any form	A1	
	Obtain correct second derivative in any form	B1 √	
	Equate second derivative to zero and solve for x	M1	
	Obtain $x = 4$	A1	
	Obtain $y = 4e^{-2}$, or equivalent	A1	[7]

Q25.

6	**(i)**	Use product rule	M1*	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve for x	M1(dep*)	
		Obtain $x = 1/e$, or exact equivalent	A1	
		Obtain $y = -1/e$, or exact equivalent	A1	[5]
	(ii)	Carry out complete method for determining the nature of a stationary point	M1	
		Show that at $x = 1/e$ there is a minimum point, with no errors seen	A1	[2]

Q26.

8	**(i)**	*EITHER*:	Substitute $x = 1$ and attempt to solve 3-term quadratic in y	M1	
			Obtain answers (1, 1) and (1, –3)	A1	
		OR:	State answers (1, 1) and (1, –3)	B1 + B1	[2]
	(ii)	State $2y\frac{dy}{dx}$ as derivative of y^2		B1	
		State $2y + 2x\frac{dy}{dx}$ as derivative of $2xy$		B1	
		Substitute for x and y, and solve for $\frac{dy}{dx}$		M1	
		Obtain $\frac{dy}{dx} = 0$ when $x = 1$ and $y = 1$		A1	
		Obtain $\frac{dy}{dx} = -2$ when $x = 1$ and $y = -3$		A1√	
		Form the equation of the tangent at (1, –3)		M1	
		Obtain answer $2x + y + 1 = 0$		A1	[7]

Q27.

4 (i) State $\frac{dx}{dt} = e^{-t}$ or $\frac{dy}{dt} = e^{t} - e^{-t}$ B1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

Obtain given answer correctly A1 [3]

(ii) Substitute $\frac{dy}{dx} = 2$ and use correct method for solving an equation of the form $e^{2t} = a$, where $a > 0$ M1

Obtain answer $t = \frac{1}{2} \ln 3$, or equivalent A1 [2]

Q28.

4 (i) State $\frac{dx}{dt} = \frac{1}{t-2}$ or $\frac{dy}{dt} = 1 - 9t^{-2}$ B1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

Obtain given answer correctly A1 [3]

(ii) Equate derivative to zero and solve for t M1

State or imply that $t = 3$ is admissible c.w.o., and note $t = -3$, 2 cases A1

Obtain coordinates (1, 6) and no others A1 [3]

Q29.

8 (i) State $2y\frac{dy}{dx}$ as derivative of y^2, or equivalent B1

State $2y + 2x\frac{dy}{dx}$ as derivative of $2xy$, or equivalent B1

Substitute $x = -2$ and $y = 2$ and evaluate $\frac{dy}{dx}$ M1

Obtain zero correctly and make correct conclusion A1 [4]

(ii) Substitute $x = -2$ into given equation and solve M1

Obtain $y = -6$ correctly A1

Obtain $\frac{dy}{dx} = 2$ correctly B1

Form the equation of the tangent at $(-2, -6)$ M1

Obtain answer $y = 2x - 2$ A1 [5]

Q30.

3	Obtain derivative of the form $k\sec^2 2x$, where $k = 1$ or $k = \frac{1}{2}$	M1	
	Obtain correct derivative $\sec^2 2x$	A1	
	Use correct method for solving $\sec^2 2x = 4$	M1	
	Obtain answer $x = \frac{1}{6}\pi$ (or 0.524 radians)	A1	
	Obtain answer $x = \frac{1}{3}\pi$ (or 1.05 radians) and no others in range	A1	[5]

Q31.

7	**(i)**	Use product rule to differentiate y	M1	
		Obtain correct derivative in any form in t for y	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain given answer correctly	A1	[4]
	(ii)	State $t = 0$	M1	
		State that $\frac{dy}{dx} = 0$ and make correct conclusion	A1	[2]
	(iii)	Substitute $t = -2$ into equation for x or y	M1	
		Obtain $(e^{-6}, 4e^{-2} + 3)$	A1	[2]

Q32.

6	**(i)**	State $\frac{dx}{dt} = 4\sin\theta\cos\theta$ or equivalent (nothing for $\frac{dy}{dx} = 4\sec^2\theta$)	B1	
		Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
		Obtain given answer correctly	A1	[3]
	(ii)	Substitute $\theta = \frac{\pi}{4}$ in $\frac{dy}{dx}$ and both parametric equations	M1	
		Obtain $\frac{dy}{dx} = 4$ and coordinates (2, 4)	A1	
		Form equation of tangent at their point	M1	
		State equation of tangent in correct form $y = 4x - 4$	A1	[4]

Q33.

1	Obtain derivative of the form $\frac{k}{5x+1}$, where $k = 1, 5$ or $\frac{1}{5}$	M1	
	Obtain correct derivative $\frac{5}{5x+1}$	A1	
	Substitute $x = 4$ into expression for derivative and obtain $\frac{5}{21}$	A1√	[3]

Q34.

8	(i)	State $2y\frac{dy}{dx}$ as derivative of y^2, or equivalent	B1	
		Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	
		Obtain given answer correctly	A1	[3]
	(ii)	Equate gradient expression to -1 and rearrange	M1	
		Obtain $y = 2x$	A1	
		Substitute into original equation to obtain an equation in x^2 (or y^2)	M1	
		Obtain $2x^2 - 3x - 2 = 0$ (or $y^2 - 3y - 4 = 0$)	A1	
		Correct method to solve their quadratic equation	M1	
		State answers $(-\frac{1}{2}, -1)$ and $(2, 4)$	A1	[6]

Q35.

4	(i)	State $\frac{dx}{dt} = \frac{-2}{1-2t}$ or $\frac{dy}{dt} = -2t^{-2}$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain given answer correctly	A1	[3]
	(ii)	Equate derivative to 3 and solve for t	M1	
		State or imply that $t = -1$ c.w.o.	A1	
		Obtain coordinates $(\ln 3, -2)$	A1	[3]

Q36.

2	Use quotient rule or product rule, correctly	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain $x = \frac{\pi}{8}$	A1	[4]

Q37.

7	(i)	State $4y\frac{dy}{dx}$ as derivative of $2y^2$, or equivalent	B1	
		State $4y + 4x\frac{dy}{dx}$ as derivative of $4xy$, or equivalent	B1	
		Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	
		Obtain given answer correctly	A1	[4]
	(ii)	State or imply that the coordinates satisfy $3x - 2y = 0$	B1	
		Obtain an equation in x^2 (or y^2)	M1	
		Solve and obtain $x^2 = 4$ (or $y^2 = 9$)	A1	
		State answer $(2, 3)$	A1	
		State answer $(-2, -3)$	A1	[5]

Q38.

3	Obtain derivative $e^{2x} - 5e^x + 4$	B1	
	Equate derivative to zero and carry out recognisable solution method for a quadratic in e^x	M1	
	Obtain $e^x = 1$ or $e^x = 4$	A1	
	Obtain $x = 0$ and $x = \ln 4$	A1	
	Use an appropriate method for determining nature of at least one stationary point	M1	
	$\left(\frac{d^2y}{dx^2} = 2e^{2x} - 5e^x, \text{when } x = 0, \frac{d^2y}{dx^2} = -(3), x = \ln 4, \frac{d^2y}{dx^2} = +(12)\right)$		
	Conclude maximum at $x = 0$ and minimum at $x = \ln 4$ (no errors seen)	A1	[6]

Q39.

5	(i)	State $\frac{dx}{d\theta} = -2\sin 2\theta + \sin\theta$ or $\frac{dy}{d\theta} = 8\sin\theta\cos\theta$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
		Use $\sin 2\theta = 2\sin\theta\cos\theta$	M1	
		Obtain given answer correctly	A1	[4]
	(ii)	Equate derivative to -4 and solve for $\cos\theta$	M1	
		Obtain $\cos\theta = ½$	A1	
		Obtain $x = -1$	A1	
		Obtain $y = 3$	A1	[4]

Q40.

2	Use quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate (numerator) of derivative to zero and solve for x	DM1	
	Obtain $x = \frac{1}{3}$	A1	
	Obtain $y = \frac{3}{2}$	A1	[5]

Q41.

5	(i)	State $\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$ or $\frac{dy}{dt} = \frac{3}{t}$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Use $y = 6$ to find t	M1	
		Obtain $t = e^2$	A1	
		Obtaind $\frac{dy}{dx} = \frac{6}{e}$	A1	[5]
	(ii)	Obtain x and form equation of the tangent at their point	M1	
		Obtain correct equation for tangent $\left(y - 6 = \frac{6}{e}(x - (1 + e))\right)$	A1	
		Show that tangent passes through (1, 0) by substitution	A1	[3]

Q42.

2	(i)	Differentiate to obtain form $k_1 \cos x + k_2 \sec^2 2x$	M1	
		Obtain correct second term $2\sec^2 2x$	A1	
		Obtain $3\cos x + 2\sec^2 2x$ and hence answer 5	A1	[3]
	(ii)	Differentiate to obtain form $ke^{2x}(1 + e^{2x})^{-2}$	M1	
		Obtain correct $-12e^{2x}(1 + e^{2x})^{-2}$ or equivalent (may be implied)	A1	
		Obtain -3	A1	[3]

Q43.

7	(i)	Obtain $3y + 3x\frac{dy}{dx}$ as derivative of $3xy$	B1	
		Obtain $2y\frac{dy}{dx}$ as derivative of y^2	B1	
		State $4x + 3y + 3x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$	B1	
		Substitute 2 and -1 to find gradient of curve (dependent on at least one B1)	M1	
		Form equation of tangent through $(2, -1)$ with numerical gradient (dependent on previous M1)	DM1	
		Obtain $5x + 4y - 6 = 0$ or equivalent of required form	A1	[6]
	(ii)	Use $\frac{dy}{dx} = 0$ to find relation between x and y (dependent on at least one B1 from part(i))	M1	
		Obtain $4x + 3y = 0$ or equivalent	A1	
		Substitute for x or y in equation of curve	M1	
		Obtain $-\frac{1}{8}y^2 = 3$ or $-\frac{2}{9}x^2 = 3$ or equivalent and conclude appropriately	A1	[4]

Q44.

4		Obtain $\frac{dx}{dt} = \frac{2}{t+1}$	B1	
		Obtain $\frac{dy}{dt} = 4e^t$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ with $t = 0$ to find gradient	M1	
		Obtain 2	A1	
		Form equation of tangent through $(0, 4)$ with numerical gradient obtained from attempt to differentiate	M1	
		Obtain $2x - y + 4 = 0$ or equivalent of required form	A1	[6]

Q45.

8	(i)	Differentiate using product rule	M1	
		Obtain $\sec^2 x \cos 2x - 2\tan x \sin 2x$	A1	
		Use $\cos 2x = 2\cos^2 x - 1$ or $\sin 2x = 2\sin x \cos x$ or both	B1	
		Express derivative in terms of $\sec x$ and $\cos x$ only	M1	
		Obtain $4\cos^2 x - \sec^2 x - 2$ with no errors seen (AG)	A1	[5]
	(ii)	State $4\cos^4 x - 2\cos^2 x - 1 = 0$	B1	
		Apply quadratic formula to a 3 term quadratic equation in terms of $\cos^2 x$ to find the least positive value of $\cos^2 x$	M1	
		Obtain or imply $\cos^2 x = \frac{1+\sqrt{5}}{4}$ or $0.809\ldots$	A1	
		Obtain 0.45	A1	[4]

Q46.

4	(i)	Differentiate to obtain form $k_1 \sin 2x + k_2 \cos x$	M1	
		Obtain correct $-6\sin 2x - 5\cos x$	A1	
		Substitute $\frac{1}{6}\pi$ to obtain $-\frac{11}{2}\sqrt{3}$ or exact equivalent	A1	[3]
	(ii)	Obtain $6y + 6x\frac{dy}{dx}$ as derivative of $6xy$	B1	
		Obtain $3y^2\frac{dy}{dx}$ as derivative of y^3	B1	
		Obtain $3x^2 + 6y + 6x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$ or equivalent	B1	
		Substitute 1 and 2 to find value of gradient dependent on at least one B1	M1	
		Obtain gradient $-\frac{15}{18}$ or $-\frac{5}{6}$	A1	[5]

Q47.

3	Obtain $6y + 6x\frac{dy}{dx}$ as derivative of $6xy$	B1	
	Obtain $2y\frac{dy}{dx}$ as derivative of y^2	B1	
	Obtain $\frac{3}{x}$ and $\frac{d}{dx}(16) = 0$	B1	
	Substitute 1 and 2 to find value of $\frac{dy}{dx}$	M1	
	Obtain value $\frac{2}{3}$ as gradient of normal following their value of $\frac{dy}{dx}$	A1√	
	Form equation of **normal** through $(1, 2)$ with numerical gradient	M1	
	Obtain $2x - 3y + 4 = 0$	A1	[7]

P3 (variant1 and 3)

Q1.

9	(i)	Use quotient or product rule to differentiate $(1 - x)/(1 + x)$	M1	
		Obtain correct derivative in any form	A1	
		Use chain rule to find $\frac{dy}{dx}$	M1	
		Obtain a correct expression in any form	A1	
		Obtain the gradient of the normal in the given form correctly	A1	[5]
	(ii)	Use product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain $x = \frac{1}{2}$	A1	[4]

Q2.

2	(i)	Obtain $\frac{k\cos 2x}{1+\sin 2x}$ for any non-zero constant k	M1	
		Obtain $\frac{2\cos 2x}{1+\sin 2x}$	A1	[2]
	(ii)	Use correct quotient or product rule	M1	
		Obtain $\frac{x\sec^2 x - \tan x}{x^2}$ or equivalent	A1	[2]

Q3.

5	(i)	Use at least one of $e^{2x} = 9$, $e^{y} = 2$ and $e^{2y} = 4$	B1	
		Obtain given result $58 + 2k = c$ **AG**	B1	[2]
	(ii)	Differentiate left-hand side term by term, reaching $ae^{2x} + be^{y}\frac{dy}{dx} + ce^{2y}\frac{dy}{dx}$	M1	
		Obtain $12e^{2x} + ke^{y}\frac{dy}{dx} + 2e^{2y}\frac{dy}{dx}$	A1	
		Substitute (ln 3, ln 2) in an attempt involving implicit differentiation at least once, where RHS = 0	M1	
		Obtain $108 - 12k - 48 = 0$ or equivalent	A1	
		Obtain $k = 5$ and $c = 68$	A1	[5]

Q4.

2	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form, e.g. $-\frac{3\ln x}{x^4} + \frac{1}{x^4}$	A1	
	Equate derivative to zero and solve for x an equation of the form $\ln x = a$, where $a > 0$	M1	
	Obtain answer $\exp(\frac{1}{3})$, or 1.40, from correct work	A1	[4]

Q5.

6	(i)	Obtain $2y\frac{dy}{dx}$ as derivative of y^2	B1	
		Obtain $-4y - 4x\frac{dy}{dx}$ as derivative of $-4xy$	B1	
		Substitute $x = 2$ and $y = -3$ and find value of $\frac{dy}{dx}$		
		(dependent on at least one B1 being earned and $\frac{d(45)}{dx} = 0$)	M1	
		Obtain $\frac{12}{7}$ or equivalent	A1	[4]
	(ii)	Substitute $\frac{dy}{dx} = 1$ in an expression involving $\frac{dy}{dx}$, x and y and obtain $ay = bx$	M1	
		Obtain $y = x$ or equivalent	A1	
		Uses $y = x$ in original equation and demonstrate contradiction	A1	[3]

Q6.

3 Obtain $\frac{dx}{d\theta} = 2\cos 2\theta - 1$ or $\frac{dy}{d\theta} = -2\sin 2\theta + 2\cos\theta$, or equivalent	B1	
Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
Obtain $\frac{dy}{dx} = \frac{-2\sin 2\theta + 2\cos\theta}{2\cos 2\theta - 1}$, or equivalent	A1	
At any stage use correct double angle formulae throughout	M1	
Obtain the given answer following full and correct working	A1	[5]

Q7.

4 (i) Use correct quotient or product rule	M1	
Obtain correct derivative in any form, e.g. $\frac{2e^{2x}}{x^3} - \frac{3e^{2x}}{x^4}$	A1	
Equate derivative to zero and solve a 2-term equation for non-zero x	M1	
Obtain $x = \frac{3}{2}$ correctly	A1	[4]
(ii) Carry out a method for determining the nature of a stationary point, e.g. test derivative either side	M1	
Show point is a minimum with no errors seen	A1	[2]

Q8.

5 (i) Use correct quotient rule or equivalent	M1	
Obtain $\frac{(1+e^{2x})2x - (1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
Substitute $x = 0$ and obtain $-\frac{1}{2}$ or equivalent	A1	[3]
(ii) Differentiate y^3 and obtain $3y^2\frac{dy}{dx}$	B1	
Differentiate $5xy$ and obtain $5y + 5x\frac{dy}{dx}$	B1	
Obtain $6x^2 + 5y + 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$	B1	

Substitute $x = 0, y = 2$ to obtain $-\frac{5}{6}$ or equivalent following correct work	B1	[4]

Q9.

4	Use product or quotient rule	M1	
	Obtain derivative in any correct form	A1	
	Equate derivative to zero and obtain an equation of the form $a\sin 2x = b$, or a quadratic in $\tan x$, $\sin^2 x$, or $\cos^2 x$	M1*	
	Carry out correct method for finding one angle	M1(dep*)	
	Obtain answer, e.g. 0.365	A1	
	Obtain second answer 1.206 and no others in the range (allow 1.21)	A1	[6]
	[Ignore answers outside the given range.]		
	[Treat answers in degrees, 20.9° and 69.1°, as a misread.]		

Q10.

2	Use of correct quotient or product rule to differentiate x or t	M1	
	Obtain correct $\dfrac{3}{(2t+3)^2}$ or unsimplified equivalent	A1	
	Obtain $-2e^{-2t}$ for derivative of y	B1	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \Big/ \dfrac{dx}{dt}$ or equivalent	M1	
	Obtain -6	cwo A1	[5]
	Alternative:		
	Eliminate parameter and attempt differentiation $\left(y = e^{\frac{-6x}{1-2x}}\right)$	B1	
	Use correct quotient or product rule	M1	
	Use chain rule	M1	
	Obtain $\dfrac{dy}{dx} = \dfrac{-6}{(1-2x)^2}e^{\frac{-6x}{1-2x}}$	A1	
	Obtain -6	cwo A1	

Q11.

2	*EITHER*:	Use chain rule	M1	
		obtain $\dfrac{dx}{dt} = 6\sin t\cos t$, or equivalent	A1	
		obtain $\dfrac{dy}{dt} = -6\cos^2 t\sin t$, or equivalent	A1	
		Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	M1	
		Obtain final answer $\dfrac{dy}{dx} = -\cos t$	A1	
	OR:	Express y in terms of x and use chain rule	M1	
		Obtain $\dfrac{dy}{dx} = k(2-\dfrac{x}{3})^{\frac{1}{2}}$, or equivalent	A1	
		Obtain $\dfrac{dy}{dx} = -(2-\dfrac{x}{3})^{\frac{1}{2}}$, or equivalent	A1	
		Express derivative in terms of t	M1	
		Obtain final answer $\dfrac{dy}{dx} = -\cos t$	A1	[5]

Q12.

2	Use correct quotient or product rule or equivalent	M1	
	Obtain $\dfrac{(1+e^{2x}).2e^{2x}-e^{2x}.2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
	Substitute $x = \ln 3$ into attempt at first derivative and show use of relevant logarithm property at least once in a correct context	M1	
	Confirm given answer $\frac{9}{50}$ legitimately	A1	[4]

Q13.

8	(i)	Differentiate y to obtain $3\sin^2 t\cos t - 3\cos^2 t\sin t$ o.e.	B1	
		Use $\dfrac{dy}{dx}=\dfrac{dy}{dt}/\dfrac{dt}{dx}$	M1	
		Obtain given result $-3\sin t\cos t$	A1cwo	[3]
	(ii)	Identify parameter at origin as $t=\dfrac{3}{4}\pi$	B1	
		Use $t=\dfrac{3}{4}\pi$ to obtain $\dfrac{3}{2}$	B1	[2]
	(iii)	Rewrite equation as equation in one trig variable e.g. $sin2t = -\frac{2}{3}$, $9\sin^4 x - 9\sin^2 x + 1 = 0$, $\tan^2 x + 3\tan x + 1 = 0$	B1	
		Find at least one value of t from equation of form $\sin 2t = k$ o.e.	M1	
		Obtain 1.9	A1	
		Obtain 2.8 and no others	A1	[4]

Q14.

7 (i) *EITHER*: State or imply $\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$ as derivative of $\ln xy$, or equivalent — B1

State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3, or equivalent — B1

Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$ — M1

Obtain the given answer — A1

OR Obtain $xy = \exp(1 + y^3)$ and state or imply $y + x\frac{dy}{dx}$ as derivative of xy — B1

State or imply $3y^2\frac{dy}{dx}\exp(1 + y^3)$ as derivative of $(1 + y^3)$ — B1

Equate derivatives and solve for $\frac{dy}{dx}$ — M1

Obtain the given answer — A1 [4]

[The M1 is dependent on at least one of the B marks being earned]

(ii) Equate denominator to zero and solve for y — M1*

Obtain $y = 0.693$ only — A1

Substitute found value in the equation and solve for x — M1(dep*)

Obtain $x = 5.47$ only — A1 [4]

Q15.

3 (i) Either Use correct quotient rule or equivalent to obtain

$\frac{dx}{dt}=\frac{4(2t+3)-8t}{(2t+3)^2}$ or equivalent — B1

Obtain $\frac{dy}{dt}=\frac{4}{2t+3}$ or equivalent — B1

Use $\frac{dy}{dx}=\frac{dy}{dt}\Big/\frac{dx}{dt}$ or equivalent — M1

Obtain $\frac{1}{3}(2t+3)$ or similarly simplified equivalent — A1

Or Express t in terms of x or y e.g. $t=\frac{3x}{4-2x}$ — B1

Obtain Cartesian equation e.g. $y=2\ln\left(\frac{6}{2-x}\right)$ — B1

Differentiate and obtain $\frac{dy}{dx}=\frac{2}{2-x}$ — M1

Obtain $\frac{1}{3}(2t+3)$ or similarly simplified equivalent — A1 [4]

(ii) Obtain $2t=3$ or $t=\frac{3}{2}$ — B1

Substitute in expression for $\frac{dy}{dx}$ and obtain 2 — B1 [2]

Q16.

1 Use correct quotient or product rule — M1

Obtain correct derivative in any form — A1

Justify the given statement — A1 **[3]**

Q17.

4	Use correct product or quotient rule at least once	M1*	
	Obtain $\frac{dx}{dt} = e^{-t}\sin t - e^{-t}\cos t$ or $\frac{dy}{dt} = e^{-t}\cos t - e^{-t}\sin t$, or equivalent	A1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain $\frac{dy}{dx} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent	A1	
	EITHER: Express $\frac{dy}{dx}$ in terms of tan t only	M1(dep*)	
	Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$	A1	
	OR: Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan t	M1	
	Show expression is identical to $\frac{dy}{dx}$	A1	**[6]**

Q18.

4	Differentiate y^3 to obtain $3y^2 \frac{dy}{dx}$	B1	
	Use correct product rule at least once	*M1	
	Obtain $6e^{2x}y + 3e^{2x}\frac{dy}{dx} + e^x y^3 + 3e^x y^2 \frac{dy}{dx}$ as derivative of LHS	A1	
	Equate derivative of LHS to zero, substitute $x = 0$ and $y = 2$ and find value of $\frac{dy}{dx}$	M1(d*M)	
	Obtain $-\frac{4}{3}$ or equivalent as **final answer**	A1	[5]

Q19.

3	Obtain $\frac{2}{2t+3}$ for derivative of x	B1	
	Use quotient of product rule, or equivalent, for derivative of y	M1	
	Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent	A1	
	Obtain $t = -1$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ in algebraic or numerical form	M1	
	Obtain gradient $\frac{5}{2}$	A1	[6]

Q20.

10	**(i)**	Use of product or quotient rule	M1	
		Obtain $-5e^{-\frac{1}{2}x}\sin 4x + 40e^{-\frac{1}{2}x}\cos 4x$	A1	
		Equate $\frac{dy}{dx}$ to zero and obtain $\tan 4z = k$ or $R\cos(4x \pm \alpha)$	M1	
		Obtain $\tan 4x = 8$ or $\sqrt{65}\cos\left(4x \pm \tan^{-1}\frac{1}{8}\right)$	A1	
		Obtain 0.362 or 20.7°	A1	
		Obtain 1.147 or 65.7°	A1	[6]
	(ii)	State or imply that x-coordinates of T_n are increasing by $\frac{1}{4}\pi$ or 45°	B1	
		Attempt solution of inequality (or equation) of form $x_1 + (n-1)k\pi$. 25	M1	
		Obtain $n > \frac{4}{\pi}(25 - 0.362) + 1$, following through on their value of x_1	A1√	
		n = 33	A1	[4]

Q21.

6	Obtain correct derivative of RHS in any form	B1	
	Obtain correct derivative of LHS in any form	B1	
	Set $\frac{dy}{dx}$ equal to zero and obtain a horizontal equation	M1	
	Obtain a correct equation, e.g. $x^2 + y^2 = 1$, from correct work	A1	
	By substitution in the curve equation, or otherwise, obtain an equation in x^2 or y^2	M1	
	Obtain $x = \frac{1}{2}\sqrt{3}$	A1	
	Obtain $y = \frac{1}{2}$	A1	**7**

Q22.

4	**(i)**	Use chain rule correctly at least once	M1	
		Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t\sec^2 t$, or equivalent	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain the given answer	A1	**[4]**
	(ii)	State a correct equation for the tangent in any form	B1	
		Use Pythagoras	M1	
		Obtain the given answer	A1	**[3]**

Q23.

2	Use correct product rule or correct chain rule to differentiate y	M1	
	Use $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M*1	
	Obtain $\frac{-4\cos\theta\sin^2\theta + 2\cos^3\theta}{\sec^2\theta}$ or equivalent	A1	
	Express $\frac{dy}{dx}$ in terms of $\cos\theta$	DM*1	
	Confirm given answer $6\cos^5\theta - 4\cos^3\theta$ legitimately	A1	[5]