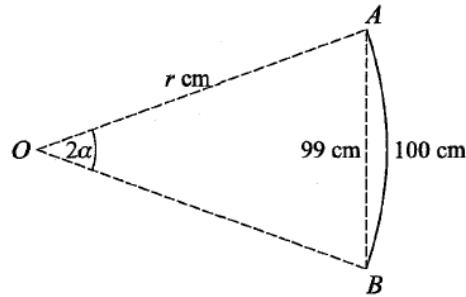


Chapter 6 Numerical Methods

Oct/Nov 2002

7



The diagram shows a curved rod AB of length 100 cm which forms an arc of a circle. The end points A and B of the rod are 99 cm apart. The circle has radius r cm and the arc AB subtends an angle of 2α radians at O , the centre of the circle.

- (i) Show that α satisfies the equation $\frac{99}{100}x = \sin x$. [3]
- (ii) Given that this equation has exactly one root in the interval $0 < x < \frac{1}{2}\pi$, verify by calculation that this root lies between 0.1 and 0.5 . [2]
- (iii) Show that if the sequence of values given by the iterative formula

$$x_{n+1} = 50 \sin x_n - 48.5x_n$$

converges, then it converges to a root of the equation in part (i). [2]

- (iv) Use this iterative formula, with initial value $x_1 = 0.25$, to find α correct to 3 decimal places, showing the result of each iteration. [2]

May/June 2003

8 The equation of a curve is $y = \ln x + \frac{2}{x}$, where $x > 0$.

- (i) Find the coordinates of the stationary point of the curve and determine whether it is a maximum or a minimum point. [5]
- (ii) The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2}{3 - \ln x_n},$$

with initial value $x_1 = 1$, converges to α . State an equation satisfied by α , and hence show that α is the x -coordinate of a point on the curve where $y = 3$. [2]

- (iii) Use this iterative formula to find α correct to 2 decimal places, showing the result of each iteration. [3]

Oct/Nov 2003

- 5 (i) By sketching suitable graphs, show that the equation

$$\sec x = 3 - x^2$$

has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

- (ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \cos^{-1}\left(\frac{1}{3 - x_n^2}\right)$$

converges, then it converges to a root of the equation given in part (i). [2]

- (iii) Use this iterative formula, with initial value $x_1 = 1$, to determine the root in the interval $0 < x < \frac{1}{2}\pi$ correct to 2 decimal places, showing the result of each iteration. [3]

May/June 2004

- 7 (i) The equation $x^3 + x + 1 = 0$ has one real root. Show by calculation that this root lies between -1 and 0 . [2]

- (ii) Show that, if a sequence of values given by the iterative formula

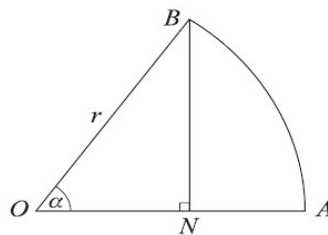
$$x_{n+1} = \frac{2x_n^3 - 1}{3x_n^2 + 1}$$

converges, then it converges to the root of the equation given in part (i). [2]

- (iii) Use this iterative formula, with initial value $x_1 = -0.5$, to determine the root correct to 2 decimal places, showing the result of each iteration. [3]

Oct/Nov 2004

5



The diagram shows a sector OAB of a circle with centre O and radius r . The angle AOB is α radians, where $0 < \alpha < \frac{1}{2}\pi$. The point N on OA is such that BN is perpendicular to OA . The area of the triangle ONB is half the area of the sector OAB .

- (i) Show that α satisfies the equation $\sin 2x = x$. [3]

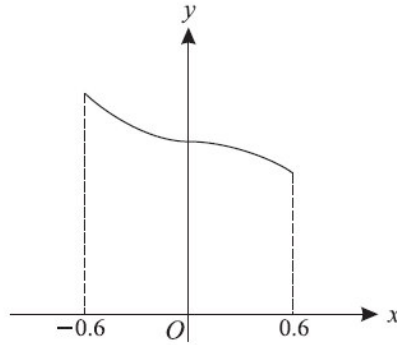
- (ii) By sketching a suitable pair of graphs, show that this equation has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

- (iii) Use the iterative formula

$$x_{n+1} = \sin(2x_n),$$

with initial value $x_1 = 1$, to find α correct to 2 decimal places, showing the result of each iteration. [3]

2



The diagram shows a sketch of the curve $y = \frac{1}{1+x^3}$ for values of x from -0.6 to 0.6 .

- (i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-0.6}^{0.6} \frac{1}{1+x^3} dx,$$

giving your answer correct to 2 decimal places. [3]

- (ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

- 7 (i) By sketching a suitable pair of graphs, show that the equation

$$\operatorname{cosec} x = \frac{1}{2}x + 1,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

- (ii) Verify, by calculation, that this root lies between 0.5 and 1. [2]

- (iii) Show that this root also satisfies the equation

$$x = \sin^{-1}\left(\frac{2}{x+2}\right). \quad [1]$$

- (iv) Use the iterative formula

$$x_{n+1} = \sin^{-1}\left(\frac{2}{x_n + 2}\right),$$

with initial value $x_1 = 0.75$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Oct/Nov 2005

4 The equation $x^3 - x - 3 = 0$ has one real root, α .

(i) Show that α lies between 1 and 2. [2]

Two iterative formulae derived from this equation are as follows:

$$x_{n+1} = x_n^3 - 3, \quad (A)$$

$$x_{n+1} = (x_n + 3)^{\frac{1}{3}}. \quad (B)$$

Each formula is used with initial value $x_1 = 1.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [5]

May/June 2006

6 (i) By sketching a suitable pair of graphs, show that the equation

$$2 \cot x = 1 + e^x,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1.0. [2]

(iii) Show that this root also satisfies the equation

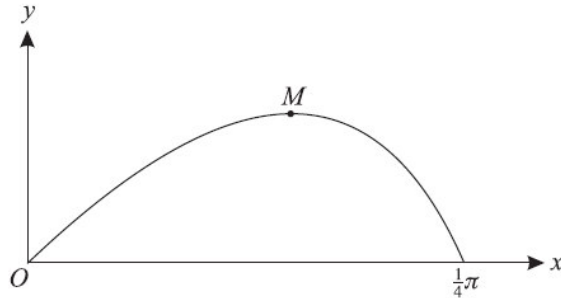
$$x = \tan^{-1}\left(\frac{2}{1 + e^x}\right). \quad [1]$$

(iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{2}{1 + e^{x_n}}\right),$$

with initial value $x_1 = 0.7$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

10



The diagram shows the curve $y = x \cos 2x$ for $0 \leq x \leq \frac{1}{4}\pi$. The point M is a maximum point.

(i) Show that the x -coordinate of M satisfies the equation $1 = 2x \tan 2x$. [3]

(ii) The equation in part (i) can be rearranged in the form $x = \frac{1}{2} \tan^{-1}\left(\frac{1}{2x}\right)$. Use the iterative formula

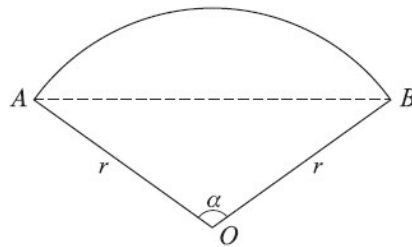
$$x_{n+1} = \frac{1}{2} \tan^{-1}\left(\frac{1}{2x_n}\right),$$

with initial value $x_1 = 0.4$, to calculate the x -coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(iii) Use integration by parts to find the exact area of the region enclosed between the curve and the x -axis from 0 to $\frac{1}{4}\pi$. [5]

May/June 2007

6



The diagram shows a sector AOB of a circle with centre O and radius r . The angle AOB is α radians, where $0 < \alpha < \pi$. The area of triangle AOB is half the area of the sector.

(i) Show that α satisfies the equation

$$x = 2 \sin x. \quad [2]$$

(ii) Verify by calculation that α lies between $\frac{1}{2}\pi$ and $\frac{2}{3}\pi$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{3}(x_n + 4 \sin x_n)$$

converges, then it converges to a root of the equation in part (i). [2]

(iv) Use this iterative formula, with initial value $x_1 = 1.8$, to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Oct/Nov 2007

- 6 (i) By sketching a suitable pair of graphs, show that the equation

$$2 - x = \ln x$$

has only one root. [2]

- (ii) Verify by calculation that this root lies between 1.4 and 1.7. [2]

- (iii) Show that this root also satisfies the equation

$$x = \frac{1}{3}(4 + x - 2 \ln x). \quad [1]$$

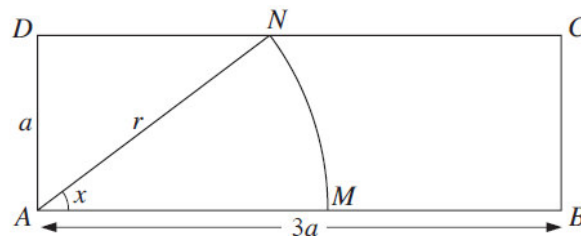
- (iv) Use the iterative formula

$$x_{n+1} = \frac{1}{3}(4 + x_n - 2 \ln x_n),$$

with initial value $x_1 = 1.5$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

May/June 2008

3



In the diagram, $ABCD$ is a rectangle with $AB = 3a$ and $AD = a$. A circular arc, with centre A and radius r , joins points M and N on AB and CD respectively. The angle MAN is x radians. The perimeter of the sector AMN is equal to half the perimeter of the rectangle.

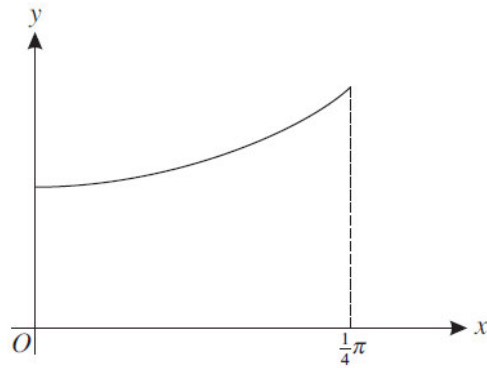
- (i) Show that x satisfies the equation

$$\sin x = \frac{1}{4}(2 + x). \quad [3]$$

- (ii) This equation has only one root in the interval $0 < x < \frac{1}{2}\pi$. Use the iterative formula

$$x_{n+1} = \sin^{-1}\left(\frac{2 + x_n}{4}\right),$$

with initial value $x_1 = 0.8$, to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]



The diagram shows the curve $y = \sqrt{1 + 2 \tan^2 x}$ for $0 \leq x \leq \frac{1}{4}\pi$.

- (i) Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{1 + 2 \tan^2 x} \, dx,$$

giving your answer correct to 2 decimal places. [3]

- (ii) The estimate found in part (i) is denoted by E . Explain, without further calculation, whether another estimate found using the trapezium rule with six intervals would be greater than E or less than E . [1]

- 4 The equation $x^3 - 2x - 2 = 0$ has one real root.

- (i) Show by calculation that this root lies between $x = 1$ and $x = 2$. [2]

- (ii) Prove that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2 - 2}$$

converges, then it converges to this root. [2]

- (iii) Use this iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Oct/Nov 2009/31

- 3 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{15}{x_n^3},$$

with initial value $x_1 = 3$, converges to α .

- (i) Use this iterative formula to find α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]
- (ii) State an equation satisfied by α and hence find the exact value of α . [2]

Oct/Nov 2009/32

2 The equation $x^3 - 8x - 13 = 0$ has one real root.

(i) Find the two consecutive integers between which this root lies. [2]

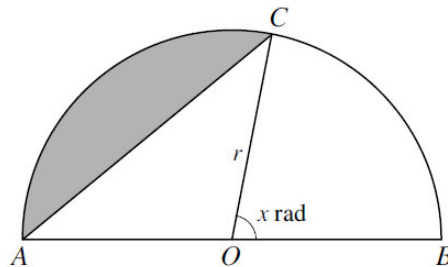
(ii) Use the iterative formula

$$x_{n+1} = (8x_n + 13)^{\frac{1}{3}}$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

May/June 2010/31

6



The diagram shows a semicircle ACB with centre O and radius r . The angle BOC is x radians. The area of the shaded segment is a quarter of the area of the semicircle.

(i) Show that x satisfies the equation

$$x = \frac{3}{4}\pi - \sin x. \quad [3]$$

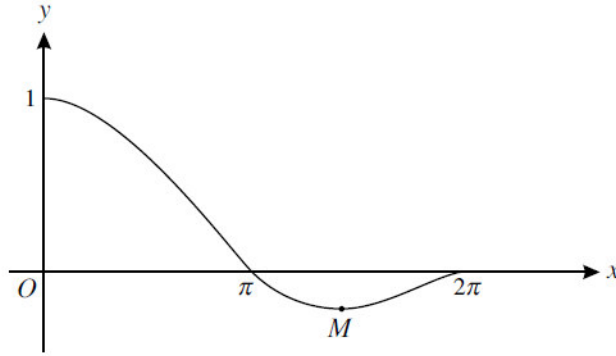
(ii) This equation has one root. Verify by calculation that the root lies between 1.3 and 1.5. [2]

(iii) Use the iterative formula

$$x_{n+1} = \frac{3}{4}\pi - \sin x_n$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4



The diagram shows the curve $y = \frac{\sin x}{x}$ for $0 < x \leq 2\pi$, and its minimum point M .

- (i) Show that the x -coordinate of M satisfies the equation

$$x = \tan x. \quad [4]$$

- (ii) The iterative formula

$$x_{n+1} = \tan^{-1}(x_n) + \pi$$

can be used to determine the x -coordinate of M . Use this formula to determine the x -coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- 6 The curve $y = \frac{\ln x}{x+1}$ has one stationary point.

- (i) Show that the x -coordinate of this point satisfies the equation

$$x = \frac{x+1}{\ln x},$$

and that this x -coordinate lies between 3 and 4. [5]

- (ii) Use the iterative formula

$$x_{n+1} = \frac{x_n + 1}{\ln x_n}$$

to determine the x -coordinate correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]