UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS
9709/11
Paper 1 Pure Mathematics 1 (P1)

May/June 2010
1 hour 45 minutes

Additional Materials: | Answer Booklet/Paper |
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| Graph Paper |
| List of Formulae (MF9) |

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 The acute angle $x$ radians is such that $\tan x=k$, where $k$ is a positive constant. Express,
(i) $\tan (\pi-x)$,
(ii) $\tan \left(\frac{1}{2} \pi-x\right)$,
(iii) $\sin x$.

2 (i) Find the first 3 terms in the expansion of $\left(2 x-\frac{3}{x}\right)^{5}$ in descending powers of $x$.
(ii) Hence find the coefficient of $x$ in the expansion of $\left(1+\frac{2}{x^{2}}\right)\left(2 x-\frac{3}{x}\right)^{5}$.

3 The ninth term of an arithmetic progression is 22 and the sum of the first 4 terms is 49 .
(i) Find the first term of the progression and the common difference.

The $n$th term of the progression is 46 .
(ii) Find the value of $n$.

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The diagram shows the curve $y=6 x-x^{2}$ and the line $y=5$. Find the area of the shaded region.

5 The function f is such that $\mathrm{f}(x)=2 \sin ^{2} x-3 \cos ^{2} x$ for $0 \leqslant x \leqslant \pi$.
(i) Express $\mathrm{f}(x)$ in the form $a+b \cos ^{2} x$, stating the values of $a$ and $b$.
(ii) State the greatest and least values of $\mathrm{f}(x)$.
(iii) Solve the equation $\mathrm{f}(x)+1=0$.

6 A curve is such that $\frac{\mathrm{d} y}{\mathrm{~d} x}=3 x^{\frac{1}{2}}-6$ and the point $(9,2)$ lies on the curve.
(i) Find the equation of the curve.
(ii) Find the $x$-coordinate of the stationary point on the curve and determine the nature of the stationary point.


The diagram shows part of the curve $y=2-\frac{18}{2 x+3}$, which crosses the $x$-axis at $A$ and the $y$-axis at $B$. The normal to the curve at $A$ crosses the $y$-axis at $C$.
(i) Show that the equation of the line $A C$ is $9 x+4 y=27$.
(ii) Find the length of $B C$.


The diagram shows a triangle $A B C$ in which $A$ is $(3,-2)$ and $B$ is $(15,22)$. The gradients of $A B, A C$ and $B C$ are $2 m,-2 m$ and $m$ respectively, where $m$ is a positive constant.
(i) Find the gradient of $A B$ and deduce the value of $m$.
(ii) Find the coordinates of $C$.

The perpendicular bisector of $A B$ meets $B C$ at $D$.
(iii) Find the coordinates of $D$.

9 The function f is defined by $\mathrm{f}: x \mapsto 2 x^{2}-12 x+7$ for $x \in \mathbb{R}$.
(i) Express $\mathrm{f}(x)$ in the form $a(x-b)^{2}-c$.
(ii) State the range of $f$.
(iii) Find the set of values of $x$ for which $\mathrm{f}(x)<21$.

The function g is defined by $\mathrm{g}: x \mapsto 2 x+k$ for $x \in \mathbb{R}$.
(iv) Find the value of the constant $k$ for which the equation $\mathrm{gf}(x)=0$ has two equal roots.


The diagram shows the parallelogram $O A B C$. Given that $\overrightarrow{O A}=\mathbf{i}+3 \mathbf{j}+3 \mathbf{k}$ and $\overrightarrow{O C}=3 \mathbf{i}-\mathbf{j}+\mathbf{k}$, find
(i) the unit vector in the direction of $\overrightarrow{O B}$,
(ii) the acute angle between the diagonals of the parallelogram,
(iii) the perimeter of the parallelogram, correct to 1 decimal place.

