Cambridge International AS & A Level

MATHEMATICS	**9709/51**
Paper 5 Probability & Statistics 1	**May/June 2024**
MARK SCHEME	
Maximum Mark: 50	

Published

This document consists of **20** printed pages.

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.	

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Mark for a correct result or statement independent of method marks.

DM or **DB**   When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT   Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

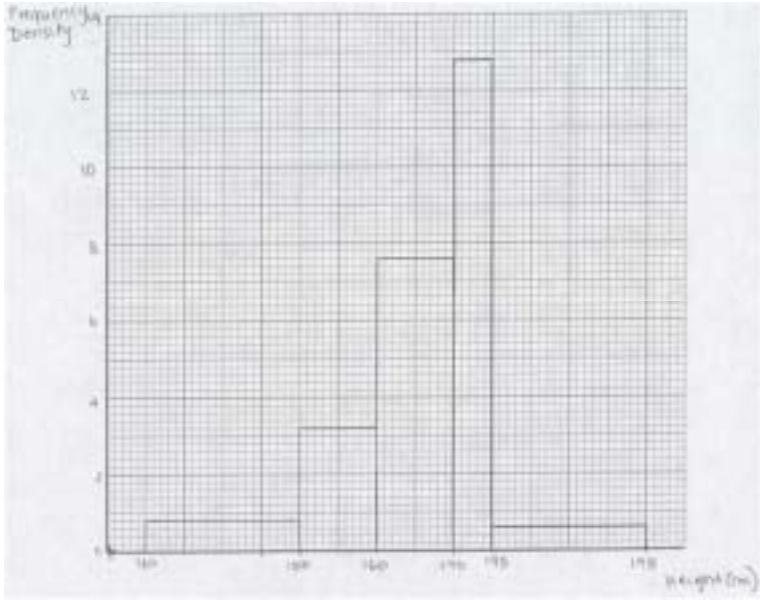|	
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	[For all 45 values Mean =] $\dfrac{439+470}{45}+30$	M1	$\dfrac{439+470}{45}$ or $\dfrac{909}{45}$ seen.
	$= 50.2$	A1	If M0 awarded, **SC B1** 50.2 WWW.
	Alternative Method for Question 1(a)		
	[For all 45 values Mean =] $\dfrac{25\times30+470+20\times30+439}{45}$	(M1)	$\dfrac{1220+1039}{45}$ or $\dfrac{2259}{45}$ seen.
	$= 50.2$	(A1)	If M0 awarded, **SC B1** 50.2 WWW.
		2	
1(b)	For all 45 values Sd^2 = $\dfrac{12405+11346}{45}-\left(\dfrac{909}{45}\right)^2$	M1	$\dfrac{their\,(12405+11346)\text{ or }23751}{45}-\left(their\,\dfrac{909}{45}\right)^2$
	sd $[=\sqrt{119.76}\ ]=10.9$	A1	If M0 awarded, **SC B1** 10.9 WWW.
		2	

Question	Answer	Marks	Guidance
2(a)	$P(23 < X < 35) = P(\dfrac{23-28}{3.3} < Z < \dfrac{35-28}{3.3})$ $[= P(-1.515 < Z < 2.121)\,]$	M1	Using ±standardisation formula once with 23 or 35, 28 and 3.3, allow σ^2, allow $\sqrt{\sigma}$, no continuity correction.
		A1	One fully correct ± standardisation formula.
	$[= \Phi(2.121) + \Phi(1.5151) - 1]$ $= 0.9830 + 0.9351 - 1$	M1	Appropriate area Φ, from final process, must be a probability.
	$= 0.918$	A1	AWRT
		4	
2(b)	$[P(X > 7.6) = P(Z > \dfrac{7.6-8.5}{\sigma}) = 0.75]$	B1	0.674 or – 0.674 seen. CAO as critical value.
	$\dfrac{7.6-8.5}{\sigma} = -0.674$	M1	Use of the ±standardisation formula with 7.6, 8.5, σ and a z-value (not 0.75, 0.25, 0.7734, 0.2266, 0.5987 nor $1 - z$-value: 0.326, 0.5987). Condone use of $\dfrac{\pm 0.9}{\sigma}$.
	$\sigma = 1.34$	A1	$1.33 \leqslant \sigma \leqslant 1.34$
		3	

Question	Answer	Marks	Guidance
3(a)	<table><tr><td>cw</td><td>20</td><td>10</td><td>10</td><td>5</td><td>20</td></tr><tr><td>fd</td><td>0.8</td><td>3.2</td><td>7.6</td><td>12.8</td><td>0.6</td></tr></table>	**M1**	At least four frequency densities calculated $\dfrac{f}{cw}$ $\left(\text{e.g. } \dfrac{16}{20}\right)$. Condone $\dfrac{f}{cw \pm 0.5}$ if unsimplified. Accept unsimplified, may be read from graph using *their* scale no lower than 1 cm = fd 2.
		A1	All bar heights correct on graph, not FT. Using their suitable linear scale with at least three values indicated, no lower than 1 cm = fd 2.
		B1	Bar ends at 150, 160, 170, 175, 195. Five bars drawn with a horizontal linear scale no lower than 1 cm = 10 cm, with at least three values indicated, $130 \leqslant$ horizontal scale $\leqslant 195$.
		B1	Axes labelled frequency density (fd) height (h) and cm, OE, or an appropriate title. (Axes may be reversed)
		4	
3(b)	[LQ:] $160 \leqslant h < 170$ [UQ:] $170 \leqslant h < 175$ $175 - 160 = 15$	**M1**	$170 \leqslant h < 175 - 160 \leqslant h < 170$ UQ and LQ classes seen.
		A1	$175 - 160 = 15$
			If M0 scored, **SC B1** for $175 - 160 = 15$.
		2	

Question	Answer	Marks	Guidance
4(a)	**Method 1**		
	[Probability of 4 in 3 throws is] $1-\left(\dfrac{3}{4}\right)^3=\dfrac{37}{64}$	**M1**	$1-\left(s\right)^3$, $\ s=\dfrac{3}{4}$ or $\dfrac{1}{4}$.
		A1	AG
	Method 2		
	[Probability of 4 in 3 throws is] $\dfrac{1}{4}+\dfrac{1}{4}\times\dfrac{3}{4}+\dfrac{1}{4}\times\left(\dfrac{3}{4}\right)^2=\dfrac{37}{64}$	**(M1)**	$t+t\left(1-t\right)+t\left(1-t\right)^2$, $t=\dfrac{1}{4}$ or $\dfrac{3}{4}$.
		(A1)	AG
	Method 3		
	$^3C_1\times\dfrac{1}{4}\times\left(\dfrac{3}{4}\right)^2+{}^3C_2\times\left(\dfrac{1}{4}\right)^2\times\dfrac{3}{4}+{}^3C_3\times\left(\dfrac{1}{4}\right)^3=\dfrac{37}{64}$	**(M1)**	
		(A1)	AG
		2	

Question	Answer	Marks	Guidance
4(b)	**Method 1**		
	$\left(1-\dfrac{37}{64}\right)^4 = 0.0317$	**B1FT**	$\left(1-\textit{their}\,(\mathbf{a})\right)^4$, accept unsimplified.
		1	
	Method 2		
	[Probability no 4s is] $\left(\dfrac{3}{4}\right)^6 \times \left(\dfrac{3}{4}\right)^6 = 0.0317$	**(B1FT)**	Accept unsimplified.
		1	
4(c)	X3 Y1 $\left(\dfrac{37}{64}\right)^3 \times \dfrac{37}{64} \times \left(\dfrac{27}{64}\right)^2 \times 3$ [= 0.059645]	**B1**	Correct probability for 1 identified scenario. Accept unsimplified.
	X2 Y0 $\left(\dfrac{37}{64}\right)^2 \times \dfrac{27}{64} \times \left(\dfrac{27}{64}\right)^3 \times 3$ [= 0.03176]	**M1**	Add values of 2 correct scenarios. Identification may be implied by correct unsimplified expressions (condone omission of × 3). Values may not be probabilities.
	Probability = 0.0914	**A1**	If A0 scored, **SC B1** for 0.0914 WWW.
		3	

Question	Answer	Marks	Guidance
5(a)	**Method 1**		
	[P(5, 6, 7) =]	**M1**	One term $^7C_x \left(p \right)^x \left(1-p \right)^{7-x}$, with $0 < p < 1$, $x \neq 0$ or 7.
	$^7C_5\, 0.7^5\, 0.3^2 + {}^7C_6\, 0.7^6\, 0.3^1 + 0.7^7$ [= 0.31765 + 0.24706 + 0.08235]	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.647	**B1**	$0.647 \leqslant p < 0.6475$
	Method 2		
	[P(5, 6, 7) = 1 – P(0, 1, 2, 3, 4) =] $1 - \{0.3^7 + {}^7C_1\, 0.7^1\, 0.3^6 + {}^7C_2\, 0.7^2\, 0.3^5 + {}^7C_3\, 0.7^3\, 0.3^4 + {}^7C_4\, 0.7^4\, 0.3^3\}$	**(M1)**	One term $^7C_x \left(p \right)^x \left(1-p \right)^{7-x}$, with $0 < p < 1$, $x \neq 0$ or 7.
		(A1)	Correct expression, accept unsimplified, no terms omitted leading to final answer. Condone omission of final bracket '}'. If other brackets omitted, allow recovery if 1 – 0.35294 seen.
	= 0.647	**(B1)**	$0.647 \leqslant p < 0.6475$
		3	

Question	Answer	Marks	Guidance
5(b)	**Method 1**		
	[1 – P(0 white weeks) =] $1 - (1 - 0.647)^3$	**M1**	$1 - p^3$, $0 < p < 1$, $p = 1 -$ *their* **(a)**, or correct.
	0.956	**A1**	
	Method 2		
	[P(1, 2, 3 white weeks) =] $3 \times 0.647 \times 0.353^2 + 3 \times 0.647^2 \times 0.353 + 0.647^3$	**(M1)**	$3 \times q \times (1-q)^2 + 3 \times q^2 \times (1-q) + q^3$, $q =$ *their* **(a)**, or correct.
	0.956	**(A1)**	
		2	

Question	Answer	Marks	Guidance
5(c)	[Mean = $60 \times 0.8 =$] 48 [Variance = $60 \times 0.8 \times 0.2 =$] 9.6	**B1**	48 and 9.6, $9\frac{3}{5}, \frac{48}{5}$ seen, allow unsimplified. May be seen in the standardisation formula ([$\sigma =$] $3.098 \leqslant \sigma \leqslant 3.1[0]$ implies correct variance). Incorrect notation penalised but values can be used as anticipated in remainder of question.
	$P(X < 47) = P\left(Z < \dfrac{46.5 - 48}{\sqrt{9.6}} \right)$	**M1**	Substituting *their* μ and σ into $\pm$ standardising formula (any number for 46.5), not *their* σ^2 or $\sqrt{their\ \sigma}$.
		M1	Use continuity correction 46.5 or 47.5 in *their* standardised formula. Note: $\dfrac{\pm 1.5}{\sqrt{9.6}}$ or $\dfrac{\pm 1.5}{3.098}$ seen gains M2 BOD.
	$[P(Z < -0.4841) = 1 - \Phi(0.4841)]$ $1 - 0.6858$	**M1**	Appropriate area Φ, from final process, must be a probability. Expect final answer < 0.5. Note: appropriate final answer implies this M1.
	$= 0.314$	**A1**	$0.314 \leqslant p < 0.3145$
		5	

Question	Answer	Marks	Guidance
6(a)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X=x)$</td><td>$\frac{24}{60}$</td><td>$\frac{26}{60}$</td><td>$\frac{9}{60}$</td><td>$\frac{1}{60}$</td></tr><tr><td></td><td>$\frac{2}{5}$</td><td>$\frac{13}{30}$</td><td>$\frac{3}{20}$</td><td>$\frac{1}{60}$</td></tr><tr><td></td><td>0.4</td><td>0.433</td><td>0.15</td><td>0.0167</td></tr></table>	**B1**	Table with correct X values and at least one probability. Values need not be in order, lines may not be drawn, may be vertical, X and $P(X)$ may be omitted. Condone any additional X values if probability stated as 0.
		B1	$P(X=1)$ or $P(X=2)$ correct and identified, need not be in table, accept unsimplified.
		B1	Two more correct and identified probabilities, need not be in table, accept unsimplified.
		B1	4 correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3sf. **SC B1** for four probabilities summing to 1 placed in a probability distribution table with the correct x values.
		4	

Question	Answer	Marks	Guidance
6(b)	$[P(Y=0)=]\ \dfrac{2}{3}\times\dfrac{3}{4}\times\dfrac{4}{5}\times(1-p)^2$; $[P(Y=5)=]\ \dfrac{1}{3}\times\dfrac{1}{4}\times\dfrac{1}{5}\times p^2$	B1	Either $\dfrac{2}{3}\times\dfrac{3}{4}\times\dfrac{4}{5}\times(1-p)^2$, not $\dfrac{2}{5}\times(1-p)^2$; or $\dfrac{1}{3}\times\dfrac{1}{4}\times\dfrac{1}{5}\times p^2$, not $\dfrac{1}{60}\times p^2$.
	$\dfrac{2}{3}\times\dfrac{3}{4}\times\dfrac{4}{5}\times(1-p)^2 = 6\times\dfrac{1}{3}\times\dfrac{1}{4}\times\dfrac{1}{5}\times p^2$ $\left[24(1-p)^2 = 6\times p^2\right]$ $3p^2 - 8p + 4 = 0$	M1	Equating and forming a 3 term quadratic equation. *Their* $P(Y=0) = 6 \times$ *their* $P(Y=5)$.
	$p = \dfrac{2}{3}$	A1	Not dependent on B1. A0 if $p = 2$ seen and not clearly rejected.
			SC B1 if $p = \dfrac{2}{3}$ obtained from a correct quadratic with more than three terms. If $p = 2$ seen and not clearly rejected, **SC B0**.
		3	

Question	Answer	Marks	Guidance
7(a)	$\left[\dfrac{8!}{2!3!}=\right]$ 3360	**B1**	
		1	
7(b)	Number of arrangements with 2s at the end – number of arrangements with 2s at the end and the 4s together 2 _ _ _ _ _ _ 2 – 2 _ (<u>444</u>) _ _ 2		
	$\dfrac{6!}{3!}$ – 4!	**M1**	$\dfrac{6!}{3!} \times r - s$, $r = 1, 2$ and s a positive integer (including 0).
		B1	4! Seen either alone or in $t - 4!$, t an integer value > 24.
		M1	$\dfrac{6!}{3!} \times r - 4! \times u$, $r = 1, 2$ and $u = 1, 2$.
	= 96	**A1**	
		4	

Question	Answer	Marks	Guidance
7(c)	**Method 1**		

Method 1 table:

2s	4s	1,3,5		
0	0	3	$^3C_3$	1
0	1	2	$^3C_1 \times {}^3C_2$	9
1	0	2	$^2C_1 \times {}^3C_2$	6
1	1	1	$^2C_1 \times {}^3C_1 \times {}^3C_1$	18

[Total 34 ways]

[Total number of selections =] $^8C_3$ [= 56]

[Probability =] $\left[\dfrac{34}{^8C_3}\right] = \dfrac{17}{28},\ 0.607$

Marks / Guidance:

M1 — One correct calculation, unsimplified for an identified scenario containing 1, 2 and/or 1, 4.

A1 — Two correct outcomes evaluated, accept unsimplified.

M1 — Four correct scenarios added.

B1 — Used as denominator of probability expression.

A1

Question	Answer	Marks	Guidance
7(c)	**Method 2**		
	Combinations of 3 numbers	**(M1)**	One correct calculation, unsimplified for an identified scenario containing 1, 2 and/or 1, 4.
		(A1)	Five correct outcomes evaluated, accept unsimplified.
		(M1)	Ten correct scenarios added.

1,2,3	$^1C_1 \times {}^2C_1 \times {}^1C_1$	2
1,2,4	$^1C_1 \times {}^2C_1 \times {}^3C_1$	6
1,2,5	$^1C_1 \times {}^2C_1 \times {}^1C_1$	2
1,3,4	$^1C_1 \times {}^1C_1 \times {}^3C_1$	3
1,3,5	$^1C_1 \times {}^1C_1 \times {}^1C_1$	1
1,4,5	$^1C_1 \times {}^3C_1 \times {}^1C_1$	3
2,3,4	$^2C_1 \times {}^1C_1 \times {}^3C_1$	6
2,3,5	$^2C_1 \times {}^1C_1 \times {}^1C_1$	2
2,4,5	$^2C_1 \times {}^3C_1 \times {}^1C_1$	6
3,4,5	$^1C_1 \times {}^3C_1 \times {}^1C_1$	3

Question	Answer	Marks	Guidance
	[Total 34 ways] [Total number of selections =] $^8C_3$ [= 56]	**(B1)**	$^8C_3$ or 56 as denominator of probability expression.
	[Probability =] $\left[\dfrac{34}{^8C_3} \right] = \dfrac{17}{28}$, 0.607	**(A1)**	

Question	Answer	Marks	Guidance
7(c)	**Method 3**		

				(M1)	One correct calculation, unsimplified for an identified scenario containing 1, 2 and/or 1, 4.
1,2,3	$\dfrac{1}{8} \times \dfrac{2}{7} \times \dfrac{1}{6} \times 3!$	$\dfrac{12}{336}$			
1,2,4	$\dfrac{1}{8} \times \dfrac{2}{7} \times \dfrac{3}{6} \times 3!$	$\dfrac{36}{336}$		**(A1)**	Five correct outcomes evaluated, accept unsimplified.
1,2,5	$\dfrac{1}{8} \times \dfrac{2}{7} \times \dfrac{1}{6} \times 3!$	$\dfrac{12}{336}$		**(M1)**	Ten correct scenarios added.
1,3,4	$\dfrac{1}{8} \times \dfrac{1}{7} \times \dfrac{3}{6} \times 3!$	$\dfrac{18}{336}$			
1,3,5	$\dfrac{1}{8} \times \dfrac{1}{7} \times \dfrac{1}{6} \times 3!$	$\dfrac{6}{336}$			
1,4,5	$\dfrac{1}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3!$	$\dfrac{18}{336}$			
2,3,4	$\dfrac{2}{8} \times \dfrac{1}{7} \times \dfrac{3}{6} \times 3!$	$\dfrac{36}{336}$			
2,3,5	$\dfrac{2}{8} \times \dfrac{1}{7} \times \dfrac{1}{6} \times 3!$	$\dfrac{12}{336}$			
2,4,5	$\dfrac{2}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3!$	$\dfrac{36}{336}$			
3,4,5	$\dfrac{2}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3!$	$\dfrac{18}{336}$			

Question	Answer			Marks	Guidance
7(c)				**(B1)**	336 or 8×7×6 seen as a denominator.
	[Probability] $= \dfrac{17}{28},\ 0.607$			**(A1)**	
	Method 4				
7(c)	444	$\dfrac{3}{8} \times \dfrac{2}{7} \times \dfrac{1}{6}$	$\dfrac{6}{336}$	**(M1)**	1–1 correct calculation, unsimplified for an identified scenario not containing three 4s.
	445	$\dfrac{3}{8} \times \dfrac{2}{7} \times \dfrac{1}{6} \times 3$	$\dfrac{18}{336}$	**(A1)**	Five correct probabilities evaluated, accept unsimplified.
	443	$\dfrac{2}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3$	$\dfrac{18}{336}$	**(M1)**	Nine correct scenarios subtracted.
	442	$\dfrac{2}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3$	$\dfrac{36}{336}$		
	441	$\dfrac{2}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3$	$\dfrac{18}{336}$		
	225	$\dfrac{2}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3$	$\dfrac{6}{336}$		
	224	$\dfrac{2}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3$	$\dfrac{18}{336}$		
	223	$\dfrac{2}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3$	$\dfrac{6}{336}$		
	221	$\dfrac{2}{8} \times \dfrac{3}{7} \times \dfrac{1}{6} \times 3$	$\dfrac{6}{336}$		

Question	Answer	Marks	Guidance
		(B1)	336 or $8 \times 7 \times 6$ seen as a denominator.
	$[\text{Probability}] = 1 - \dfrac{132}{336}, \ \dfrac{204}{336}, \ 0.607$	**(A1)**	
		5	