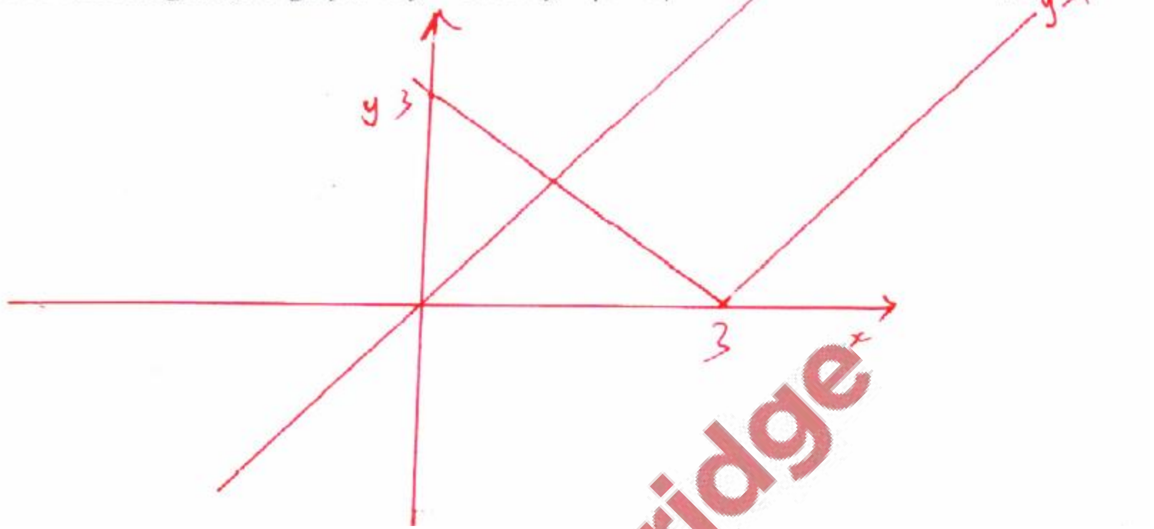
Algebra – 2021 A2 Nov P2 Math

1. Nov/2021/Paper_9709/21/No.2

(a) Sketch, on the same diagram, the graphs of $y = 3x$ and $y = |x - 3|$. [2]



(b) Find the coordinates of the point where the two graphs intersect. [3]

$3x = |x-3|$

$\Rightarrow 3x = \sqrt{(x-3)^2}$

$(3x)^2 = \left(\sqrt{(x-3)^2}\right)^2$

$\Rightarrow 9x^2 = x^2 - 3x - 3x + 9$

$9x^2 = x^2 - 6x + 9$

$\Rightarrow 8x^2 + 6x - 9 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(8)(-9)}}{2(8)}$

$x = 3/4$ or $-3/2$

ignore $-3/2$ $\Rightarrow$

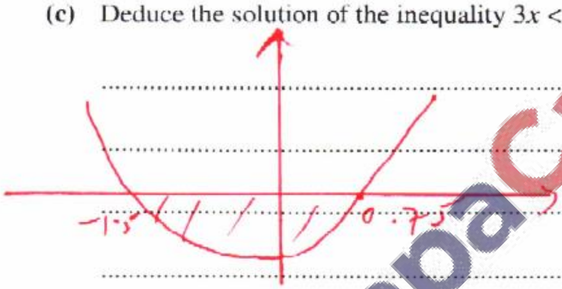
$x = 3/4$

But $y = 3x$

$\Rightarrow y = 3(3/4) = 9/4$

Point $(3/4, 9/4)$

(c) Deduce the solution of the inequality $3x < |x - 3|$. [1]



$\Rightarrow$ using graphical method

solution set

$x < 3/4$

2. Nov/2021/Paper_9709/21/No.6

The polynomials f(x) and g(x) are defined by

$$f(x) = 4x^3 + ax^2 + 8x + 15 \qquad \text{and} \qquad g(x) = x^2 + bx + 18,$$

where a and b are constants.

(a) Given that $(x + 3)$ is a factor of f(x), find the value of a. [2]

$(x+3)$ is a factor, then by Factor theorem $\Rightarrow f(-3) = 0$

$4(-3)^3 + a(-3)^2 + 8(-3) + 15 = 0$

$\Rightarrow -108 + 9a - 24 + 15 = 0$

$\Rightarrow 9a - 117 = 0$

$\frac{9a}{9} = \frac{117}{9}$ 13

$\Rightarrow a = 13$

(b) Given that the remainder is 40 when g(x) is divided by $(x - 2)$, find the value of b. [2]

Using Remainder Theorem

$\Rightarrow g(2) = 40$

$\Rightarrow (2)^2 + b(2) + 18 = 40$

$4 + 2b + 18 = 40$

$\Rightarrow 2b + 22 = 40$

$\Rightarrow \frac{2b}{2} = \frac{18}{2}$ 9

$b = 9$

$\Rightarrow b = 9.$

(c) When a and b have these values, factorise $\mathrm{f}(x) - \mathrm{g}(x)$ completely. [3]

$f(x) - g(x)$

$= 4x^3 + 3x^2 + 8x + 15 - [x^2 + 9x + 18]$

$= 4x^3 + 3x^2 + 8x + 15 - x^2 - 9x - 18$

$= 4x^3 + 12x^2 - x - 3$

$(x+3)$ is a factor of $f(x) - g(x)$. Now, using Long division method

$\Rightarrow$

$$\begin{array}{r|l} & 4x^2 - 1 \\ \hline (x+3) & 4x^3 + 12x^2 - x - 3 \\ & 4x^3 + 12x^2 \\ \hline & 0 - x - 3 \\ & -x - 3 \\ \hline & 0 \end{array}$$

$f(x) - g(x) =$ (quotient $\times$ divisor) + Remainder

$= (4x^2 - 1)(x+3) + 0$

$= (2x+1)(2x-1)(x+3)$

(d) Hence solve the equation $\mathrm{f}(\operatorname{cosec}\theta) - \mathrm{g}(\operatorname{cosec}\theta) = 0$ for $0 < \theta < 2\pi$. [3]

Let $x = \operatorname{cosec}\theta$

$\Rightarrow \operatorname{cosec}\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{\sin\theta} = \frac{1}{2}$

When $x = -3$

$\Rightarrow \operatorname{cosec}\theta = -3$

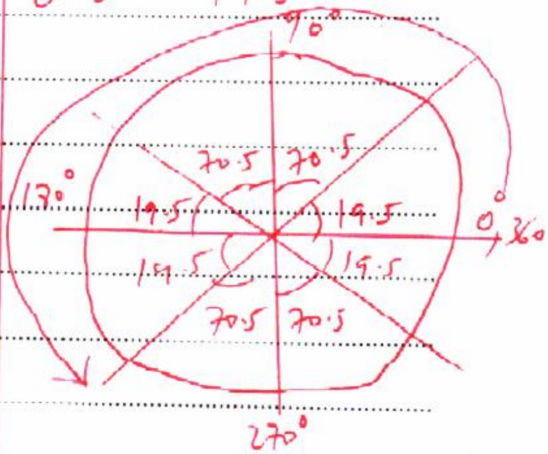
$\frac{1}{\sin\theta} = \frac{-3}{1}$

$\Rightarrow \frac{-3\sin\theta}{-3} = \frac{1}{-3}$

$\Rightarrow \sin\theta = -\frac{1}{3}$

$\Rightarrow \theta = \sin^{-1}\left(-\frac{1}{3}\right)$

$\theta = -19.5^\circ$



$180 + 19.5 = 199.5^\circ$

Solution set

$\theta = \{199.5^\circ\}$

3. Nov/2021/Paper_9709/22/No.1

The polynomial $p(x)$ is defined by

$$p(x) = ax^3 + bx - 10,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$ and that the remainder is -55 when $p(x)$ is divided by $(x + 3)$.

(a) Find the values of a and b. [5]

$p(x) = ax^3 + bx - 10$

$(x+2)$ is a factor of $p(x) \Rightarrow$ using factor Theorem $\Rightarrow p(-2) = 0$

$\Rightarrow p(-2) = a(-2)^3 + b(-2) - 10 = 0$

$\Rightarrow -8a - 2b - 10 = 0$

$\Rightarrow 8a + 2b = -10$

$\Rightarrow \frac{8a}{2} + \frac{2b}{2} = \frac{-10}{2}$

$\Rightarrow 4a + b = -5$ --- (i)

using Remainder Theorem

$\Rightarrow p(-3) = -55$

$\Rightarrow a(-3)^3 + b(-3) - 10 = -55$

$\Rightarrow -27a - 3b - 10 = -55$

$-27a - 3b - 10 = -55$

$\Rightarrow \frac{-27a}{-3} - \frac{3b}{-3} = \frac{-45}{-3}$

$\Rightarrow 9a + b = 15$ --- (ii)

Now solving both Equations --- (i) & (ii) Simultaneously $\Rightarrow$

$9a + (-5 - 4a) = 15$

$9a - 5 - 4a = 15$

$9a - 4a = 15 + 5$

$\frac{5a}{5} = \frac{20}{5}$

$\boxed{a = 4}$

But we know

$b = -5 - 4a$

$b = -5 - 4(4)$

$b = -5 - 16$

$\boxed{b = -21}$

and

$\boxed{a = 4}$

(b) Hence factorise p(x) completely. [3]

$(x+2)$ is a factor of $p(x)$.
Now using long division method of polynomial

$\Rightarrow p(x) = 4x^3 - 21x - 10$

$$
\begin{array}{r|l}
 & 4x^2 - 8x - 5 \\
\hline
(x+2) & 4x^3 - 21x - 10 \\
 & 4x^3 + 8x^2 \quad - \\
\hline
 & 0 - 8x^2 - 21x - 10 \\
 & -8x^2 - 16x \quad - \\
\hline
 & 0 - 5x - 10 \\
 & -5x - 10 \quad - \\
\hline
 & 0
\end{array}
$$

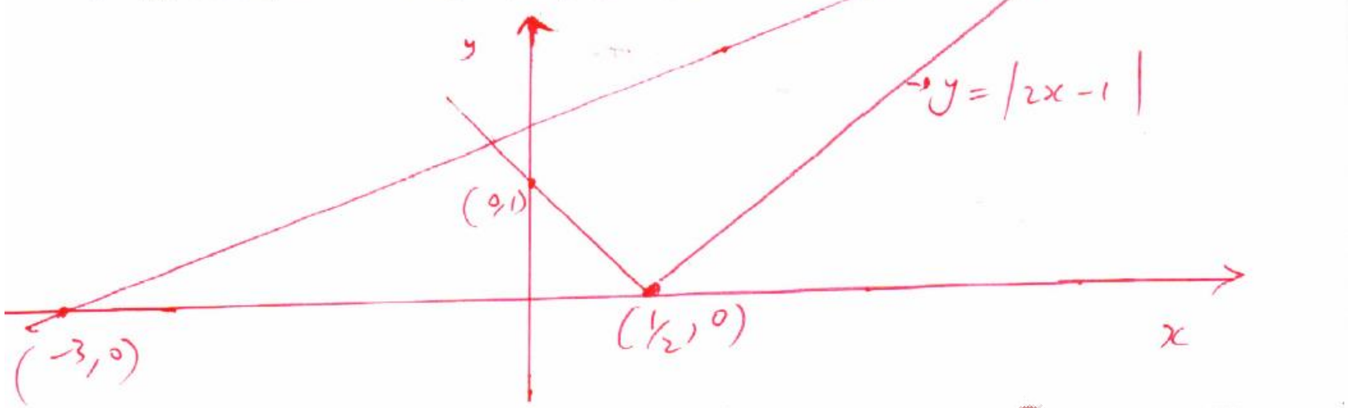
$p(x) = (\text{quotient} \times \text{divisor}) + \text{Remainder}$

$p(x) = (4x^2 - 8x - 5)(x+2) = 0$

$p(x) = (2x+1)(2x-5)(x+2)$.

4. Nov/2021/Paper_9709/22/No.2

2 **(a)** Sketch, on the same diagram, the graphs of $y = x + 3$ and $y = |2x - 1|$. [2]



(b) Solve the equation $x + 3 = |2x - 1|$. [3]

Recall $\sqrt{a^2} = |a|$

$\Rightarrow (x+3) = \sqrt{(2x-1)^2}$

$\Rightarrow (x+3)^2 = \left(\sqrt{(2x-1)^2}\right)^2$

$(x+3)(x+3) = (2x-1)(2x-1)$

$x(x+3) + 3(x+3) = 2x(2x-1) - 1(2x-1)$

$x^2 + 3x + 3x + 9 = 4x^2 - 2x - 2x + 1$

$x^2 + 6x + 9 = 4x^2 - 4x + 1$

$\Rightarrow 4x^2 - x^2 - 4x - 6x + 1 - 9 = 0$

$3x^2 - 10x - 8 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(-8)}}{2(3)}$

$x = \frac{10 \pm \sqrt{196}}{6} = \frac{10 \pm 14}{6}$

$x = -\frac{4}{6}$ or $\frac{24}{6}$

$\Rightarrow x = -\frac{2}{3}$ or 4.

(c) Find the value of y such that $5^{\frac{1}{2}y} + 3 = |2 \times 5^{\frac{1}{2}y} - 1|$. Give your answer correct to 3 significant figures. [2]

$5^{\frac{1}{2}y} + 3 = |2 \times 5^{\frac{1}{2}y} - 1|$

Let $5^{\frac{1}{2}y} = x$

$\Rightarrow x + 3 = |2x - 1|$

but $x = -\frac{2}{3}$ or 4

$-\frac{2}{3}$ is Impossible

$\Rightarrow 5^{\frac{1}{2}y} = 4$ introduce ln on both sides

$\Rightarrow \ln 5^{\frac{1}{2}y} = \ln 4$

$\frac{1}{2}y \ln 5 = \ln 4$

$\frac{0.5y \ln 5}{\ln 5} = \frac{\ln 4}{\ln 5}$

$\frac{0.5y}{0.5} = \frac{0.86135}{0.5}$

$y = 1.7227$

$\Rightarrow y = 1.7227$

$y = 1.72$