1. Nov/2020/Paper_9709/31/No.3

The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \qquad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\dfrac{dy}{dx} = \cot\theta$. [5]

$\dfrac{dx}{dx} = 2\sin 2\theta \qquad \dfrac{dy}{dx} = 2 + 2\ \ 2\theta$

$\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \times \dfrac{d\theta}{dx}$

$\dfrac{2 + 2\cos 2\theta}{2\sin 2\theta}$

$\dfrac{\cancel{2}(1 + \cos 2\theta)}{\cancel{2}\sin 2\theta}$

$\cos 2\theta = 2\cos^2\theta - 1$

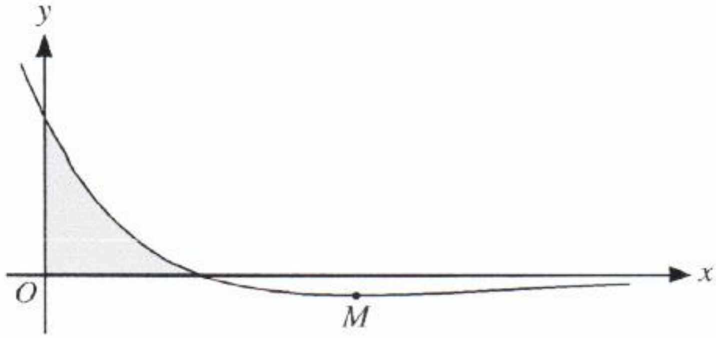
$\dfrac{1 + 2\cos^2\theta - 1}{2\sin\theta\cos\theta}$

$$= \frac{\cancel{x}\cos^2\theta}{\cancel{x}\sin\theta\cos\theta}$$

$$\frac{\cos\theta \cdot \cancel{\cos\theta}}{\sin\theta\,\cancel{\cos\theta}}$$

$$\frac{dy}{dx} = \cot\theta$$

2. Nov/2020/Paper_9709/31/No.10a



The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

Product rule $u'v + uv'$

$u = 2 - x$

$\frac{du}{dx} = -1$ $\quad v = e^{-\frac{1}{2}x}$

$dv/dx = -\frac{1}{2}e^{-\frac{1}{2}x}$

$-e^{-\frac{1}{2}x} + (2-x)(-\frac{1}{2}e^{-\frac{1}{2}x})$

$-e^{-\frac{1}{2}x} - e^{-\frac{1}{2}x} + \frac{1}{2}xe^{-\frac{1}{2}x} = 0$

max / min $\frac{dy}{dx} = 0$

$-2e^{-\frac{1}{2}x} + \frac{1}{2}xe^{-\frac{1}{2}x} = 0$

$$e^{-\frac{1}{2}x}\left(-2+\frac{x}{2}\right)=0$$

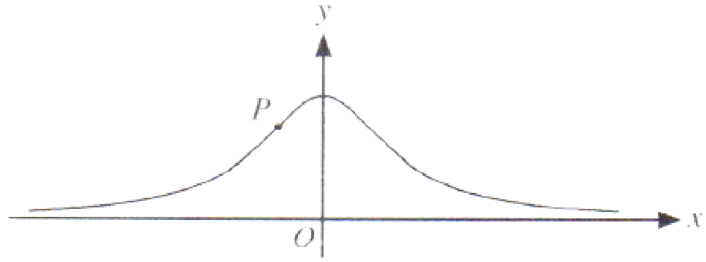
$$\frac{x}{2}=2 \qquad x=4$$

$$y=(2-4)e^{-\frac{1}{2}(4)}$$

$$y=-2e^{-2} \text{ or } \frac{-2}{e^{2}}$$

$$M\left(4, -\frac{2}{e^{2}}\right)$$

3. Nov/2020/Paper_9709/32/No.5



The diagram shows the curve with parametric equations

$$x = \tan\theta, \qquad y = \cos^2\theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(a) Show that the gradient of the curve at the point with parameter θ is $-2\sin\theta\cos^3\theta$. [3]

$\frac{dx}{d\theta} = \sec^2\theta \qquad \frac{dy}{d\theta} = -2\cos\theta\sin\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{-2\sin\theta\cos\theta \times}{\sec^2\theta}$

$= -2\sin\theta\cos\theta \times \cos^2\theta$

$= -2\sin\theta\cos^3\theta$

The gradient of the curve has its maximum value at the point P.

(b) Find the exact value of the x-coordinate of P. [4]

$m = -2\sin\theta\cos^3\theta$

$u = -2\sin\theta \qquad v = \cos^3\theta$

$\frac{\delta u}{\delta x} = -2\cos\theta \qquad \frac{\delta v}{\delta x} = -3\sin\theta\cos^2\theta$

$\frac{\delta m}{\delta\theta} = -2\cos^4\theta + 6\sin^2\theta\cos^2\theta = 0$

$\frac{6\sin^2\theta\cos^2\theta}{\cos^2\theta} = \frac{2\cos^4\theta}{\cos^2\theta}$

$6\sin^2\theta = 2\cos^2\theta$

$\frac{6\sin^2\theta}{\cos^2\theta} = 2$

$\tan^2\theta = 2/6$

$\tan\theta = \pm\sqrt{1/3}$

$\theta = -\frac{1}{6}\pi$

$x = \tan\left(-\frac{1}{6}\pi\right)$

$x = -\frac{1}{\sqrt{3}}$

4. June/2020/Paper_9709/31/No.4

The curve with equation $y = e^{2x}(\sin x + 3\cos x)$ has a stationary point in the interval $0 \leqslant x \leqslant \pi$.

(a) Find the x-coordinate of this point, giving your answer correct to 2 decimal places. [4]

$u = e^{2x}$ $\quad v = \sin x + 3\cos x$

$u' = 2e^{2x}$ $\quad v' = \cos x - 3\sin x$

using product rule

$u'v + uv'$

$2e^{2x}(\sin x + 3\cos x) + e^{2x}(\cos x - 3\sin x)$

$2e^{2x}\sin x + 6e^{2x}\cos x + e^{2x}\cos x - 3e^{2x}\sin x$

$\frac{dy}{dx} = 7e^{2x}\cos x - e^{2x}\sin x = 0$

$e^{2x}(7\cos x - \sin x) = 0$

$7\cos x = \sin x$ (divide by $\cos x$)

$7 = \tan x$

$x = \tan^{-1} 7 = 1.429$

$x = 1.43$

(b) Determine whether the stationary point is a maximum or a minimum. [2]

$\frac{d^2y}{dx^2} = 2e^{2x}(7\cos x - \sin x) + e^{2x}(-7\sin x - \cos x)$ $u = e^{2x}$ $u' = 2e^{2x}$

$v = 7\cos x - \sin x$

$v' = -7\sin x - \cos x$

$= 2e^{2x}(7\cos x - \sin x) + e^{2x}(-7\sin x - \cos x)$

when $x = 1.43$

$\frac{d^2y}{dx^2} < 0$ maximum.

5. June/2020/Paper_9709/32/No.4

A curve has equation $y = \cos x \sin 2x$.

Find the x-coordinate of the stationary point in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

use product rule $\quad u = \cos x \quad v = \sin 2x$

$u' = -\sin x \quad v' = 2\cos 2x$

$$\frac{dy}{dx} = -\sin x \sin 2x + 2\cos x \cos 2x$$

$$= -\sin x(2\sin x \cos x) + 2\cos x(2\cos^2 x - 1)$$

$$= -2\sin^2 x \cos x + 4\cos^3 x - 2\cos x$$

$$= -2\cos x(1 - \cos^2 x) + 4\cos^3 x - 2\cos x$$

$$= -2\cos x + 2\cos^3 x + 4\cos^3 x - 2\cos x$$

$$6\cos^3 x - 4\cos x = 0$$

$$\frac{6\cos^3 x}{6\cos x} = \frac{4\cos x}{6\cos x}$$

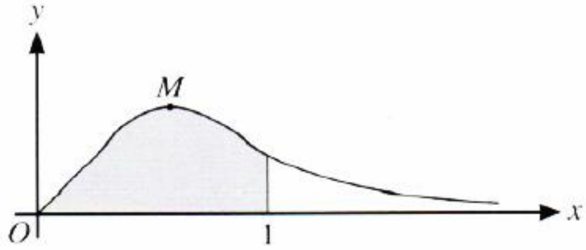
$$\cos^2 x = 2/3$$

$$\cos x = \sqrt{2/3}$$

$$x = \cos^{-1}\sqrt{2/3}$$

$$x = 0.615$$

6. June/2020/Paper_9709/32/No.6



The diagram shows the curve $y = \dfrac{x}{1+3x^4}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [4]

$y = \dfrac{x}{1+3x^4}$ use goutient

$u = x$

$u' = \dfrac{du}{dx} = 1$

$\dfrac{u'v - uv'}{v^2}$

$v = 1+3x^4$

$v' = \dfrac{dv}{dx} = 12x^3$

$\dfrac{dy}{dx} = \dfrac{1+3x^4 - 12x^4}{(1+3x^4)^2} = 0$

so $-12x^4 + 3x^4 + 1 = 0$

$-9x^4 + 1 = 0$

$x^4 = 1/9$

$x = \sqrt[4]{1/9}$

$\boxed{x = 0.577}$

(b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 1$. [5]

$u = \sqrt{3}\,x^2$

$\frac{du}{dx} = 2\sqrt{3}\,x$

$du = 2\sqrt{3}\,x\,dx$

$\text{Area} = \int_0^1 \frac{x}{1+3x^4}\,dx$ $\qquad \frac{du}{2\sqrt{3}} = \boxed{x\,dx}$

$\int_0^{\sqrt{3}} \frac{1}{1+u^2}\,\frac{du}{2\sqrt{3}}$

$\left[\frac{1}{2\sqrt{3}}\tan^{-1}u\right]_0^{\sqrt{3}}$

$\left[\frac{1}{2\sqrt{3}} \times \tan^{-1}\sqrt{3}\right] - \left[\frac{1}{2\sqrt{3}}\tan^{-1}0\right]$

limits
$u = \sqrt{3}\,(1)^2$
$u = \sqrt{3}$
$u = \sqrt{3}\,(0)^2$
$u = 0$

$\frac{1}{2\sqrt{3}} \times \frac{1}{3}\pi - 0$

$\frac{1 \times \sqrt{3}}{2\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{6} \times \frac{1}{3}\pi$

$\frac{\sqrt{3}}{18}\pi$

7. June/2020/Paper_9709/33/No.4

The equation of a curve is $y = x\tan^{-1}(\frac{1}{2}x)$.

(a) Find $\frac{dy}{dx}$. [3]

$u = x \quad v = \tan^{-1}\left(\frac{1}{2}x\right) = \tan^{-1}\left(\frac{x}{2}\right)$

$u' = 1 \quad v' = \frac{2}{x^2+4}$

$u'v + uv'$

$\frac{dy}{dx} = \tan^{-1}\left(\frac{1}{2}x\right) + \frac{2x}{x^2+4}$

(b) The tangent to the curve at the point where $x = 2$ meets the y-axis at the point with coordinates $(0, p)$.

Find p. [3]

$y = x\tan^{-1}\left(\frac{1}{2}x\right)$ when $x = 2$

$y = 2\tan^{-1}\left(\frac{1}{2}\times 2\right) = \frac{1}{2}\pi$

$\frac{dy}{dx} = \tan^{-1}(1) + \frac{2\times 2}{2^2+4} = \frac{\pi}{4} + \frac{4}{8} = \frac{\pi+2}{4}$

$y - \frac{1}{2}\pi = \frac{\pi+2}{4}(x-2)$

$y - \frac{1}{2}\pi = \frac{\pi+2}{4}x - \frac{\pi+2}{2}$ when $x = 0$

$y = -\frac{\pi+2}{2} + \frac{1}{2}\pi \quad y = -1$

$p = -1$

8. March/2020/Paper_9709/32/No.7

The equation of a curve is $x^3 + 3xy^2 - y^3 = 5$.

(a) Show that $\frac{dy}{dx} = \frac{x^2 + y^2}{y^2 - 2xy}$. [4]

$x^3 + 3xy^2 - y^3 = 5$

$3x^2 + 3y^2 + 6xy\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$

$3x^2 + 3y^2 = 3y^2 dy/dx - 6xy\frac{dy}{dx}$

$3x^2 + 3y^2 = dy/dx\,(3y^2 - 6xy)$

$dy/dx = \frac{3x^2 + 3y^2}{3y^2 - 6xy}$

$\frac{dy}{dx} = \frac{3(x^2+y^2)}{3(y^2-2xy)} = \frac{x^2+y^2}{y^2-2xy}$

(b) Find the coordinates of the points on the curve where the tangent is parallel to the y-axis. [5]

parallel to y axis equate denominator to zero.

$y^2 = 2xy$

$y = 2x$

$x^3 + 3x(2x)^2 - (2x)^3 = 5$

$x^3 + 12x^3 - 8x^3 = 5$

$5x^3 = 5$

$x^3 = 1$ $\quad x = 1$

$y = 2x$

$y = 2$

$(1, 2)$

Using $y^2 - 2xy = 0$

$y(y - 2x) = 0$

$y = 0$

$\therefore x^3 + 3x(0)^2 - (0)^3 = 5$

$x^3 = 5 \quad x = \sqrt[3]{5}$

another point

$(\sqrt[3]{5}, 0)$