

1. June/2021/Paper\_9709/21/No.4

A curve has parametric equations

$$x = \ln(2t + 6) - \ln t, \quad y = t \ln t.$$

(a) Find the value of  $t$  at the point  $P$  on the curve for which  $x = \ln 4$ .

[3]

$$x = \ln 4$$

$$\ln 4 = \ln(2t + 6) - \ln t$$

$$\ln 4 = \ln \left( \frac{2t + 6}{t} \right)$$

$$4 = \frac{2t + 6}{t}$$

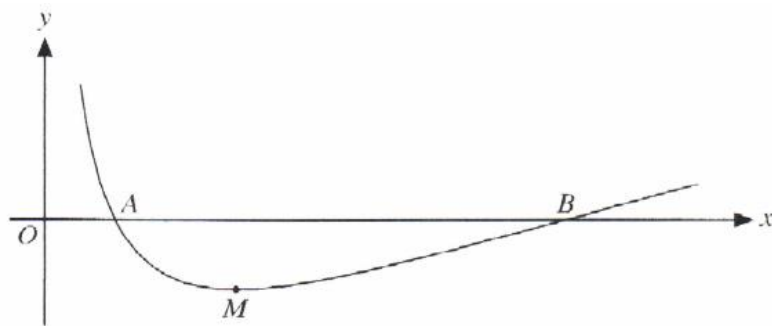
$$4t = 2t + 6$$

$$2t = 6$$

$$t = \frac{6}{2} = 3$$

$$t = 3$$





The diagram shows the curve with equation

$$y = (\ln x)^2 - 2 \ln x.$$

The curve crosses the  $x$ -axis at the points  $A$  and  $B$ , and has a minimum point  $M$ .

- (a) Find the exact value of the gradient of the curve at each of the points  $A$  and  $B$ . [6]

$$\frac{dy}{dx} = 2(\ln x)' \times \frac{1}{x} - 2 \times \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{2 \ln x}{x} - \frac{2}{x}$$

Coordinates of  $A$  and  $B$

$$(\ln x)^2 - 2 \ln x = 0$$

$$\downarrow$$

$$u - 2u = 0$$

$$u(u - 2) = 0$$

$$u = 0 \quad u = 2$$

$$\ln x = 0$$

$$1 = e^0 \quad x = 1$$

$$\ln x = 2 \quad x = e^2$$

$$A(1, 0)$$

$$B(e^2, 0)$$

$$m_A = \frac{2 \ln 1}{1} - \frac{2}{1}$$
$$= -2$$

$$m_B = \frac{2 \ln e^2}{e^2} - \frac{2}{e^2}$$

$$M_B = \frac{4}{e^2} - \frac{2}{e^2} = \frac{2}{e^2}$$

(b) Find the exact  $x$ -coordinate of  $M$ .

[2]

$$\frac{2 \ln x}{x} - \frac{2}{x} = 0$$

$$\frac{2 \ln x}{x} = \frac{2}{x}$$

$$2 \ln x = 2$$

$$\ln x^2 = 2$$

$$x^2 = e^2$$

$$x = e$$

$$y = (\ln e)^2 - 2 \ln e$$

$$y = 1 - 2 = -1$$

$$(e, -1)$$

The parametric equations of a curve are

$$x = e^{2t} \cos 4t, \quad y = 3 \sin 2t.$$

Find the gradient of the curve at the point for which  $t = 0$ .

[5]

$$\frac{dx}{dt} \Rightarrow \text{product rule}$$

$$u = e^{2t}$$

$$\frac{du}{dt} = 2e^{2t}$$

$$v = \cos 4t$$

$$\frac{dx}{dt} = 2e^{2t} \cos 4t - 4e^{2t} \sin 4t \quad \frac{dv}{dt} = -4 \sin 4t$$

$$\frac{dy}{dt} = 6 \cos 2t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{6 \cos 2t}{2e^{2t} \cos 4t - 4e^{2t} \sin 4t}$$

$$\text{when } t=0 \quad m = \frac{6 \cos(2 \times 0)}{2e^0 \cos 0 - 4e^0 \sin 0}$$

$$= \frac{6}{2} = 3$$