Hypothesis Tests – 2021 A2

1. June/2021/Paper_9709/61/No.8

At a certain large school it was found that the proportion of students not wearing correct uniform was 0.15. The school sent a letter to parents asking them to ensure that their children wear the correct uniform. The school now wishes to test whether the proportion not wearing correct uniform has been reduced.

(a) It is suggested that a random sample of the students in Grade 12 should be used for the test.

Give a reason why this would not be an appropriate sample. [1]

Not representative of all the students in the school

A suitable sample of 50 students is selected and the number not wearing correct uniform is noted. This figure is used to carry out a test at the 5% significance level.

(b) State suitable null and alternative hypotheses. [1]

$h_0: p = 0.15$

$h_1: p < 0.15$

(c) Use a binomial distribution to find the probability of a Type I error. You must justify your answer fully. [5]

$X \sim B(50, 0.15)$

Tng $P(X \leq 3) = {}^{50}C_0 (0.15)^0 (0.85)^{50} + {}^{50}C_1 (0.15)^1 (0.85)^{49} +$

${}^{50}C_2 (0.15)^2 (0.85)^{48} + {}^{50}C_3 (0.15)^3 (0.85)^{47} = 0.04605$

$P(X \leq 3) = 0.04605 < 0.05$

$P(X \leq 4) = 0.04605 + {}^{50}C_4 (0.15)^4 (0.85)^{46}$

$P(X \leq 4) = 0.112 > 0.05$

Type I error $= 0.0460$

(d) In fact 4 students out of the 50 are not wearing correct uniform.

State the conclusion of the test, explaining your answer. [2]

4 is outside the Critical

$(\le 3) = 0.04605 < 0.05$

$P(x \le 4) = 0.112 > 0.05$

No evidence that proportion not weaving the correct uniform decrease

Accept H_0

(e) State, with a reason, which of the errors, Type I or Type II, may have been made. [2]

H_0 not rejected (Accept H_0)

type II error

2. June/2021/Paper_9709/62/No.1

In a game, a ball is thrown and lands in one of 4 slots, labelled A, B, C and D. Raju wishes to test whether the probability that the ball will land in slot A is $\frac{1}{4}$.

(a) State suitable null and alternative hypotheses for Raju's test. [1]

$H_0 : p = 1/4$

$H_1 : p \neq 1/4$

The ball is thrown 100 times and it lands in slot A 15 times.

(b) Use a suitable approximating distribution to carry out the test at the 2% significance level. [5]

$X \sim B(100, 1/4)$

$np = 100 \times 1/4 = 25 > 5$ so Normal dist.

$npq = 25 \times 3/4 = \frac{75}{4}$

$X \sim N(25, 75/4)$

$P(X \leq 15)$ $\quad P\left(Z \leq \frac{15.5 - 25}{\sqrt{75/4}}\right)$

$P(Z \leq -2.194) = 1 - \phi(2.194)$

$= 0.0141 > 0.01$

ie 2% two tailed $0.02 \div 2 = 0.01$

Accept H_0; no evidence that probability is not $1/4$

3. June/2021/Paper_9709/62/No.5

The time, in minutes, spent by customers at a particular gym has the distribution $N(\mu, 38.2)$. In the past the value of μ has been 42.4. Following the installation of some new equipment the management wishes to test whether the value of μ has changed.

(a) State what is meant by a Type I error in this context. [1]

Conclude that mean time has changed although it has not

(b) The mean time for a sample of 20 customers is found to be 45.6 minutes.

Test at the 2.5% significance level whether the value of μ has changed. [5]

$X \sim N\left(42.4, \frac{38.2}{20}\right)$

$P(\bar{X} < 45.6) = $ $z = \frac{45.6 - 42.4}{\sqrt{38.2/20}}$

$z = 2.315$

at 2.5% $z = 2.240$ two tailed

$H_0: \mu = 42.4$

$H_1: \mu \neq 42.4$

$2.240 < 2.315$

There is no evidence that the mean has changed.

4. June/2021/Paper_9709/63/No.2

In the past, the time, in hours, for a particular train journey has had mean 1.40 and standard deviation 0.12. Following the introduction of some new signals, it is required to test whether the mean journey time has decreased.

(a) State what is meant by a Type II error in this context. [1]

Conclude that the mean journey time has not decreased when in fact it has.

(b) The mean time for a random sample of 50 journeys is found to be 1.36 hours.

Assuming that the standard deviation of journey times is still 0.12 hours, test at the 2.5% significance level whether the population mean journey time has decreased. [5]

$H_0: \mu = 1.4$

$H_1: \mu < 1.4$

$n = 50$ $\bar{x} = 1.36$ $\sigma = \frac{0.12}{\sqrt{50}}$

$z \text{ value} = \frac{1.36 - 1.4}{\frac{0.12}{\sqrt{50}}}$

$z = -2.357$ at 2.5% $z = \pm 1.96$

$-2.357 < -1.96$ There is evidence that the mean journey has changed.

(c) State, with a reason, which of the errors, Type I or Type II, might have been made in the test in part **(b)**. [2]

H_0 was rejected

Type I error

5. June/2021/Paper_9709/63/No.3

The local council claims that the average number of accidents per year on a particular road is 0.8. Jane claims that the true average is greater than 0.8. She looks at the records for a random sample of 3 recent years and finds that the total number of accidents during those 3 years was 5.

(a) Assume that the number of accidents per year follows a Poisson distribution.

(i) State null and alternative hypotheses for a test of Jane's claim. [1]

$H_0: \lambda = 2.4$ for 3 years $3 \times 0.8 = 2.4$

$H_1: \lambda > 2.4$

(ii) Test at the 5% significance level whether Jane's claim is justified. [4]

$P(X \geq 5) = 1 - P(X = 0, 1, 2, 3, 4)$

$1 - e^{-2.4}\left(\frac{2.4^0}{0!} + \frac{2.4}{1!} + \frac{2.4^2}{2!} + \frac{2.4^3}{3!} + \frac{2.4^4}{4!}\right)$

$1 - e^{-2.4}\left(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!} + \frac{2.4^4}{4!}\right)$

$= 0.0959$

at 5%

$0.0959 > 0.05$

There is insufficient evidence to support Jane's claim.

(b) Jane finds that the number of accidents per year has been gradually increasing over recent years.

State how this might affect the validity of the test carried out in part **(a)(ii)**. [1]

Mean not constant; so Poisson model is not valid.

6. March/2021/Paper_9709/62/No.3

An architect wishes to investigate whether the buildings in a certain city are higher, on average, than buildings in other cities. He takes a large random sample of buildings from the city and finds the mean height of the buildings in the sample. He calculates the value of the test statistic, z, and finds that $z = 2.41$.

(a) Explain briefly whether he should use a one-tail test or a two-tail test. [1]

One tailed because and the architect is investigation whether it's 'higher'

(b) Carry out the test at the 1% significance level. [3]

H_0: pop' mean (μ) in city same as other

H_1: pop mean in city is greather than others

$2.41 > 2.326 \rightarrow 1\%$

There is evidence that the buildings are higher

7. March/2021/Paper_9709/62/No.6

It is known that 8% of adults in a certain town own a Chantor car. After an advertising campaign, a car dealer wishes to investigate whether this proportion has increased. He chooses a random sample of 25 adults from the town and notes how many of them own a Chantor car.

(a) He finds that 4 of the 25 adults own a Chantor car.

Carry out a hypothesis test at the 5% significance level. [5]

8/100

H_0: pop' proportion $= 0.08$

H_1: pop proportion > 0.08

$P(X \geq 4) = 1 - P(X \leq 3) \rightarrow$ binomial

$X \sim B(25, 0.08)$ $p = 0.08$

$q = 0.92$

$1 - \left(0.92^{25} + {}^{25}C_1 \times 0.92^{24} \times 0.08 + {}^{25}C_2 \times 0.92^{23} \times 0.08^2 + {}^{25}C_3 \times 0.92^{22} \times 0.08^3\right) = 0.1351$

0.135 3 significant figures

$0.135 > 0.05$

There is no evidence that the proportion owning chantor has increased

(b) Explain which of the errors, Type I or Type II, might have been made in carrying out the test in part **(a)**. [2]

Ho was not rejected
Hence type II might have been made

Later, the car dealer takes another random sample of 25 adults from the town and carries out a similar hypothesis test at the 5% significance level.

(c) Find the probability of a Type I error. [3]

$P(X \geq 4) = 0.1351$

$P(X \geq 5) = 1 - P(X \leq 4)$

$1 - \{P(X \leq 3) + P(X = 4)\}$

$1 - [(1 - 0.1351) + {}^{25}C_4 \times 0.92^{21} \times 0.08^4]$

$= 0.0451$ which less than 0.5

$0.0451 < 0.5$

$P(\text{Type I error}) = 0.0451$