The Poisson distribution – 2020 A2 Math

1. Nov/2020/Paper_9709/61/No.1

It is known that, on average, 1 in 300 flowers of a certain kind are white. A random sample of 200 flowers of this kind is selected.

(a) Use an appropriate approximating distribution to find the probability that more than 1 flower in the sample is white. [3]

$X \sim Po(\frac{2}{3})$ $\mu = np = 200 \times \frac{1}{300} = \frac{2}{3}$

$P(X > 1) = 1 - P(X = 0, 1)$.

$1 - e^{-\frac{2}{3}}(1 + \frac{2}{3})$

$= 0.144$

(b) Justify the approximating distribution used in part **(a)**. [1]

$n > 50$ $np = \frac{2}{3} < 5$; $p = \frac{1}{300} < 0.1$

The probability that a randomly chosen flower of another kind is white is 0.02. A random sample of 150 of these flowers is selected.

(c) Use an appropriate approximating distribution to find the probability that the total number of white flowers in the two samples is less than 4. [3]

$\mu = np = 150 \times 0.02 = 3$

$E(X+Y) = 3 + 2/3 = 11/3$

$X \sim Po(11/3) \qquad P(X < 4) \Rightarrow X = 0, 1, 2, 3$

$e^{-11/4}\left\{1 + \frac{11}{3} + \frac{(11/3)^2}{2!} + \frac{(11/3)^3}{3!}\right\}$

$= 0.501$

2. Nov/2020/Paper_9709/61/No.5a,5b

The number of absences per week by workers at a factory has the distribution Po(2.1).

(a) Find the standard deviation of the number of absences per week. [1]

$\sqrt{2.1} = 1.45$

(b) Find the probability that the number of absences in a 2-week period is at least 2. [3]

2-Week Period $\lambda = 4.2$

$P(X \geqslant 2) : 1-(X=0,1)$

$1 - e^{-4.2}(1+4.2)$

$= 0.922$

3. Nov/2020/Paper_9709/62/No.1

On average, 1 in 50 000 people have a certain gene.

Use a suitable approximating distribution to find the probability that more than 2 people in a random sample of 150 000 have the gene. [3]

$\lambda = np = \frac{1}{50{,}000} \times 150{,}000 = 3$

$X \sim Po(3)$

$P(X>2) = 1 - P(X=0,1,2)$

$1 - e^{-3}\left[1 + 3 + \frac{3^2}{2!}\right]$

$= 0.577$

4. Nov/2020/Paper_9709/62/No.5

Customers arrive at a shop at a constant average rate of 2.3 per minute.

(a) State another condition for the number of customers arriving per minute to have a Poisson distribution. [1]

Customers arrive independently or singly or at random.

It is now given that the number of customers arriving per minute has the distribution Po(2.3).

(b) Find the probability that exactly 3 customers arrive during a 1-minute period. [2]

$\lambda = 2.3$

$P(X=3) = e^{-2.3} \times \frac{2.3^3}{3!}$

$= 0.203$

(c) Find the probability that more than 3 customers arrive during a 2-minute period. [3]

$\lambda = 2 \times 2.3 = 4.6$

$P(X>3) = 1 - P(X=0,1,2,3)$

$1 - e^{-4.6}\left(1 + 4.6 + \frac{4.6^2}{2!} + \frac{4.6^3}{3!}\right)$

$= 0.674$

(d) Five 1-minute periods are chosen at random. Find the probability that no customers arrive during exactly 2 of these 5 periods. [3]

$P(\text{no Customer arrive}) = e^{-2.3} = 0.10026.$

$p = 0.10026 \quad q = 0.89974$

${}^5C_2 (0.10026)^2 \times (0.89974)^3$

$= 0.0732$

5. June/2020/Paper_9709/61/No.5

Each week a sports team plays one home match and one away match. In their home matches they score goals at a constant average rate of 2.1 goals per match. In their away matches they score goals at a constant average rate of 0.8 goals per match. You may assume that goals are scored at random times and independently of one another.

(a) A week is chosen at random.

(i) Find the probability that the team scores a total of 4 goals in their two matches. [2]

$\lambda = 2.1 + 0.8 = 2.9 \quad \sim Po(2.9)$

$P(X=r) = e^{-\lambda} \times \frac{\lambda^r}{r!}$

$= e^{-2.9} \times \frac{2.9^4}{4!} = 0.162$

(ii) Find the probability that the team scores a total of 4 goals, with more goals scored in the home match than in the away match. [3]

for $Po(2.1) \rightarrow P(X=3, 4)$ for $Po(0.8) \rightarrow P(X=1, 0)$

$e^{-2.1} \times \frac{2.1^4}{4!} \times e^{-0.8} \times \frac{0.8^0}{0!} +$

$e^{-2.1} \times \frac{2.1^3}{3!} \times e^{-0.8} \times \frac{0.8^1}{1!}$

$0.0679 + 0.0446$

$= 0.1125$

(b) Use a suitable approximating distribution to find the probability that the team scores fewer than 25 goals in 10 randomly chosen weeks. [4]

$\lambda = 10 \times 2.1 + 10 \times 0.8 = 21 + 8 = 29$

$X \sim N(29, 29)$

$P(X < 25) \simeq P(X < 24.5)$ approx.

$P\left(Z < \frac{24.5 - 29}{\sqrt{29}}\right) = P(Z < -0.8356$

$P(Z < -a) = 1 - \phi(a)$

$P(Z < -0.836) = 1 - \phi(0.836)$

$1 - 0.7983$

$= \underline{\underline{0.2017}}$

(c) Justify the use of the approximating distribution used in part **(b)**. [1]

$np > 15$ $29 > 15$

6. June/2020/Paper_9709/62/No.3

In the data-entry department of a certain firm, it is known that 0.12% of data items are entered incorrectly, and that these errors occur randomly and independently.

(a) A random sample of 3600 data items is chosen. The number of these data items that are incorrectly entered is denoted by X.

(i) State the distribution of X, including the values of any parameters. [1]

$X \sim B(3600, 0.0012)$

(ii) State an appropriate approximating distribution for X, including the values of any parameters.

Justify your choice of approximating distribution. [3]

$X \sim Po(4.32)$ $\lambda = np = 3600 \times 0.0012$

n is very large $(n = 3600)$

$np = 3600 \times 0.0012 = 4.32 < 5$

normal not possible.

(iii) Use your approximating distribution to find $P(X > 2)$. [2]

$P(X > 2) = 1 - P(X = 0, 1, 2)$

$1 - e^{-4.32}\left(1 + 4.32 + \frac{4.32^2}{2!}\right)$

$1 - 0.1949$

$= 0.8051$

(b) Another large random sample of n data items is chosen. The probability that the sample contains no data items that are entered incorrectly is more than 0.1.

Use an approximating distribution to find the largest possible value of n. [3]

$e^{-\lambda} > 0.1$

$-\lambda \ln e > \ln 0.1$

$\lambda < \ln 10$

$\lambda = np \qquad p = 0.0012$

$\lambda = 0.0012n$

$0.0012n < \ln 10$

$n < \dfrac{\ln 10}{0.0012}$

$n < 1918.8$

$n < 1918$

7. June/2020/Paper_9709/62/No.5

(a) The random variable X has the distribution $\mathrm{Po}(\lambda)$.

(i) State the values that X can take. [1]

$0, 1, 2, 3 \ldots$

It is given that $\mathrm{P}(X = 1) = 3 \times \mathrm{P}(X = 0)$.

(ii) Find λ. [1]

$e^{-\lambda} \times \frac{\lambda^1}{1!} = 3 \times e^{-\lambda} \times \frac{\lambda^0}{0!}$

$e^{-\lambda} \times \lambda = 3 \times e^{-\lambda}$

$\lambda = 3$

(iii) Find $\mathrm{P}(4 \leqslant X \leqslant 6)$. [2]

$e^{-3}\left(\frac{3^4}{4!} + \frac{3^5}{5!} + \frac{3^6}{6!}\right)$

0.3193

(b) The random variable Y has the distribution $\mathrm{Po}(\mu)$ where μ is large. Using a suitable approximating distribution, it is found that $\mathrm{P}(Y < 46) = 0.0668$, correct to 4 decimal places.

Find μ. [5]

approximation $Y \sim N(\mu, \mu)$

$P\left(Z < \frac{45.5 - \mu}{\sqrt{\mu}}\right) = 0.0668$

$\frac{4 \quad - \mu}{\sqrt{\mu}} = -\phi^{-1}(1 - 0.0668)$

$\frac{45.5 - \mu}{\sqrt{\mu}} = -\phi^{-1}(0.9332)$

$\frac{45.5 - \mu}{\sqrt{\mu}} = -1.5$

$4.55 - \mu = -1.5\sqrt{\mu}$.

$\mu - 1.5\sqrt{\mu} - 4 \quad = 0$

let x be $\sqrt{\mu}$.

$x^2 - 1.5x - 45.5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{1.5 \pm \sqrt{1.5^2 - 4(1)(-45.5}}{2}$

$x = \frac{1.5 \pm 13.57}{2}$. $\quad x = 7.537$

$\mu = x^2 = 56.8$

8. March/2020/Paper_9709/62/No.1

The booklets produced by a certain publisher contain, on average, 1 incorrect letter per 30 000 letters, and these errors occur randomly. A randomly chosen booklet from this publisher contains 12 500 letters.

Use a suitable approximating distribution to find the probability that this booklet contains at least 2 errors. [3]

$\lambda = \frac{1}{30{,}000} \times 12{,}500$

$\Rightarrow \lambda = 5/12$ or 0.417

$X \sim Po(5/12)$ $\quad P(X \geq 2) = 1 - P(X = 0, 1)$

$P(X=r) = e^{-\lambda}\frac{\lambda^r}{r!}$

$1 - e^{-5/12}\left(1 + 5/12\right)$

$1 - 0.9339$

$= 0.0661$

9. March/2020/Paper_9709/62/No.4

The number of accidents on a certain road has a Poisson distribution with mean 0.4 per 50-day period.

(a) Find the probability that there will be fewer than 3 accidents during a year (365 days). [3]

$\lambda = \frac{0.4 \times 365}{50} = 2.92$

$P(X < 3) = P(X = 0, 1, 2)$ $\quad P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$

$e^{-2.92}\left(1 + 2.92 + \frac{2.92^2}{2!}\right)$

$= 0.44135$

$= 0.441$

(b) The probability that there will be no accidents during a period of n days is greater than 0.95.

Find the largest possible value of n. [4]

$e^{-\lambda} > 0.95$

introduce ln

$-\lambda \ln e > \ln 0.95$

$\lambda < 0.051293$

so for number of days

$0.051293 \times 50 \div 0.4$

(0.4 per 50 day period)

$= 6.411$

largest n is 6