## Integration – 2022 A2 Nov Math

1. Nov/2022/Paper_9709_21/No.6(b)

**(b)** Hence find $\int_2^{12}\left(\frac{p(x)}{4x-3}-3x^2\right)dx$, giving your answer in the form $a+\ln b$. [6]

From (a) $\frac{p(x)}{4x-3} = 3x^2 + 2 + \frac{2}{4x-3}$

$\Rightarrow \int_2^{12}\left(\frac{p(x)}{4x-3} - 3x^2\right)dx = \int_2^{12}\left(3x^2 + 2 + \frac{2}{4x-3} - 3x^2\right)dx$

$= \int_2^{12}\left(2 + \frac{2}{4x-3}\right)dx$

$$= 2x + \frac{2\ln(4x-3)}{4}\Big|_2^{12}$$

$$= 2x + \frac{1}{2}\ln(4x-3)\Big|_2^{12}$$

$$= 2(12) + \frac{1}{2}\ln(4(12)-3) - \left(2(2) + \frac{1}{2}\ln(4(2)-3)\right)$$

$$= 24 + \frac{1}{2}\ln 45 - 4 - \frac{1}{2}\ln 5$$

$$= 20 + \frac{1}{2}\ln 45 - \frac{1}{2}\ln 5$$

Recall that $\ln a^b = b\ln a$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$$\Rightarrow 20 + \ln 45^{\frac{1}{2}} - \ln 5^{\frac{1}{2}}$$

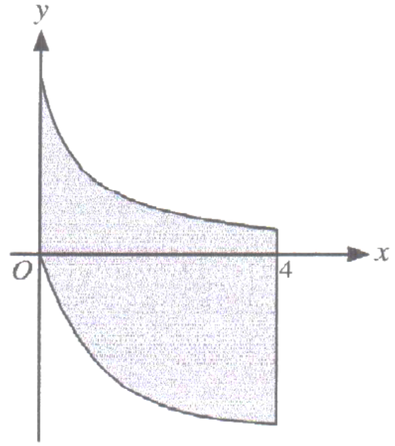
$$= 20 + \ln\sqrt{45} - \ln\sqrt{5}$$

$$= 20 + \ln\left(\frac{\sqrt{45}}{\sqrt{5}}\right)$$

$$= 20 + \ln 3$$

$$\therefore \int_2^{12}\left(\frac{p(x)}{4x-3} - 3x^2\right)dx = 20 + \ln 3$$

**2.** Nov/2022/Paper_9709_22/No.6




The diagram shows the curves $y = \frac{6}{3x+2}$ and $y = 3e^{-x} - 3$ for values of $x$ between 0 and 4. The shaded region is bounded by the two curves and the lines $x = 0$ and $x = 4$.

Find the exact area of the shaded region, giving your answer in the form $\ln a + b + ce^{d}$. [9]

Area $= \int_0^4 \frac{6}{3x+2} \, dx - \int_0^4 (3e^{-x} - 3) \, dx$

Using u- substitution let $u = 3x + 2$

$\frac{du}{dx} = 3$

$\Rightarrow dx = \frac{du}{3}$

$\int_0^4 \frac{6}{u} \frac{du}{3} - \int_0^4 (3e^{-x} - 3) \, dx$

$2 \int_0^4 \frac{1}{u} du - \int_0^4 (3e^{-x} - 3) \, dx$

$2 [\ln u]_0^4 - \left[ \frac{3e^{-x}}{-1} - 3x \right]_0^4$

$2 [\ln(3x+2)]_0^4 - \left[ -3e^{-x} - 3x \right]_0^4$