Algebra – 2023 June A2 Math 9709

1. June/2023/Paper_9709/21/No.4

The polynomial $p(x)$ is defined by

$$p(x) = 2x^3 + 3x^2 + kx - 30,$$

where k is a constant. It is given that $(x-3)$ is a factor of $p(x)$.

(a) Find the value of k. [2]

By factor theorem if $(x-3)$ is a factor of $p(x)$ then $p(3) = 0$.

$p(3) = 2(3)^3 + 3(3)^2 + k(3) - 30 = 0$

$\Rightarrow 54 + 27 + 3k - 30 = 0$

$3k + 51 = 0 \Rightarrow \frac{3k}{3} = \frac{-51}{3}$

$\therefore k = -17$

(b) Hence find the quotient when $p(x)$ is divided by $(x-3)$ and factorise $p(x)$ completely. [3]

Using long division method

$$\begin{array}{r|l} & 2x^2 + 9x + 10 \\ \hline x-3 & 2x^3 + 3x^2 - 17x - 30 \\ & -(2x^3 - 6x^2) \\ \hline & 9x^2 - 17x \\ & -(9x^2 - 27x) \\ \hline & 10x - 30 \\ & -(10x - 30) \\ \hline & 0 \end{array}$$

$\therefore$ quotient $= 2x^2 + 9x + 10$

Factorise $2x^2 + 9x + 10$

$2x^2 + 4x + 5x + 10$

$2x(x+2) + 5(x+2)$

$= (x+2)(2x+5)$

$\therefore p(x) = (x-3)(x+2)(2x+5)$.

(c) It is given that a is one of the roots of the equation $p(x) = 0$.

Given also that the equation $|4y - 5| = a$ is satisfied by two real values of y, find these two values of y. [3]

From part (b) one of the roots of $p(x) = 0$ is $x = 3$.

$\Rightarrow \ |4y - 5| = 3$

Recall that $|a| = \sqrt{a^2} = \pm a$

$\Rightarrow \ 4y - 5 = 3$

$4y = 3 + 5$

$\frac{\cancel{4}y}{\cancel{4}} = \frac{8}{4} \Rightarrow y = 2$

$-(4y - 5) = 3$

$-4y + 5 = 3$

$-4y = 3 - 5$

$\frac{-\cancel{4}y}{-\cancel{4}} = \frac{-2}{-4} \Rightarrow y = \frac{1}{2}$

$\therefore \ y = 2, \ \frac{1}{2}$