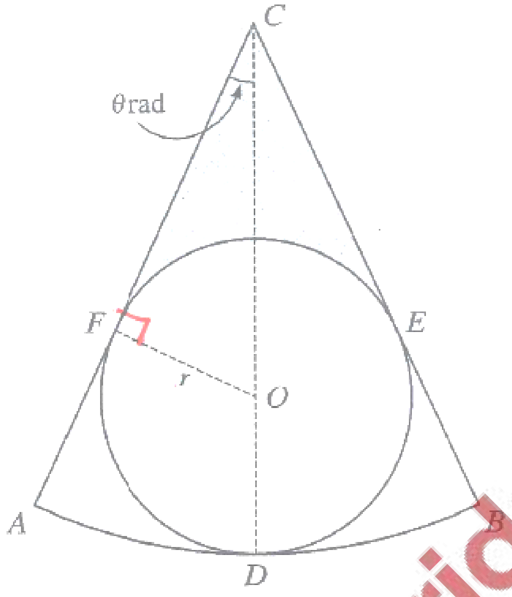
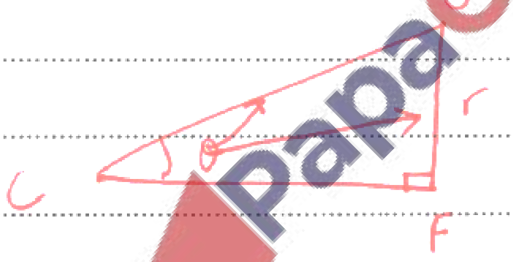
Circular Measure – 2020 AS

1. Nov/2020/Paper_9709/11/No.10



The diagram shows a sector CAB which is part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E and F, where COD is a straight line and angle ACD is θ radians.

(a) Find CD in terms of r and $\sin\theta$. [3]



$$\sin\theta = \frac{r}{OC}$$

$$OC = \frac{r}{\sin\theta}$$

$$CD = CO + OD$$

$$\frac{r}{\sin\theta} + r$$

It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

(b) Find the perimeter of sector CAB in terms of π. [3]

$AB = r\theta \Rightarrow r = \frac{4}{\sin \frac{1}{6}\pi} + 4 = 12$

2θ

$AB = 12 \times \frac{1}{3}\pi = 4\pi$

$AC = CB = 12$

$12 + 12 + 4\pi$

$\boxed{24 + 4\pi}$

(c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

$\tan \frac{1}{6}\pi = \frac{FO}{FC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4}{FO}$

$FO = 4\sqrt{3}$

Area of $OFC = \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$

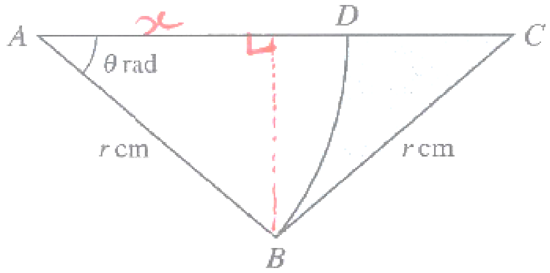
Area of $OFCE = 2 \times 8\sqrt{3} = 16\sqrt{3}$

Area of sector $OFE = \frac{1}{2} \times 4^2 \times \frac{2}{3}\pi$

$= \frac{16\pi}{3}$

hence $16\sqrt{3} - \frac{16}{3}\pi$

2. Nov/2020/Paper_9709/12/No.8



In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

$\cos\theta = \frac{x}{r}$ $x = r\cos\theta$

$AC = 2r\cos\theta$

Area of $ABC = \frac{1}{2}ab\sin C$

$\frac{1}{2} \times r \times 2r\cos\theta\sin\theta$

$r^2\sin\theta\cos\theta$

Area of the sector $\frac{1}{2}r^2\theta$

$r^2\sin\theta\cos\theta - \frac{1}{2}r^2\theta$.

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

Perimeter $= BD + DC + BC$

$BD = r\theta = 10 \times 0.6 = 6$ cm.

$BC = 6$ cm.

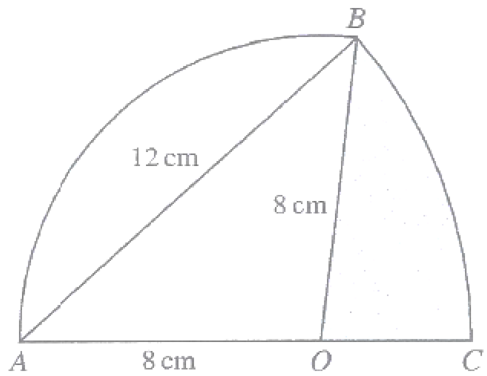
$AC = 2r\cos\theta = 2 \times 10 \cos 0.6$

$DC = 2r\cos\theta - r = 20\cos 0.6 - 10$

$= 6.506$

$10 + 6 + 6.506 = 22.5$

3. Nov/2020/Paper_9709/13/No.9



In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

Using cosine rule.

$\cos A = \dfrac{12^2 + 8^2 - 8^2}{2 \times 12 \times 8}$

$\cos A = 0.75$

angle $BAO = 0.7227$ radians

(b) Find the area of the shaded region. [4]

Area of the sector ABC $= \frac{1}{2}r^2\theta$

$\frac{1}{2} \times 12 \times 12 \times 0.7227$

$= 52.0344$

Area of the triangle OAB

$\frac{1}{2} \times 12 \times 8 \sin 0.7227$ $\left(\frac{1}{2}ab\sin C\right)$

$= 31.75$

$52.0344 - 31.75 = 20.2844$

$\underline{\underline{20.3}}$ cm^2.

(c) Find the perimeter of the shaded region. [3]

$P = BO + OC + BC$

arc BC $= r\theta = 12 \times 0.7227$

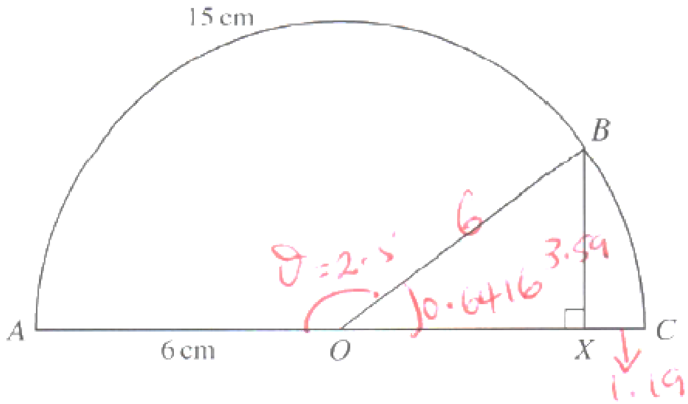
$= 8.6724$

$OC = 12 - 8 = 4$ cm.

$P = 8.67 + 8 + 4$

$= 20.7$ cm.

4. June/2020/Paper_9709/11/No.8



In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

arc length = $r\theta$ $\quad 15 = 6\theta$

$\theta = \frac{15}{6} = 2.5$

$\pi - 2.5 = 0.6416$

$\sin 0.6416 = \frac{BX}{6}$

$BX = 6 \sin 0.6416 = 3.59$ cm.

$\cos 0.6416 = \frac{OX}{6}$

$OX = 6 \cos 0.6416 = 4.81$

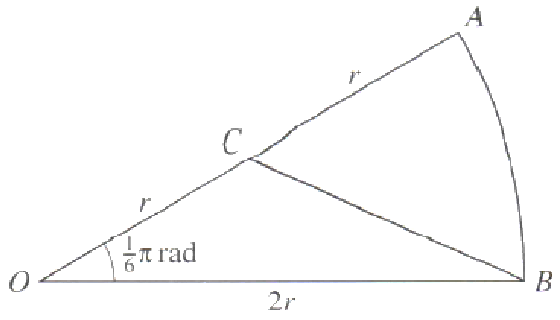
$6 - 4.81 = 1.19$

$BC = r\theta = 6 \times 0.6416 = 3.85$

$3.59 + 1.19 + 3.85$

$= 8.63$ cm.

5. June/2020/Paper_9709/12/No.7



In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

using cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

$= r^2 + (2r)^2 - 2 \times 2r \times r \cos \frac{1}{6}\pi$

$= 5r^2 - 4r^2 \cos \frac{1}{6}\pi$

$= 5r^2 - \overset{2}{\cancel{4}}r^2 \times \frac{\sqrt{3}}{\cancel{2}_1}$

$a^2 = 5r^2 - 2\sqrt{3}\,r^2$

$a = \sqrt{5r^2 - 2\sqrt{3}\,r^2} = \sqrt{r^2(5 - 2\sqrt{3})}$

$a = r\sqrt{5 - 2\sqrt{3}}$

(b) Find the exact perimeter of the shaded region. [2]

$P = AC + CB + AB$

$= r + r\sqrt{5-2\sqrt{3}} + r\theta$

$= r + r\sqrt{5-2\sqrt{3}} + 2r \times \pi/6$

$= r + r\sqrt{5-2\sqrt{3}} + \frac{\pi r}{3}$

(c) Find the exact area of the shaded region. [3]

Area of the sector OAB − area of OCB

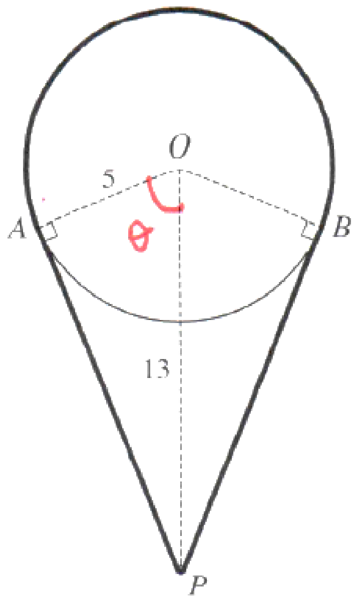
$\frac{1}{2} \times (2r)^2 \times \pi/6 - \frac{1}{2} \times r \times 2r \sin \pi/6$

$\frac{1}{2} \times 4r^2 \times \pi/6 - \frac{1}{2} r \times 2r \times \frac{1}{2}$

$\frac{1}{3}\pi r^2 - \frac{1}{2} r^2$

$r^2\left(\frac{\pi}{3} - \frac{1}{2}\right)$

6. June/2020/Paper_9709/13/No.5



The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

$\cos\theta = \frac{5}{13}$

$\theta = \cos^{-1}(5/13) = 1.176 \times 2$

$= 2.352$

length of major arc AC $= (2\pi - 2.356) \times 5$
$(r\theta)$

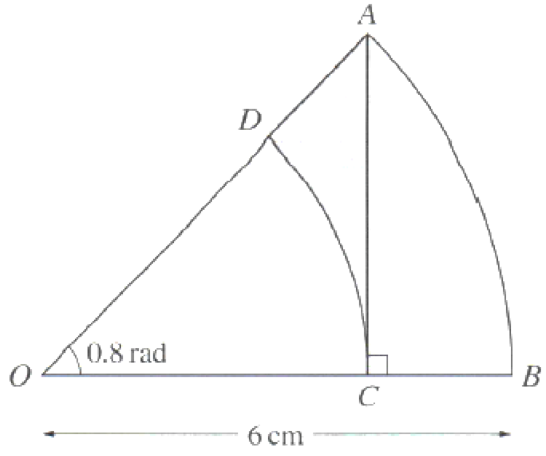
$3.931 = 19.66$

length of AP or BP $\Rightarrow \sqrt{13^2 - 5^2} = 12$
(Pythagoras theorem

length of the cord $= 19.66 + 12 + 12$

$\boxed{= 43.7}$

7. March/2020/Paper_9709/12/No.7



The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA.

Find the area of the shaded region. [6]

$\cos 0.8 = \frac{OC}{OA}$ $OA = 6$; radius

$OC = 6\cos 0.8 = 4.18$

Area of the sector $OCD \Rightarrow \frac{1}{2} r^2 \theta$

radius $= 4.18$

$\frac{1}{2} \times 4.18^2 \times 0.8 = 6.98896.$

Area of the triangle $OAC = \frac{1}{2} ab \sin C$

$\frac{1}{2} \times 6 \times 4.18 \sin 0.8 = 8.9956$

Required area $= 8.9956 - 6.98896$

≈ 2.01 (3 s.f.)