Probability Distribution – 2020 AS

1. Nov/2020/Paper_9709/51/No.4

The random variable X takes each of the values 1, 2, 3, 4 with probability $\frac{1}{4}$. Two independent values of X are chosen at random. If the two values of X are the same, the random variable Y takes that value. Otherwise, the value of Y is the larger value of X minus the smaller value of X.

(a) Draw up the probability distribution table for Y. [4]

	1	2	3	4
1	1	1	2	3
2	1	2	1	2
3	2	1	3	1
4	3	2	1	4

y	1	2	3	4
$P(Y=y)$	$7/16$	$5/16$	$3/16$	$1/16$

(b) Find the probability that $Y = 2$ given that Y is even. [2]

$$\frac{P(Y=2)}{P(E)} = \frac{5/16}{5/16 + 1/16}$$

$$\frac{5/16}{6/16}$$

$$\underline{\underline{5/6}}$$

2. Nov/2020/Paper_9709/52/No.2

A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

$P(1\text{ red}) = \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} \times 3$

$P(RBB)$ or $P(BRB)$ or $P(BBR)$

or $\frac{{}^5C_1 \times {}^3C_2}{{}^8C_3} = \frac{15}{56}$

$\frac{15}{56}$

(b) Draw up the probability distribution table for X. [3]

X	0	1	2	3	total
$P(X=x)$	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$	1

$P(X=0)$ $\frac{{}^5C_0 \times {}^3C_3}{{}^8C_3} = \frac{1}{56}$

$P(X=2)$ $\frac{{}^5C_2 \times {}^3C_1}{{}^8C_3}$

(c) Given that $\mathrm{E}(X) = \frac{15}{8}$, find $\mathrm{Var}(X)$. [2]

$$\left(0^2 \times \frac{1}{56}\right) + \left(1^2 \times \frac{15}{56}\right) + \left(2^2 \times \frac{30}{56}\right) + \left(3^2 \times \frac{10}{56}\right) - \left(\frac{15}{8}\right)^2$$

$$\frac{15 + 120 + 90}{56} - \frac{225}{64}$$

$$\frac{225}{56} - \frac{225}{64}$$

$$\frac{225}{448} \text{ or } 0.502$$

3. June/2020/Paper_9709/51/No.3

A company produces small boxes of sweets that contain 5 jellies and 3 chocolates. Jemeel chooses 3 sweets at random from a box.

(a) Draw up the probability distribution table for the number of jellies that Jemeel chooses. [4]

J (5)	C (3)
0	3
1	2
2	1
3	0

$P(X=0) = \frac{{}^5C_0 \times {}^3C_3}{{}^8C_3} = \frac{1}{56}$

$P(X=1) = \frac{{}^5C_1 \times {}^3C_2}{{}^8C_3} = \frac{15}{56}$

$P(X=2) = \frac{{}^5C_2 \times {}^3C_1}{{}^8C_3} = 30/56$

$P(X=3) = \frac{{}^5C_3 \times {}^3C_0}{{}^8C_3} = \frac{10}{56}$

X	0	1	2	3
$P(X=x)$	$1/56$	$15/56$	$30/56$	$10/56$

The company also produces large boxes of sweets. For any large box, the probability that it contains more jellies than chocolates is 0.64. 10 large boxes are chosen at random.

(b) Find the probability that no more than 7 of these boxes contain more jellies than chocolates. [3]

Binomial distribution

$X \sim B(10, 0.64)$

$P(X \leq 7) = 1 - P(X = 8, 9, 10)$

$1 - \left\{ {}^{10}C_8 (0.64)^8 (0.36)^2 + {}^{10}C_9 (0.64)^9 \times 0.36 + {}^{10}C_{10} (0.64)^{10} \times 0.36^0 \right\}$

$\left\{ 1 - (0.164156 + 0.064852 + 0.11529) \right\}$

$= 0.759$

4. June/2020/Paper_9709/52/No.5

A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands is noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same.

(a) Show that $P(X = 3) = \frac{7}{15}$. [2]

	1	2	3
1	1	2	3
1	1	2	3
2	2	2	3
2	2	2	3
3	3	3	3

$P(X=3) = 7/15$

(b) Draw up the probability distribution table for X. [3]

X	1	2	3	total
$P(X=x)$	$2/15$	$6/15$	$7/15$	1

(c) Find E(X) and Var(X). [3]

$E(X) = \sum x_i p_i = \left(1 \times \frac{2}{15}\right) + \left(2 \times \frac{6}{15}\right) + \left(3 \times \frac{7}{15}\right)$

$= \frac{2 + 12 + 21}{15} = \frac{35}{15} = \frac{7}{3}$

$\mathrm{Var}(X) = \sum x_i^2 p_i - \left(\sum x_i p_i\right)^2$

$\left(1^2 \times \frac{2}{15}\right) + \left(2^2 \times \frac{6}{15}\right) + \left(3^2 \times \frac{7}{15}\right) - \left(\frac{7}{3}\right)^2$

$\left(\frac{2 + 24 + 63}{15}\right) - \frac{49}{9}$

$\frac{89}{15} - \frac{49}{9} = \frac{22}{45}$

5. June/2020/Paper_9709/53/No.4

A fair four-sided spinner has edges numbered 1, 2, 2, 3. A fair three-sided spinner has edges numbered -2, -1, 1. Each spinner is spun and the number on the edge on which it comes to rest is noted. The random variable X is the sum of the two numbers that have been noted.

(a) Draw up the probability distribution table for X. [3]

	-2	-1	1
1	-1	0	2
2	0	1	3
2	0	1	3
3	1	2	4

x	-1	0	1	2	3	4
$P(X=x)$	$1/12$	$3/12$	$3/12$	$2/12$	$2/12$	$1/12$

(b) Find Var(X). [3]

$Var(X) = \sum x_i^2 p_i - (E(X))^2$

$E(X) = \sum x_i p_i = \left(-1 \times \frac{1}{12}\right) + \left(0 \times \frac{3}{12}\right) + \left(1 \times \frac{3}{12}\right) + \left(2 \times \frac{2}{12}\right) + \left(3 \times \frac{2}{12}\right) + \left(4 \times \frac{1}{12}\right)$

$\frac{-1 + 0 + 3 + 4 + 6 + 4}{12} = \frac{16}{12} = \frac{4}{3}$

$Var(X) = (-1)^2 \times \frac{1}{12} + 0^2 \times \frac{3}{12} + 1^2 \times \frac{3}{12} + 2^2 \times \frac{2}{12} + 3^2 \times \frac{2}{12} + 4^2 \times \frac{1}{12}$

$= \frac{1 + 0 + 3 + 8 + 18 + 16}{12} - \left(\frac{4}{3}\right)^2$

$= 37/18 = 2.06$

6. March/2020/Paper_9709/52/No.2b,2c

An ordinary fair die is thrown repeatedly until a 1 or a 6 is obtained.

On another occasion, this die is thrown 3 times. The random variable X is the number of times that a 1 or a 6 is obtained.

(b) Draw up the probability distribution table for X. [3]

Using $P(X=r) = pq^{r-1}$

$P(X=0) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$P(X=1) = \left(\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}\right) \times {}^3C_2$

for order

ie $\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}$

or

$\frac{2}{3} \times \frac{1}{3} \times \frac{2}{3}$

$= \frac{12}{27}$

$P(X=2) = \left(\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3}\right) \times {}^3C_1 = \frac{6}{27}$

$P(X=3) = \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{1}{27}$

X	0	1	2	3	total
$P(X=x)$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$	1

(c) Find E(X). [2]

$E(X) = \left(0 \times \frac{8}{27}\right) + \left(1 \times \frac{12}{27}\right) + \left(2 \times \frac{6}{27}\right) + \left(3 \times \frac{1}{27}\right)$

$= 1$

$E(X) = \Sigma x_i p_i$