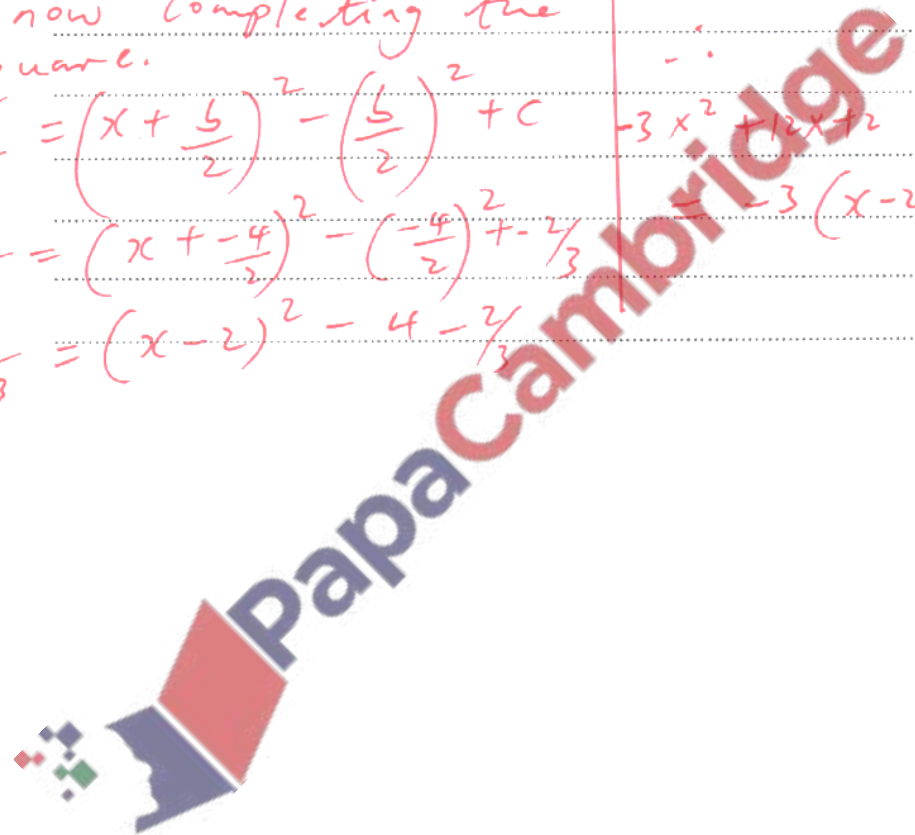


**1. Nov/2021/Paper\_9709/11/No.8(a)**

(a) Express  $-3x^2 + 12x + 2$  in the form  $-3(x - a)^2 + b$ , where  $a$  and  $b$  are constants. [2]

$$\begin{aligned}
 y &= -3x^2 + 12x + 2 \\
 \Rightarrow \frac{y}{-3} &= \frac{-3x^2}{-3} + \frac{12x}{-3} + \frac{2}{-3} \\
 \Rightarrow \frac{y}{-3} &= x^2 - 4x - \frac{2}{3} \\
 &\Rightarrow \text{now completing the square.} \\
 \Rightarrow \frac{y}{-3} &= \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \\
 \frac{y}{-3} &= \left(x + \frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + \frac{-2}{3} \\
 \frac{y}{-3} &= (x - 2)^2 - 4 - \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \frac{y}{-3} &= (x - 2)^2 - \frac{14}{3} \\
 \frac{1}{-3} \left(\frac{y}{-3}\right) &= -3(x - 2)^2 - \frac{14(-3)}{3} \\
 y &= -3(x - 2)^2 + 14 \\
 \therefore -3x^2 + 12x + 2 &= -3(x - 2)^2 + 14
 \end{aligned}$$



(a) Express  $5y^2 - 30y + 50$  in the form  $5(y + a)^2 + b$ , where  $a$  and  $b$  are constants.

[2]

$$\text{Let } t = 5y^2 - 30y + 50$$

$$\frac{t}{5} = \frac{5y^2}{5} - \frac{30y}{5} + \frac{50}{5}$$

$$\frac{t}{5} = y^2 - 6y + 10$$

Now complete square

$$\Rightarrow \frac{t}{5} = \left(y + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

$$\frac{t}{5} = \left(y + \frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 + 10$$

$$\frac{t}{5} = (y - 3)^2 - 9 + 10$$

$$\frac{t}{5} = (y - 3)^2 + 1$$

$$5\left(\frac{t}{5}\right) = 5(y - 3)^2 + (1 \times 5)$$

$$t = 5(y - 3)^2 + 5$$

$$\Rightarrow 5y^2 - 30y + 50$$

$$= 5(y - 3)^2 + 5$$

$$\text{Where } a = -3, b = 5$$



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3. June/2021/Paper\_9709/12/No.1

The gradient of a curve is given by  $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$ , where  $k$  is a constant. The curve has a stationary point at  $(2, -3.5)$ .

(a) Find the value of  $k$ .

[2]

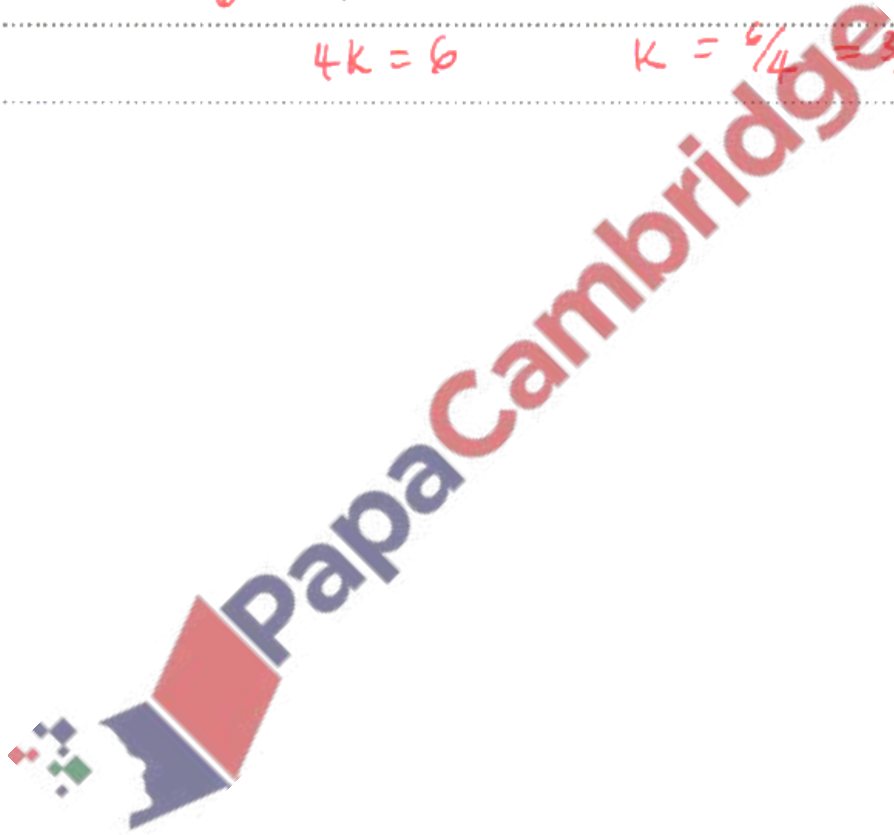
$$x=2 \quad \frac{dy}{dx} = 0$$

$$6(3 \times 2 - 5)^3 - k(2)^2 = 0$$

$$6 - 4k = 0$$

$$4k = 6$$

$$k = \frac{6}{4} = \frac{3}{2} = 1.5$$



(b) Find the equation of the curve.

[4]

$$y = \int (6(3x-5)^3 - \frac{3}{2}x^2) dx$$

$$y = \frac{6(3x-5)^4}{4 \times 3} - \frac{\frac{3}{2}x^3}{\frac{3}{3}} + C$$

$$y = \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3 + C$$

at  $(2, -3.5)$

$$-3.5 = \frac{1}{2} - \frac{1}{2} \times 8 + C$$

$$-3.5 = \frac{1}{2} - 4$$

$$C = 0$$

$$y = \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$$

(c) Find  $\frac{d^2y}{dx^2}$ .

[2]

$$\frac{dy}{dx} = 6(3x-5)^3 - \frac{3}{2}x^2$$

$$\frac{d^2y}{dx^2} = 3 \times 6(3x-5)^2 \times 3 - 2 \times \frac{3}{2}x$$

$$= 54(3x-5)^2 - 3x$$

(d) Determine the nature of the stationary point at (2, -3.5).

[2]

$$54(3 \times 2 - 5)^2 - 3 \times 2$$

$$54(1)^2 - 6$$

$$\frac{d^2y}{dx^2} = 48 > 0$$

Minimum

By using a suitable substitution, solve the equation

$$(2x-3)^2 - \frac{4}{(2x-3)^2} - 3 = 0.$$

[4]

let  $u = 2x-3$

$$u^2 - \frac{4}{u^2} - 3 = 0$$

multiply by  $u^2$  (every term)

$$u^4 - 4 - 3u^2 = 0$$

$$u^4 - 3u^2 - 4 = 0$$

factorize

$$(u^2 - 4)(u^2 + 1) = 0$$

$$u^2 = 4 \quad u^2 = -1$$

N/A

$$u = 2 \text{ or } -2$$

$$2x - 3 = 2$$

$$x = \frac{5}{2}$$

$$2x - 3 = -2$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$x = \frac{5}{2}$$

$$x = \frac{1}{2}$$