Quadratics – 2021 AS Nov/June

1. Nov/2021/Paper_9709/11/No.8(a)

(a) Express $-3x^2 + 12x + 2$ in the form $-3(x-a)^2 + b$, where a and b are constants. [2]

$y = -3x^2 + 12x + 2$

$\Rightarrow \frac{y}{-3} = \frac{-3x^2}{-3} + \frac{12x}{-3} + \frac{2}{-3}$

$\Rightarrow \frac{y}{-3} = x^2 - 4x - \frac{2}{3}$

$\Rightarrow$ now completing the square.

$\Rightarrow \frac{y}{-3} = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

$\frac{y}{-3} = \left(x + \frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + -\frac{2}{3}$

$\frac{y}{-3} = (x-2)^2 - 4 - \frac{2}{3}$

$\Rightarrow \frac{y}{-3} = (x-2)^2 - \frac{14}{3}$

$-3\left(\frac{y}{-3}\right) = -3(x-2)^2 - \frac{14}{3}(-3)$

$y = -3(x-2)^2 + 14$

$\therefore -3x^2 + 12x + 2 = -3(x-2)^2 + 14$

2. Nov/2021/Paper_9709/13/No.3(a)

(a) Express $5y^2 - 30y + 50$ in the form $5(y+a)^2 + b$, where a and b are constants. [2]

Let $t = 5y^2 - 30y + 50$

$\frac{t}{5} = \frac{5y^2}{5} - \frac{30y}{5} + \frac{50}{5}$

$\frac{t}{5} = y^2 - 6y + 10$

Now complete square

$\Rightarrow \frac{t}{5} = \left(y + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

$\frac{t}{5} = \left(y + -\frac{6}{2}\right)^2 - \left(-\frac{6}{2}\right)^2 + 10$

$\frac{t}{5} = (y-3)^2 - 9 + 10$

$\frac{t}{5} = (y-3)^2 + 1$

$5\left(\frac{t}{5}\right) = 5(y-3)^2 + (1 \times 5)$

$t = 5(y-3)^2 + 5$

$\Rightarrow$

$5y^2 - 30y + 50$

$= 5(y-3)^2 + 5$

where $a = -3$, $b = 5$

3. June/2021/Paper_9709/12/No.1

The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

$x = 2 \quad dy/dx = 0$

$6(3 \times 2 - 5)^2 - k(2)^2 = 0$

$6 - 4k = 0$

$4k = 6 \qquad k = 6/4 = 3/2 = 1.5$

(b) Find the equation of the curve. [4]

$$y = \int \left(6(3x-5)^3 - \tfrac{3}{2}x^2\right) dx$$

$$y = \frac{6(3x-5)^4}{4 \times 3} - \frac{\frac{3}{2}x^3}{3} + C$$

$$y = \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3 + C$$

at $(2, -3.5)$

$$-3.5 = \frac{1}{2} - \frac{1}{2} \times 8 + C$$

$$-3.5 = \frac{1}{2} - 4$$

$C = 0$

$$y = \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$$

(c) Find $\frac{d^2y}{dx^2}$. [2]

$\frac{dy}{dx} = 6(3x-5)^3 - \frac{3}{2}x^2$

$\frac{d^2y}{dx^2} = 3 \times 6(3x-5)^2 \times 3 - 2 \times \frac{3}{2}x$

$= 54(3x-5)^2 - 3x$

(d) Determine the nature of the stationary point at (2, −3.5). [2]

$54(3 \times 2 - 5)^2 - 3 \times 2$

$54(1)^2 - 6$

$\frac{d^2y}{dx^2} = 48 > 0$

minimum

4. March/2021/Paper_9709/12/No.2

By using a suitable substitution, solve the equation

$$(2x-3)^2 - \frac{4}{(2x-3)^2} - 3 = 0. \qquad [4]$$

let $u = 2x - 3$

$$u^2 - \frac{4}{u^2} - 3 = 0$$

multiply by u^2 (every term)

$$u^4 - 4 - 3u^2 = 0$$

$$u^4 - 3u^2 - 4 = 0$$

factorize

$$(u^2 - 4)(u^2 + 1) = 0$$

$u^2 = 4$ $u^2 = -1$

N/A

$u = 2, -2$

$2x - 3 = 2$

$x = \frac{5}{2}$

$2x - 3 = -2$

$2x = 1$

$x = \frac{1}{2}$

$\boxed{x = \frac{5}{2}}$ $\boxed{x = \frac{1}{2}}$