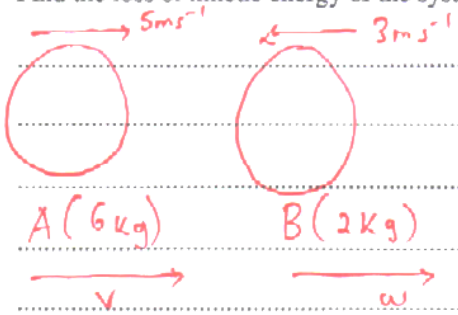
# Impulse and Momentum – 2022 Nov AS

**1.** Nov/2022/Paper_9709_41/No.2

Small smooth spheres $A$ and $B$, of equal radii and of masses 6 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially $A$ is moving towards $B$ with speed $5\,\mathrm{m\,s^{-1}}$ and $B$ is moving towards $A$ with speed $3\,\mathrm{m\,s^{-1}}$. After the spheres collide, both $A$ and $B$ move in the same direction and the difference in the speeds of the spheres is $2\,\mathrm{m\,s^{-1}}$.

Find the loss of kinetic energy of the system due to the collision. [5]




Let $v$ = speed of $A$ immediately after collision

$w$ = speed of $B$ immediately after collision.

Using Conservation of Linear momentum

$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2$

$(6 \times 5) + (2 \times -3) = 6v + 2w$

But the difference in the speeds of the two spheres after collision is $2\,\mathrm{ms^{-1}}$ $\Rightarrow w - v = 2$

$w = 2 + v$

$\Rightarrow 30 - 6 = 6v + 2(2 + v)$

$24 = 6v + 4 + 2v$

$24 - 4 = 8v$

$20 = 8v \Rightarrow v = 20/8 = 2.5$

$\Rightarrow w = 2 + 2.5 = 4.5$

Loss in KE = Initial KE − Final KE

$= \left(\tfrac{1}{2}m_1U_1^2 + \tfrac{1}{2}m_2U_2^2\right) - \left(\tfrac{1}{2}m_1V_1^2 + \tfrac{1}{2}m_2V_2^2\right)$

$= \left(\tfrac{1}{2} \times 6 \times 5^2 + \tfrac{1}{2} \times 2 \times 3^2\right) - \left(\tfrac{1}{2} \times 6 \times 2.5^2 + \tfrac{1}{2} \times 2 \times 4.5^2\right)$

$= (75 + 9) - (18.75 + 20.25)$

$= 84 - 39 = 45$

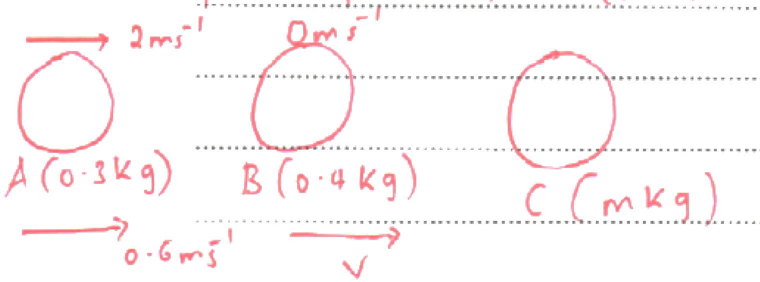
$\therefore$ Loss in KE = 45 J

**2.** Nov/2022/Paper_9709_42/No.6

Three particles $A$, $B$ and $C$ of masses 0.3 kg, 0.4 kg and $m$ kg respectively lie at rest in a straight line on a smooth horizontal plane. The distance between $B$ and $C$ is 2.1 m. $A$ is projected directly towards $B$ with speed $2\,\text{m s}^{-1}$. After $A$ collides with $B$ the speed of $A$ is reduced to $0.6\,\text{m s}^{-1}$, still moving in the same direction.

(a) Show that the speed of $B$ after the collision is $1.05\,\text{m s}^{-1}$. [2]

Let V = Speed of B after collision




By Conservation of Linear momentum:

$m_1 U_1 + m_2 U_2 = m_1 V_1 + m_2 V_2$

$(0.3 \times 2) + (0.4 \times 0) = (0.3 \times 0.6) + (0.4 \times V)$

$0.6 = 0.18 + 0.4V$

$0.6 - 0.18 = 0.4V$

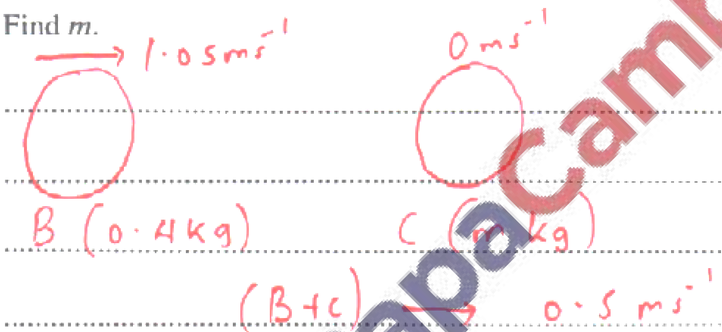
$0.42 = 0.4V$

$\Rightarrow V = \dfrac{0.42}{0.4} = 1.05\,\text{ms}^{-1}$

$\therefore$ Speed of B after the collision $= 1.05\,\text{ms}^{-1}$

After the collision between $A$ and $B$, $B$ moves directly towards $C$. Particle $B$ now collides with $C$. After this collision, the two particles coalesce and have a combined speed of $0.5\,\text{m s}^{-1}$.

(b) Find $m$. [2]




By CLM:

$m_1 U_1 + m_2 U_2 = m_1 V_1 + m_2 V_2$

$(0.4 \times 1.05) + (m \times 0) = (0.4 + m)\,0.5$

$0.42 + 0 = 0.2 + 0.5m$

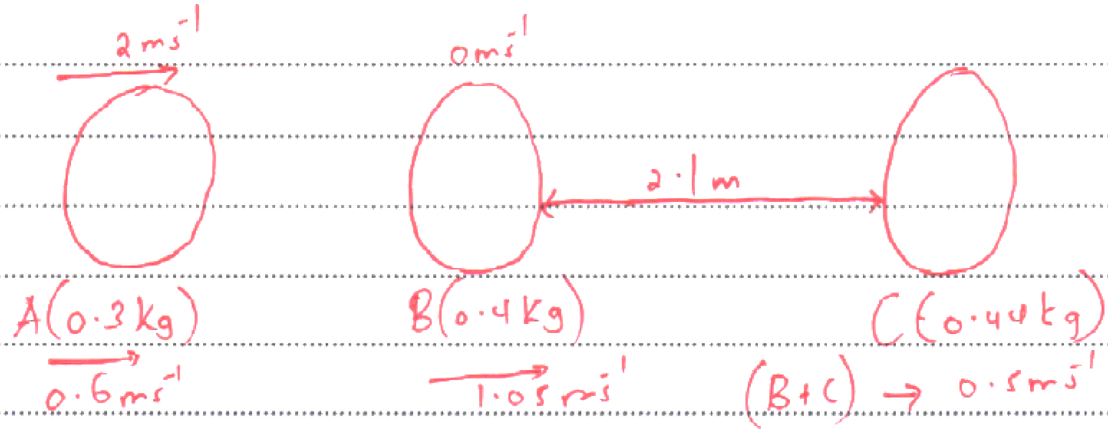
$0.42 - 0.2 = 0.5m$

$0.22 = 0.5m$

$\Rightarrow m = \dfrac{0.22}{0.5}$

$\therefore m = 0.44\,\text{kg}$

(c) Find the time that it takes, from the instant when $B$ and $C$ collide, until $A$ collides with the combined particle. [5]




Let $T$ = time taken

Distance = Speed × time

Let $d$ = distance of the centre of mass

$d = \left(\dfrac{m_C}{m_B + m_C}\right) 2 \cdot 1$

$d = \left(\dfrac{0 \cdot 44}{0 \cdot 4 + 0 \cdot 44}\right) 2 \cdot 1$

$= 1 \cdot 1 \text{ m}$

Time taken before B and C collide $= \dfrac{2 \cdot 1}{1 \cdot 05} = 2$ seconds

Distance from A = $\text{Speed}_A \times T$

$= 0 \cdot 6T$

Distance from C = $\text{Speed}_{(B+C)} \times T$

$= 0 \cdot 5T$

$\Rightarrow$ $0 \cdot 6T - 1 \cdot 1 = 0 \cdot 5T$

$0 \cdot 6T - 0 \cdot 5T = 1 \cdot 1 \Rightarrow 0 \cdot 1T = 1 \cdot 1$

$T = \dfrac{1 \cdot 1}{0 \cdot 1} = 11$ seconds.

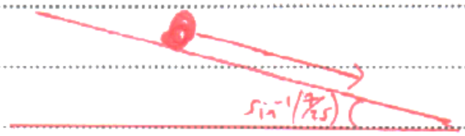
$\therefore$ Time taken from when B and C collide $= (11 - 2)$ seconds

$= 9$ seconds.

**3.** March/2022/Paper_9709/42/No.7

A bead, $A$, of mass 0.1 kg is threaded on a long straight rigid wire which is inclined at $\sin^{-1}\left(\frac{7}{25}\right)$ to the horizontal. $A$ is released from rest and moves down the wire. The coefficient of friction between $A$ and the wire is $\mu$. When $A$ has travelled 0.45 m down the wire, its speed is $0.6\,\text{m s}^{-1}$.

(a) Show that $\mu = 0.25$. [6]




Now using Energy Equation

$\Delta G.p.E$ = change in gravitational potential energy.

$\Rightarrow \Delta K.E$ = change in kinetic energy

$\Rightarrow \Delta G.p.E = \Delta K.E + W_f$

$W_f$ = work done against friction

$\Delta G.p.E = \Delta K.E + W_f$

$mgh = \frac{1}{2}m(v^2 - u^2) + W_f$

$\Rightarrow (0.1)(10)\left(\frac{7}{25} \times 0.45\right) = \frac{1}{2}(0.1)[0.6^2 - 0^2] + (F_f \times 0.45)$

$F_f$ = frictional force

$\Rightarrow 0.126 = 0.018 + \mu R(0.45)$

Recall $F_f = \mu R$ ; where $R = mg\cos\theta$

$\Rightarrow F_f = \mu mg\cos\theta$

$0.126 = 0.018 + (\mu \times R)$

$0.126 = 0.018 + \left(\mu \times 0.1 \times 10 \times \frac{24}{25}\right)0.45$

$0.126 = 0.018 + F_f$

$0.108 = 0.432\mu$

$\frac{0.432\mu}{0.432} = \frac{0.108}{0.432}$

$\Rightarrow \mu = 0.25$

$\mu = 0.25$

Shown.



 9709/42/F/M/22

Another bead, $B$, of mass 0.5 kg is also threaded on the wire. At the point where $A$ has travelled 0.45 m down the wire, it hits $B$ which is instantaneously at rest on the wire. $A$ is brought to instantaneous rest in the collision. The coefficient of friction between $B$ and the wire is 0.275.

**(b)** Find the time from when the collision occurs until $A$ collides with $B$ again. [6]

using Law of Conservation of momentum

$\Rightarrow m_A u_A + \cancel{m_B u_B} = \cancel{m_A v_A} + m_B v_B$

- We're cancelling $m_B u_B$ because initially B is not moving
- Again we're cancelling A because after collision A is brought into rest.

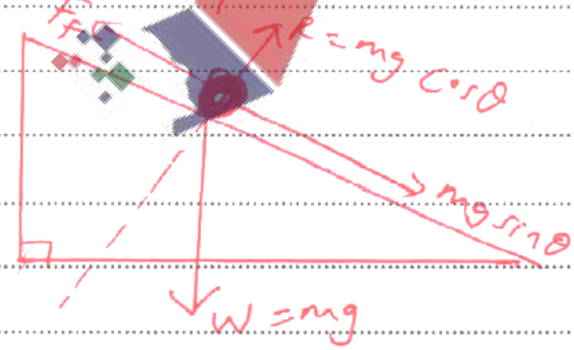
$(0.1)(0.6) = (0.5) v_B$

$\frac{0.06}{0.5} = \frac{0.5 v_B}{0.5}$

$v_B = \frac{0.06}{0.5} = 0.12$ m/s

Now, using Newton's 2nd Law of motion

$F = ma$




for bead A

$W\sin\theta - F_f = ma$

$mg\sin\theta - F_f = ma$

$\left[(0.1)\times 10\times \frac{7}{25}\right] - F_f = ma$

$\frac{7}{25} - (0.25)(0.1)\times 10\times\frac{24}{25} = 0.1a$

$\frac{7}{25} - \frac{6}{25} = 0.1a$

$\frac{1}{25} = 0.1a$

$\frac{0.04}{0.1} = \frac{0.1a}{0.1}$

$\Rightarrow a = 0.4\text{ m/s}^2$

for Bead B

$mg\sin\theta - F_f = ma$

$\left(0.5\times10\times\frac{7}{25}\right) - (0.275)\times\left(0.5\times10\times\frac{24}{25}\right) = 0.5a$

$0.5a = 1.4 - 1.32$

$\frac{0.5a}{0.5} = \frac{0.08}{0.5}$

$a = 0.16\text{ m/s}^2$

$s = s_0 + ut + \frac{1}{2}at^2$

$s_0$ initial s

$s_A = \frac{1}{2}(0.4)t^2$

$s_A = 0.2t^2$

$s_B = 0.12t + \frac{1}{2}(0.16)t^2$

$s_B = 0.12t + 0.08t^2$

$s_A = s_B$

$0.2t^2 = 0.12t + 0.08t^2$

$0.12t^2 - 0.12t = 0$

$0.12t(t-1) = 0$

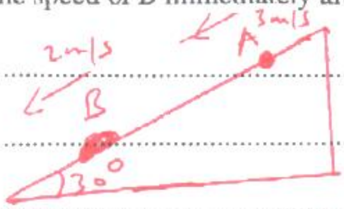
$t = 0$ or $1$

$t = 1$ sec

**4.** June/2022/Paper_9709/41/No.7

Two particles $A$ and $B$, of masses 0.4 kg and 0.2 kg respectively, are moving down the same line of greatest slope of a smooth plane. The plane is inclined at 30° to the horizontal, and $A$ is higher up the plane than $B$. When the particles collide, the speeds of $A$ and $B$ are $3\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ respectively. In the collision between the particles, the speed of $A$ is reduced to $2.5\,\mathrm{m\,s^{-1}}$.

**(a)** Find the speed of $B$ immediately after the collision. [2]




$A = 0.4\,kg$ $\quad B = 0.2\,kg$

using Law of Conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.4(3) + (0.2)(2) = (0.4)(2.5) + m_2v_2$

$(0.4)(3) + 0.4 = 1 + 0.2v_2$

$1.2 + 0.4 = 1 + 0.2v_2$

$1.6 - 1 = 0.2v_2$

$\frac{0.6}{0.2} = \frac{0.2v_2}{0.2}$

$v_2 = \frac{0.6}{0.2} = 3\,m/s$

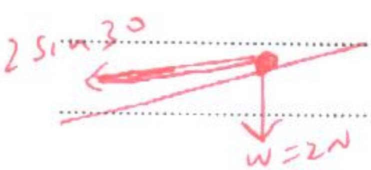
$= 3\,m/s$

After the collision, when $B$ has moved 1.6 m down the plane from the point of collision, it hits a barrier and returns back up the same line of greatest slope. $B$ hits the barrier 0.4 s after the collision, and when it hits the barrier, its speed is reduced by 90%. The two particles collide again 0.44 s after their previous collision, and they then coalesce on impact.

**(b)** Show that the speed of $B$ immediately after it hits the barrier is $0.5\,\mathrm{m\,s^{-1}}$. Hence find the speed of the combined particle immediately after the second collision between $A$ and $B$. [7]


$S = 1.6\,m$ $\quad t = 0.4\,sec$ $\quad v = ?$ $\quad u = 3\,m/s$

$v = u + at$




But $ma = F_{app} - F_R$

$0.2a = 2\sin 30$

$0.2a = 1$

$\Rightarrow \frac{0.2a}{0.2} = \frac{1}{0.2}$

$a = 5\,m/s^2$

But $v = u + at$

$v = 3 + 5(0.4)$

$v = 3 + 2 = 5\,m/s$

After hitting the barrier

$\Rightarrow \frac{10}{100} \times 5 = 0.5 m/s$

using Law of conservation of momentum

$\Rightarrow m_1u_1 + m_2u_2 = (m_1+m_2)V$

V = common Velocity

$0.4u_1 + m_2u_2 = (m_1+m_2)V$


$V = u + at$

$V = 2.5 + 5(0.44)$

$= 2.5 + 2.2$

$= 4.7 m/s$

$\Rightarrow 0.4(4.7) + 0.2(u_2) = (m_1+m_2)V$

$\uparrow$ $u = 0.5 m/s$  $a = -5 m/s^2$

$t = 0.44 - 0.4 = 0.04 sec$

$V = u + at$

$V = 0.5 + -5(0.04)$

$V = 0.5 + -0.2 = 0.3 m/s$

$\Rightarrow 0.4u_1 + m_2u_2 = (m_1+m_2)V$

$0.4(4.7) + 0.2(-0.3) = (0.4+0.2)V$

$1.88 - 0.06 = 0.6V$

$1.82 = 0.6V$

$0.6V = 1.82$

$\frac{0.6V}{0.6} = \frac{1.82}{0.6}$

$\Rightarrow V = 3.03 m/s$

$\therefore$ Combined speed of A and B after collision $= 3.03 m/s$

**5.** June/2022/Paper_9709/42/No.1

Small smooth spheres $A$ and $B$, of equal radii and of masses 5 kg and 3 kg respectively, lie on a smooth horizontal plane. Initially $B$ is at rest and $A$ is moving towards $B$ with speed $8.5\,\mathrm{m\,s^{-1}}$. The spheres collide and after the collision $A$ continues to move in the same direction but with a quarter of the speed of $B$.

**(a)** Find the speed of $B$ after the collision. [3]


smooth surface means frictional force is zero.

After collision. $\frac{1}{4}V$ $V$ A B ⇒ AB

Coalesce and move in the same direction.

Now using Law of Conservation of momentum

$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$5(8.5) + 3(0) = 5(\frac{1}{4}V) + 3V$

$\Rightarrow 42.5 + 0 = \frac{17}{4}V$

$\Rightarrow \frac{17}{4}V = 42.5$

$4.25V = 42.5$

$\Rightarrow \frac{4.25V}{4.25} = \frac{42.5}{4.25}$

$V = 10\,\mathrm{m/s}$

Speed of B after collision $= 10\,\mathrm{m/s}$

**(b)** Find the loss of kinetic energy of the system due to the collision. [2]

K.E = Kinetic Energy

$\Rightarrow$ K.E before collision $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

$= \frac{1}{2}(5)(8.5)^2 + \frac{1}{2}(3)(0)^2$

$= \frac{1}{2}(5)(8.5)^2 + \frac{1}{2}(3)(0)^2$

$= 180.625 + 0$

$= 180.625\,\mathrm{J}$

K.E after Collision =

K.E after Collision

$= \frac{1}{2}(5)(2.5)^2 + \frac{1}{2}(3)(10)^2$

$= 15.625 + 150$

$= 165.625\,\mathrm{J}$

$\Rightarrow$ loss in K.E

$= 180.625 - 165.625$

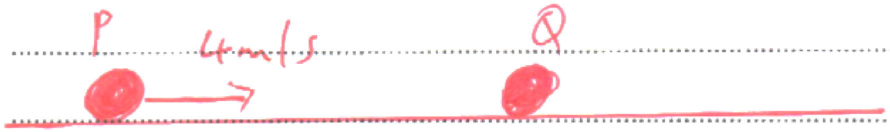
$= 15\,\mathrm{J}$

$= 15\,\mathrm{J}$.

**6.** June/2022/Paper_9709/43/No.1

Two particles $P$ and $Q$, of masses 0.3 kg and 0.2 kg respectively, are at rest on a smooth horizontal plane. $P$ is projected at a speed of $4\,\mathrm{m\,s^{-1}}$ directly towards $Q$. After $P$ and $Q$ collide, $Q$ begins to move with a speed of $3\,\mathrm{m\,s^{-1}}$.

**(a)** Find the speed of $P$ after the collision. [2]




$v_p = ?$

$v_Q = 3\,m/s$

$m_p = 0.3\,kg$ $\quad m_Q = 0.2\,kg$

at rest implies $u_Q = 0$ i.e initial velocity of $Q = 0m/s$

using Law of conservation of Linear momentum we know $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow (0.3) \times (4) + (0.2) \times (0) = (0.3) \times (v_p) + (0.2)(3)$

$1.2 = 0.3 v_p + 0.6$

$\Rightarrow 1.2 - 0.6 = 0.3 v_p$

$0.6 = 0.3 v_p$

$\Rightarrow v_p = \frac{0.6}{0.3}$

$\Rightarrow v_p = 2\,m/s$

speed of $P$ after collision $= 2\,m/s$.

After the collision, $Q$ moves directly towards a third particle $R$, of mass $m$ kg, which is at rest on the plane. The two particles $Q$ and $R$ coalesce on impact and move with a speed of $2\,\mathrm{m\,s^{-1}}$.

**(b)** Find $m$. [2]

Recall when two bodies or particles coalesce it implies that the two bodies stick together and they move with a common velocity ($V$).

$\Rightarrow m_1 u_1 + m_2 u_2 = (m_1 + m_2) V$

Where $V$ = common velocity

$\Rightarrow (0.2)(3) + 0 = (0.2 + m)\,2$

$0.6 = 2(0.2 + m)$

$\frac{\cancel{0.6}^{\,0.3}}{\cancel{2}_{1}} = \frac{\cancel{2}(0.2 + m)}{\cancel{2}}$

$\Rightarrow 0.3 = 0.2 + m$

$\Leftrightarrow 0.2 + m = 0.3$

$\Rightarrow m = 0.3 - 0.2$

$\Rightarrow m = 0.1\,kg$