Normal Distribution – 2022 AS Nov/June Math S1

1. Nov/2022/Paper_9709_51/No.4

In a large population, the systolic blood pressure (SBP) of adults is normally distributed with mean 125.4 and standard deviation 18.6.

(a) Find the probability that the SBP of a randomly chosen adult is less than 132. [2]

Let Y = SBP of adults, $Y \sim N(125.4, 18.6^2)$

$P(Y < 132) = P\left(Z < \frac{132 - 125.4}{18.6}\right)$

$= P(Z < 0.3548)$

$= 0.639$

The SBP of 12-year-old children in the same population is normally distributed with mean 117. Of these children 88% have SBP more than 108.

(b) Find the standard deviation of this distribution. [3]

Let X = SBP of 12-year-old children

$X \sim N(117, \sigma^2)$

$P(X > 108) = P\left(Z > \frac{108-117}{\sigma}\right) = \frac{88}{100}$

$P\left(Z > \frac{108-117}{\sigma}\right) = 0.88$

$\Rightarrow 1 - P\left(Z < \frac{108-117}{\sigma}\right) = 0.88$

$\Rightarrow P\left(Z < \frac{108-117}{\sigma}\right) = 1 - 0.88$

$P\left(Z < \frac{108-117}{\sigma}\right) = 0.12$

From the tables, the Z-inverse of 0.12 is -1.175

$\Rightarrow \Phi^{-1}(0.12) = -1.175$

$\Rightarrow \frac{108-117}{\sigma} = -1.175$

$\frac{-9}{\sigma} = -1.175$

$\Rightarrow \sigma = \frac{-9}{-1.175}$

$\therefore \sigma = 7.66$

Three adults are chosen at random from this population.

(c) Find the probability that each of these three adults has SBP within 1.5 standard deviations of the mean. [4]

$P(-1.5 < Z < 1.5)$

$= P(Z < 1.5) - P(Z < -1.5)$

$= P(Z < 1.5) - (1 - P(Z < 1.5))$

$\phi(1.5) - (1 - \phi(1.5))$

From the tables $\phi(1.5) = 0.9332$

$= 0.9332 - (1 - 0.9332)$

$= 0.9332 - 0.0668$

$= 0.8664$ For 1 adult.

For 3 adults $= (0.8664)^3$

$= 0.65036$

$\simeq 0.650$ (3 sf).

2. Nov/2022/Paper_9709_53/No.2

In a large college, 32% of the students have blue eyes. A random sample of 80 students is chosen.

Use an approximation to find the probability that fewer than 20 of these students have blue eyes. [5]

Probability of success (P) $= 32\% = \frac{32}{100} = 0.32$

Let X = Number of students with blue eyes

$X \sim B(80, 0.32)$

$\Rightarrow$ Since the sample size is large and p is close to 0.5, the normal distribution is used as an approximation to the binomial distribution.

Mean = np = $80 \times 0.32 = 25.6$

Variance = npq = $80 \times 0.32 \times (1 - 0.32)$

$= 80 \times 0.32 \times 0.68$

$= 17.408$

$\Rightarrow X \sim B(80, 0.32) \approx N(25.6, \sqrt{17.408}^2)$

Applying continuity correction

$P(X < 20) = P\left(Z < \frac{19.5 - 25.6}{\sqrt{17.408}}\right)$

$\boxed{Z = \frac{X - \mu}{\sigma}}$

$= P(Z < -1.462)$

$= 1 - P(Z < 1.462)$

$= 1 - \Phi(1.462)$

$= 1 - 0.9282$

$= 0.0718$

3. Nov/2022/Paper_9709_53/No.5

Company A produces bags of sugar. An inspector finds that on average 10% of the bags are underweight.

10 of the bags are chosen at random.

(a) Find the probability that fewer than 3 of these bags are underweight. [3]

$p = \frac{10}{100} = 0.1$

Let X = Number of bags of sugar produced by company A.

$X \sim B(10, 0.1)$

$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

$P(X < 3) = P(X=0) + P(X=1) + P(X=2)$

$= \binom{10}{0}(0.1)^0(0.9)^{10} + \binom{10}{1}(0.1)^1(0.9)^9 + \binom{10}{2}(0.1)^2(0.9)^8$

$= 0.34878 + 0.38742 + 0.19371$

$= 0.9298$

$= 0.930$ (3sf)

The weights of the bags of sugar produced by company B are normally distributed with mean 1.04 kg and standard deviation 0.06 kg.

(b) Find the probability that a randomly chosen bag produced by company B weighs more than 1.11 kg. [3]

Let Y = Weight of bags of sugar produced by company B.

$Y \sim N(1.04, 0.06^2)$

$P(Y > 1.11) = P\left(Z > \frac{1.11 - 1.04}{0.06}\right)$

$= P(Z > 1.167)$

$= 1 - P(Z < 1.167)$

$= 1 - \phi(1.167)$

$= 1 - 0.8784$

$= 0.1216$

$= 0.122 \quad (3sf)$

81% of the bags of sugar produced by company B weigh less than w kg.

(c) Find the value of w. [3]

$Y \sim N(1.04,\ 0.06^2)$

$P(Y < w) = P\left(Z < \frac{w - 1.04}{0.06}\right) = 0.81$

The Z - inverse value of 0.81 $(\phi^{-1}(0.81) = 0.878)$

$\Rightarrow \frac{w - 1.04}{0.06} = 0.878$

$w - 1.04 = 0.06 \times 0.878$

$w - 1.04 = 0.05268$

$w = 0.05268 + 1.04$

$w = 1.09268$

$\therefore w = 1.09 \quad (3sf)$

4. Nov/2022/Paper_9709_52/No.2

The lengths of the rods produced by a company are normally distributed with mean 55.6 mm and standard deviation 1.2 mm.

(a) In a random sample of 400 of these rods, how many would you expect to have length less than 54.8 mm? [4]

Recall $Z = \frac{X - \mu}{\sigma}$

Let X - lengths of rods

$X \sim N(55.6, 1.2^2)$

$P(X < 54.8) = P\left(Z < \frac{54.8 - 55.6}{1.2}\right)$

$= P(Z < -0.6667)$

$= 1 - P(Z < 0.6667)$

$= 1 - \phi(0.6667)$

$= 1 - 0.7477$

$= 0.2523$

$\therefore$ Expected number of 400 rods $= 400 \times 0.2523$

$= 100.92$

$\therefore$ Expected number $=$ 100 or 101 rods.

(b) Find the probability that a randomly chosen rod produced by this company has a length that is within half a standard deviation of the mean. [3]

$P(-0.5 < Z < 0.5)$

$= P(Z < 0.5) - P(Z < -0.5)$

$= P(Z < 0.5) - (1 - P(Z < 0.5))$

$= \phi(0.5) - (1 - \phi(0.5))$

$= 0.6915 - (1 - 0.6915)$

$= 0.6915 - 0.3085$

$= 0.383$

5. March/2022/Paper_9709/52/No.4

The weights of male leopards in a particular region are normally distributed with mean 55 kg and standard deviation 6 kg.

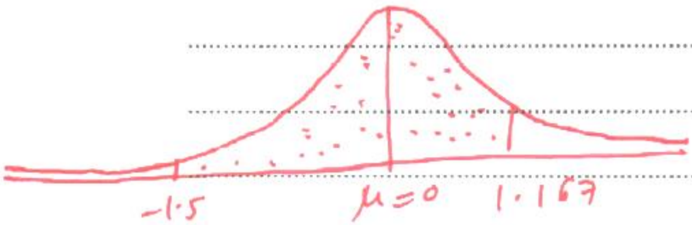
(a) Find the probability that a randomly chosen male leopard from this region weighs between 46 and 62 kg. [4]

$M \sim N(55, 6^2)$

$\Rightarrow P(46 < M < 62) =$

$= P\left(\frac{46-55}{6} < Z < \frac{62-55}{6}\right)$

$= P(-1.5 < Z < 1.167)$



$= P(Z < 1.167) - P(Z \leq -1.5)$

$= 0.8784 - [1 - P(Z \geq 1.5)]$

We have used the concept of symmetry.

$= 0.8784 - [1 - P(Z \geq 1.5)]$

$= 0.8784 - 1 + 0.9332$

$= 0.812$

$= 0.812$

The weights of female leopards in this region are normally distributed with mean 42 kg and standard deviation σ kg. It is known that 25% of female leopards in the region weigh less than 36 kg.

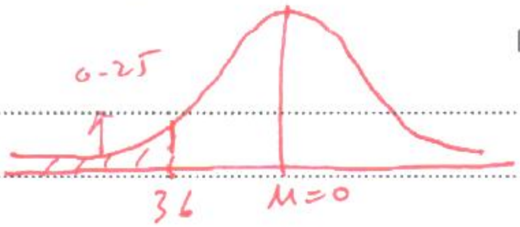
(b) Find the value of σ. [3]

$F \sim N(42, \sigma^2)$

$\mu = 42$

$\Rightarrow P(F < 36) = 0.25$

Since $P[F < 36] < 0.5$ then it implies that Z value will be on the left tail of the standard normal distribution



$\Rightarrow Z = -0.674$

$P\left(\frac{F-42}{\sigma} < \frac{36-42}{\sigma}\right) = 0.25$

$P(Z < a) = 0.25$

let $a = \frac{36-42}{\sigma}$

$\Phi(a) = 0.25$

$a = -0.674$

$\Rightarrow a = \frac{36-42}{\sigma}$

$\Rightarrow -0.674 = \frac{36-42}{\sigma}$

$\sigma \times -0.674 = \frac{-6}{\cancel{\sigma}} \times \cancel{\sigma}$

$\Rightarrow -0.674\sigma = -6$

$\frac{\cancel{-0.674}\,\sigma}{\cancel{-0.674}} = \frac{-6}{-0.674}$

$\Rightarrow \sigma = 8.902$

$\Rightarrow \sigma = 8.90$

The distributions of the weights of male and female leopards are independent of each other. A male leopard and a female leopard are each chosen at random.

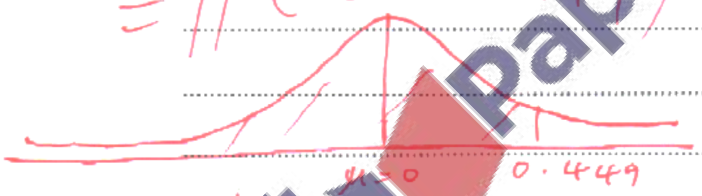
(c) Find the probability that both the weights of these leopards are less than 46 kg. [4]

$P(M < 46) = 1 - 0.9332$

$= 0.0668$

$P(F < 46) = P(Z < \frac{46-42}{8.90})$

$= P(Z < 0.449)$



$= \Phi(0.449)$

$= 0.6732$

Since the weights of male and female Leopards are independent of Each other

$\Rightarrow P(W < 46) =$

$= 0.0668 \times 0.6732$

$= 0.0450$

0.0668×0.6732

$= 0.0450$

6. June/2022/Paper_9709/52/No.4

The weights, in kg, of bags of rice produced by Anders have the distribution $N(2.02, 0.03^2)$.

(a) Find the probability that a randomly chosen bag of rice produced by Anders weighs between 1.98 and 2.03 kg. [3]

Let $W \sim N(2.02, 0.03^2)$

$\Rightarrow \mu = \text{mean} = 2.02$

$\text{Variance} = \sigma^2 = 0.03^2$

$\Rightarrow P(1.98 < W < 2.03)$

$\Rightarrow = P\left(\frac{1.98-2.02}{\sqrt{0.03^2}} < Z < \frac{2.03-2.02}{\sqrt{0.03^2}}\right)$

$= P\left(\frac{1.98-2.02}{0.03} < Z < \frac{2.03-2.02}{0.03}\right)$

$= P\left(\frac{-0.04}{0.03} < Z < \frac{0.01}{0.03}\right)$

$= P(-1.333 < Z < 0.333)$

using the concept of symmetry: $\Rightarrow$

$\Rightarrow P(-1.333 < Z < 0.333)$

$= [\Phi(1.333) - \Phi(0)] + [\Phi(0.333) - \Phi(0)]$

$= [\Phi(1.333) - 0.5] + [\Phi(0.333) - 0.5]$

$= [0.9087 - 0.5] + [0.6304 - 0.5]$

$= 0.4087 + 0.1304$

$= 0.5391$

$= \underline{0.539}$

The weights of bags of rice produced by Binders are normally distributed with mean 2.55 kg and standard deviation σ kg. In a random sample of 5000 of these bags, 134 weighed more than 2.6 kg.

(b) Find the value of σ. [4]

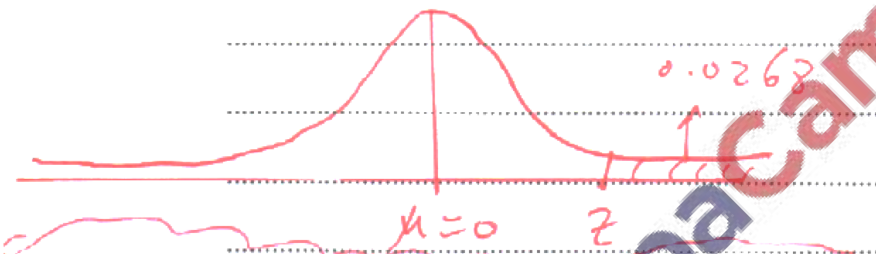
$W \sim N(2.55, \sigma^2)$

$W \sim N(2.55, \sigma^2)$

μ = mean = 2.55 and
Variance = σ^2

$\Rightarrow P(W > 2.6) = \frac{134}{5000}$

$P(W > 2.6) = 0.0268$



Note: the area of the upper tail of the normal distribution = 0.0268

$P(W \le 2.6) = 1 - 0.0268$

$\Rightarrow P(W \le 2.6) = 0.9732$

$\Rightarrow z = 1.93$

Recall $z = \frac{W - \mu}{\sigma}$

$\Rightarrow$

$1.93 = \frac{2.6 - 2.55}{\sigma}$

$\Rightarrow \sigma \times 1.93 = \frac{(2.6 - 2.55)}{\cancel{\sigma}} \times \cancel{\sigma}$

$1.93\sigma = 2.6 - 2.55$

$\frac{1.93\sigma}{1.93} = \frac{0.05}{1.93}$

$\Rightarrow \sigma = \frac{0.05}{1.93}$

$\sigma = 0.0259$

$\Rightarrow$

$\sigma = 0.0259.$

7. June/2022/Paper_9709/53/No.5

Farmer Jones grows apples. The weights, in grams, of the apples grown this year are normally distributed with mean 170 and standard deviation 25. Apples that weigh between 142 grams and 205 grams are sold to a supermarket.

(a) Find the probability that a randomly chosen apple grown by Farmer Jones this year is sold to the supermarket. [4]

$X \sim N(170, 25^2)$ $\mu = 170$ and $\sigma^2 = 25^2$

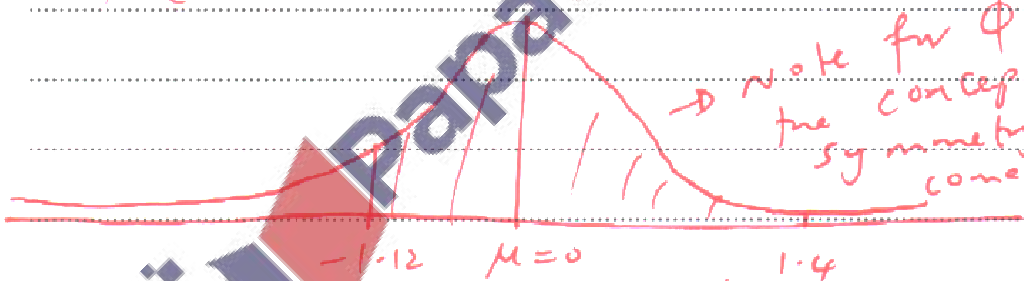
$\Rightarrow P(142 < X < 205) =$

$= P\left(\frac{142 - 170}{25} < Z < \frac{205 - 170}{25}\right)$

$= P\left(\frac{-28}{25} < Z < \frac{35}{25}\right)$

$= P(-1.12 < Z < 1.4)$

$= P(-1.12 < Z < 1.4)$



→ note for $\Phi(-1.12)$ of the concept of symmetry has to come in handy.

$= [\Phi(1.4) - 0.5] + [\Phi(1.12) - 0.5]$

$= [0.9192 - 0.5] + [0.8686 - 0.5]$

$= [0.9192 - 0.5] + [0.8686 - 0.5]$

$= 0.4192 + 0.3686$

$= 0.7878$

Farmer Jones sells the apples to the supermarket at \$0.24 each. He sells apples that weigh more than 205 grams to a local shop at \$0.30 each. He does not sell apples that weigh less than 142 grams.

The total number of apples grown by Farmer Jones this year is 20 000.

(b) Calculate an estimate for his total income from this year's apples. [3]

$p(x > 205) = 1 - p(x \le 205)$

$= 1 - p\left[z \le \frac{205-170}{25}\right]$

$= 1 - p(z \le 1.4)$

$= 1 - \phi(1.4)$

$= 1 - 0.9192$

$= 0.0808$

Estimate $= (0.788 \times 20{,}000 \times 0.24) + (0.0808 \times 20{,}000 \times 0.30)$

$= 3782.40 + 484.80$

$= \$\,4267.20$

$= \$\,4267.20$

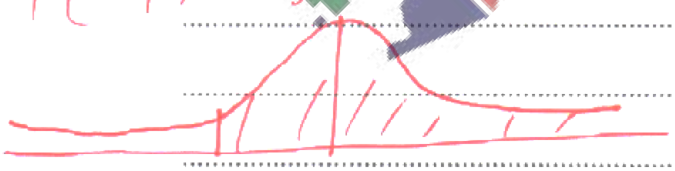
Farmer Tan also grows apples. The weights, in grams, of the apples grown this year follow the distribution $N(182, 20^2)$. 72% of these apples have a weight more than w grams.

(c) Find the value of w. [3]

$Y \sim N(182, 20^2)$

$P(Y > w) = 72\%$

$P(Y > w) = 0.72$



$-w$

$z = -0.583$

$z = \frac{y - \mu}{\sigma}$

$-0.583 = \frac{w - 182}{20}$

$(-0.583)(20) = w - 182$

$-11.66 = w - 182$

$\Rightarrow w = -11.66 + 182$

$w = 170.34$

$\underline{\underline{w = 170}}$