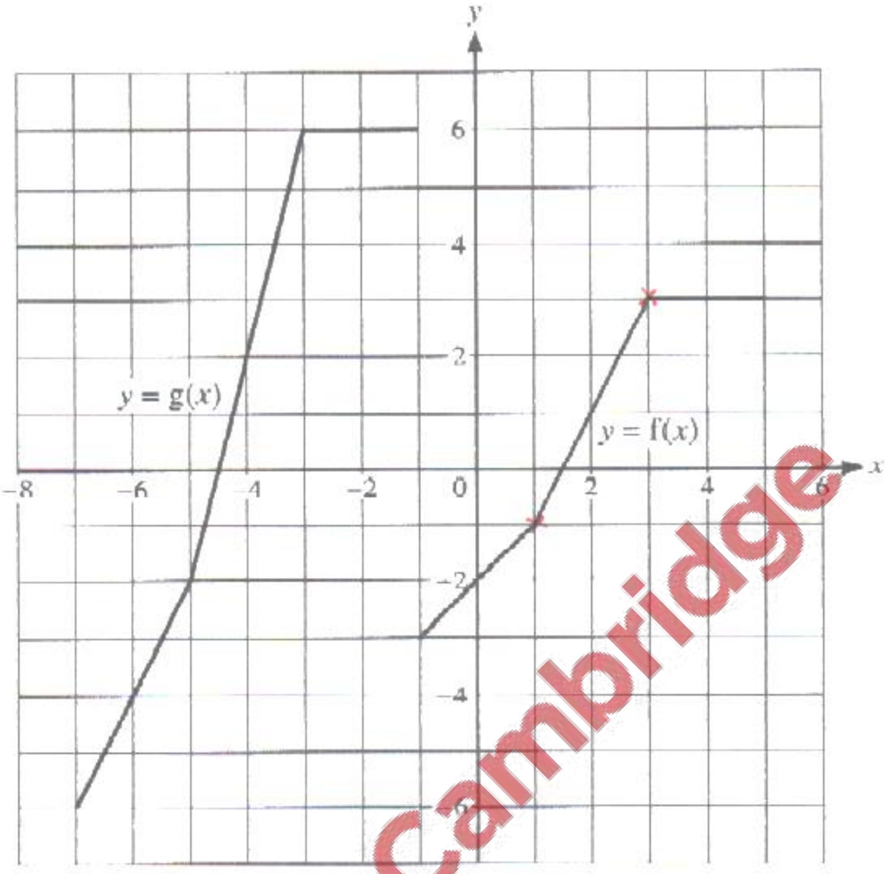
1. June/2023/Paper_9709/11/No.3



The diagram shows graphs with equations $y = f(x)$ and $y = g(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to $y = g(x)$. [4]

(Multiply by 2)

(3, 3) → (−3, 6)

(1, −1) → (−5, −2)

(−6)

A vertical stretch of scale factor 2 followed by a translation by vector $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$

2. June/2023/Paper_9709/11/No.8

The functions f and g are defined as follows, where a and b are constants.

$$f(x) = 1 + \frac{2a}{x-a} \text{ for } x > a$$

$$g(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

(a) Given that $f(7) = \frac{5}{2}$ and $gf(5) = 4$, find the values of a and b. [4]

$f(7) = 1 + \dfrac{2a}{7-a} = \dfrac{5}{2}$

$\dfrac{2a}{7-a} = \dfrac{5}{2} - 1 \Rightarrow \dfrac{2a}{7-a} = \dfrac{3}{2}$

$\Rightarrow 2(2a) = 3(7-a)$

$4a = 21 - 3a$

$4a + 3a = 21$

$\dfrac{\cancel{7}a}{\cancel{7}} = \dfrac{21}{7}$

$\therefore \quad a = 3$

$gf(5) = 4$

$f(x) = 1 + \dfrac{2(3)}{x-3} \Rightarrow f(x) = 1 + \dfrac{6}{x-3}$

$f(5) = 1 + \dfrac{6}{5-3} = 1 + 3 = 4$

$gf(5) = b(4) - 2 = 4$

$4b - 2 = 4$

$4b = 4 + 2$

$\dfrac{\cancel{4}b}{\cancel{4}} = \dfrac{6}{4} \Rightarrow b = \dfrac{3}{2}$

$\therefore \quad a = 3$ and $b = \dfrac{3}{2}$

For the rest of this question, you should use the value of a which you found in (a).

(b) Find the domain of f^{-1}. [1]

$f(x) = 1 + \frac{6}{x-3}$

Domain of f^{-1} : $x > 1$

(c) Find an expression for $f^{-1}(x)$. [3]

let $y = 1 + \frac{6}{x-3}$

$\Rightarrow y - 1 = \frac{6}{x-3}$

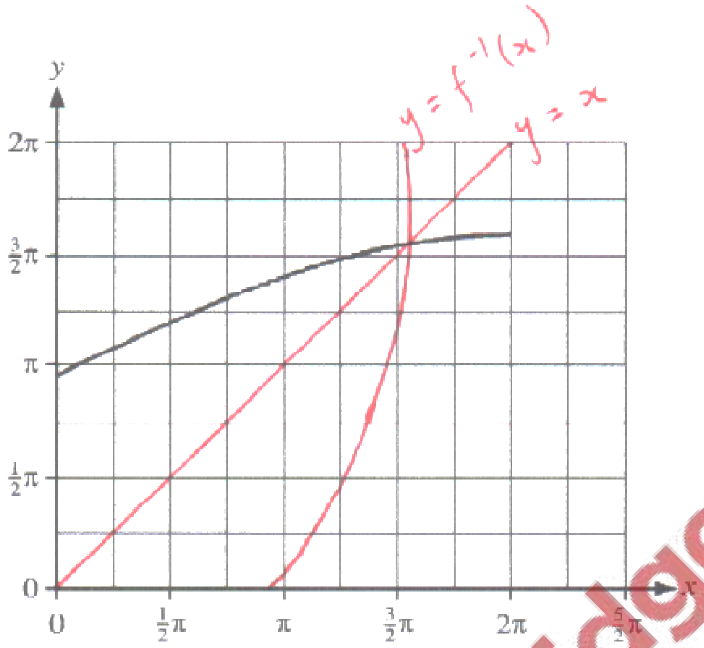
$\frac{\cancel{(y-1)}(x-3)}{\cancel{y-1}} = \frac{6}{y-1}$

$\Rightarrow x - 3 = \frac{6}{y-1}$

$x = 3 + \frac{6}{y-1}$

$\therefore f^{-1}(x) = 3 + \frac{6}{x-1}$

3. June/2023/Paper_9709/12/No.8



The diagram shows the graph of $y = f(x)$ where the function f is defined by

$$f(x) = 3 + 2\sin\tfrac{1}{4}x \text{ for } 0 \leqslant x \leqslant 2\pi.$$

(a) On the diagram above, sketch the graph of $y = f^{-1}(x)$. [2]

The graph of $y = f^{-1}(x)$ is a reflection of the graph of $y = f(x)$ on the line $y = x$.

(b) Find an expression for $f^{-1}(x)$. [2]

Let $y = 3 + 2\sin\frac{1}{4}x$ (Make x the subject)

$\Rightarrow \frac{\cancel{2}\sin\frac{1}{4}x}{\cancel{2}} = \frac{y-3}{2}$

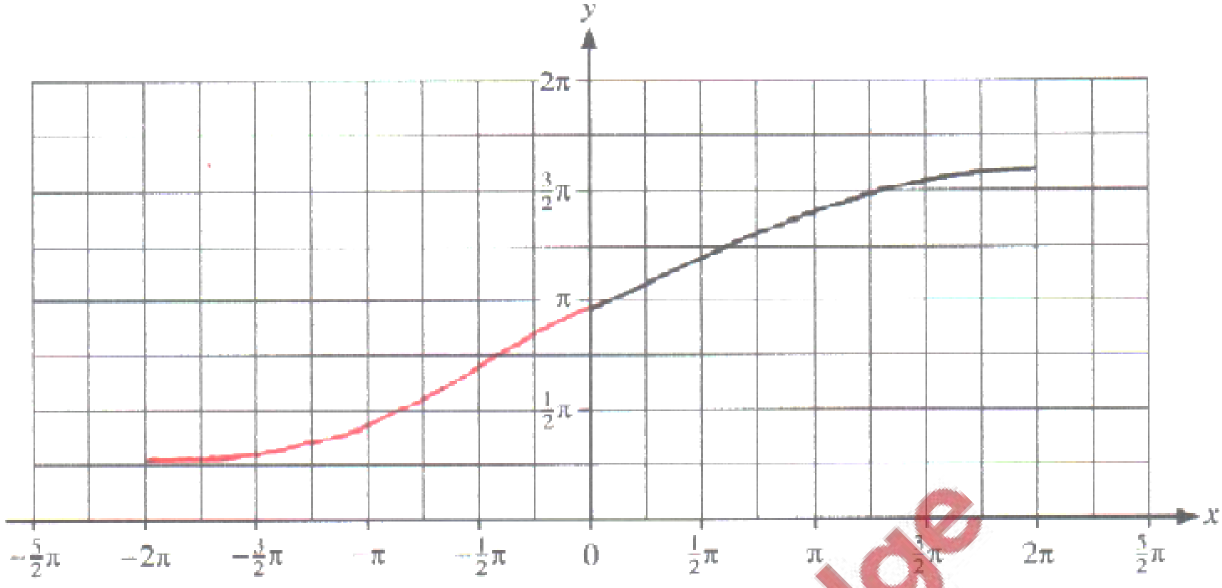
$\sin\frac{1}{4}x = \frac{y-3}{2}$

$\frac{1}{4}x = \sin^{-1}\left(\frac{y-3}{2}\right)$

$\Rightarrow x = 4\sin^{-1}\left(\frac{y-3}{2}\right)$

$\therefore f^{-1}(x) = 4\sin^{-1}\left(\frac{x-3}{2}\right)$

(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2\sin\frac{1}{4}x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

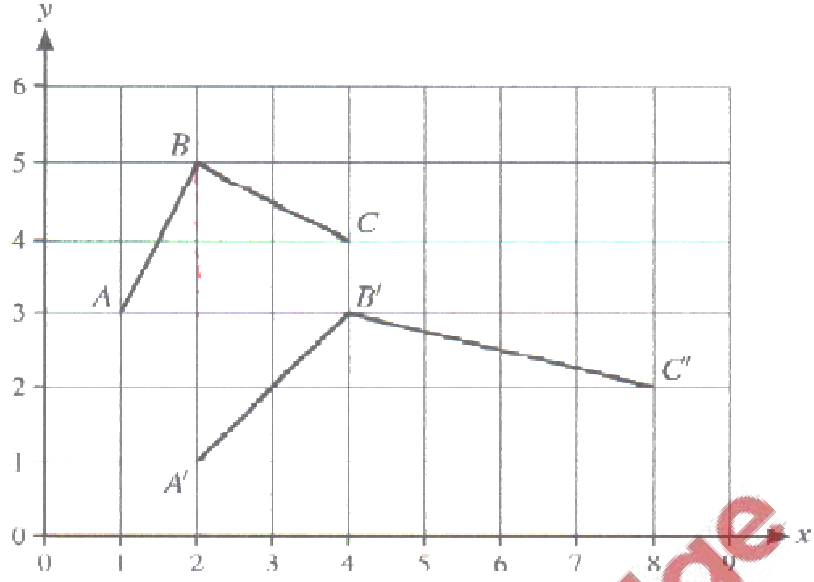
function g has an inverse because its one-

(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

$y = \sin x \longrightarrow y = 3 + 2\sin\frac{1}{4}x$

Stretch of scale factor 4 in x - direction followed by a stretch of scale factor 2 in y - direction then a translation by vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$.

4. June/2023/Paper_9709/13/No.1



The diagram shows the graph of $y = f(x)$, which consists of the two straight lines AB and BC. The lines $A'B'$ and $B'C'$ form the graph of $y = g(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = f(x)$.

State fully the two transformations. [4]

$A(1, 3) \longrightarrow A'(2, 1)$

$B(2, 5) \longrightarrow B'(4, 3)$

$C(4, 4) \longrightarrow C'(8, 2)$

A translation by vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ followed by a sketch of scale factor 2 parallel to the x-axis.

5. June/2023/Paper_9709/13/No.2

The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 - 6x + c$, where c is a constant. It is given that $f(x) > 2$ for all values of x.

Find the set of possible values of c. [4]

$f(x) = x^2 - 6x + c$

$f(x) > 2 \Rightarrow x^2 - 6x + c > 2$

Using Completing square method :

$x^2 - 6x + \left(\frac{1}{2}(-6)\right)^2 - \left(\frac{1}{2}(-6)\right)^2 + c > 2$

$\underbrace{x^2 - 6x + (-3)^2} - (-3)^2 + c > 2$

$(x-3)^2 - 9 + c > 2$

$\Rightarrow c > 2 + 9 - (x-3)^2$

$c > 11 - (x-3)^2$

But $(x-3)^2 \geq 0$ for all $x \in \mathbb{R}$

$\therefore c > 11$

6. June/2023/Paper_9709/13/No.7

The function f is defined by $f(x) = 2 - \dfrac{5}{x+2}$ for $x > -2$.

(a) State the range of f. [1]

$f(x) < 2$

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

Let $y = 2 - \dfrac{5}{x+2}$

$y = \dfrac{2(x+2) - 5}{x+2}$

$y = \dfrac{2x+4-5}{x+2} \Rightarrow y = \dfrac{2x-1}{x+2}$

$\Rightarrow y(x+2) = 2x-1$

$xy + 2y = 2x - 1$

$\Rightarrow 2y + 1 = 2x - xy$

$\dfrac{2y+1}{2-y} = \dfrac{x(2-y)}{2-y}$

$\Rightarrow x = \dfrac{2y+1}{2-y}$

$\therefore f^{-1}(x) = \dfrac{2x+1}{2-x}$

Domain is $x < 2$

The function g is defined by $g(x) = x + 3$ for $x > 0$.

(c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax + b}{cx + d}$, where a, b, c and d are integers. [3]

$f(x) = 2 - \frac{5}{x+2}$

$fg(x) = 2 - \frac{5}{x+3+2}$

$= 2 - \frac{5}{x+5}$

$= \frac{2(x+5) - 5}{x+5}$

$= \frac{2x + 10 - 5}{x+5}$

$\therefore fg(x) = \frac{2x+5}{x+5}$