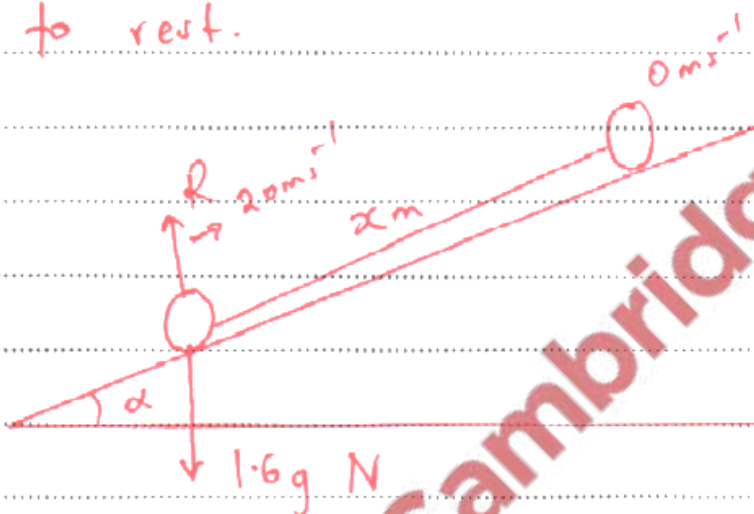


1. Nov/2023/Paper_9709/41/No.1

A particle of mass 1.6 kg is projected with a speed of 20 m s^{-1} up a line of greatest slope of a smooth plane inclined at α to the horizontal, where $\tan \alpha = \frac{3}{4}$.

Use an energy method to find the distance the particle moves up the plane before coming to instantaneous rest. [3]

Let x be the distance the particle moves before coming to rest.



$$\text{KE lost} = \text{PE gained}$$

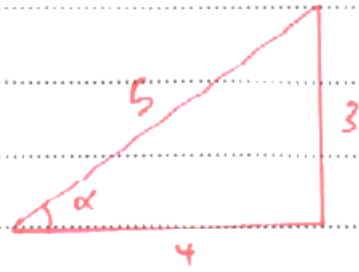
$$\frac{1}{2} m u^2 - \frac{1}{2} m v^2 = mgh$$



$$\left(\frac{1}{2} \times 1.6 \times 20^2\right) - \left(\frac{1}{2} \times 1.6 \times 0\right) = 1.6 \times g \times x \sin \alpha$$

But $g = 10 \text{ m s}^{-2}$

Given $\tan \alpha = \frac{3}{4}$

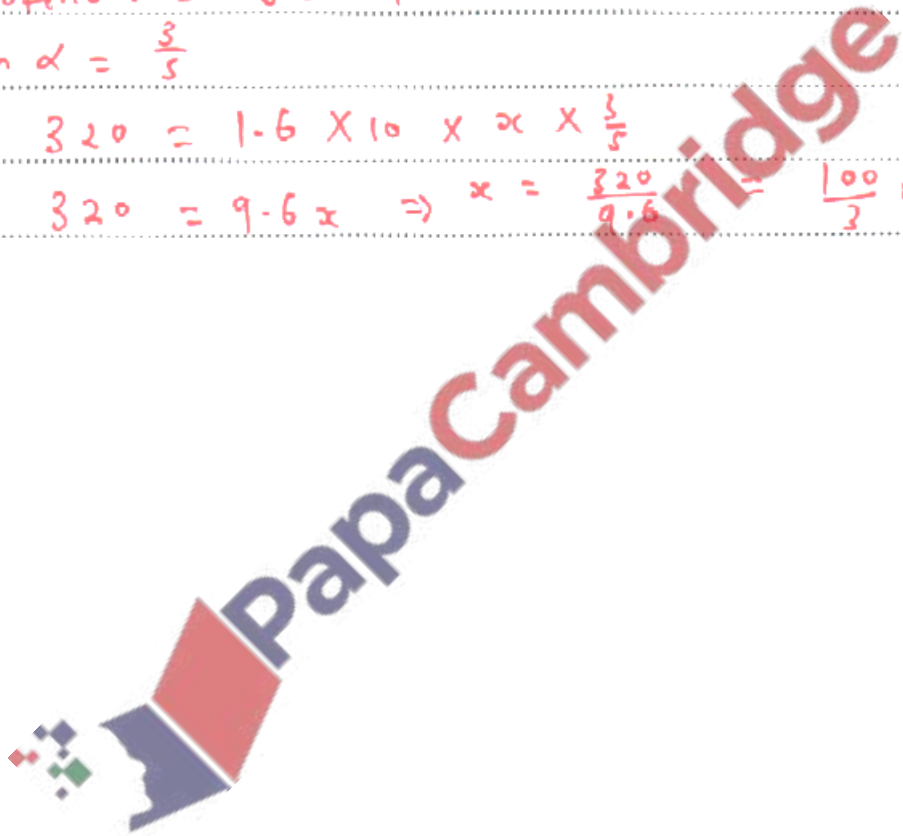


$$\text{Hypotenuse} = \sqrt{3^2 + 4^2} = 5$$

$$\Rightarrow \sin \alpha = \frac{3}{5}$$

$$\Rightarrow 320 = 1.6 \times 10 \times x \times \frac{3}{5}$$

$$320 = 9.6x \Rightarrow x = \frac{320}{9.6} = \frac{100}{3} \text{ m}$$



A car of mass 1300 kg is moving on a straight road.

- (a) On a horizontal section of the road, the car has a constant speed of 30 m s^{-1} and there is a constant force of 650 N resisting the motion.

- (i) Calculate, in kW, the power developed by the engine of the car.

[2]

Since the car is moving at a constant speed

Driving force = Resistance force = 650 N

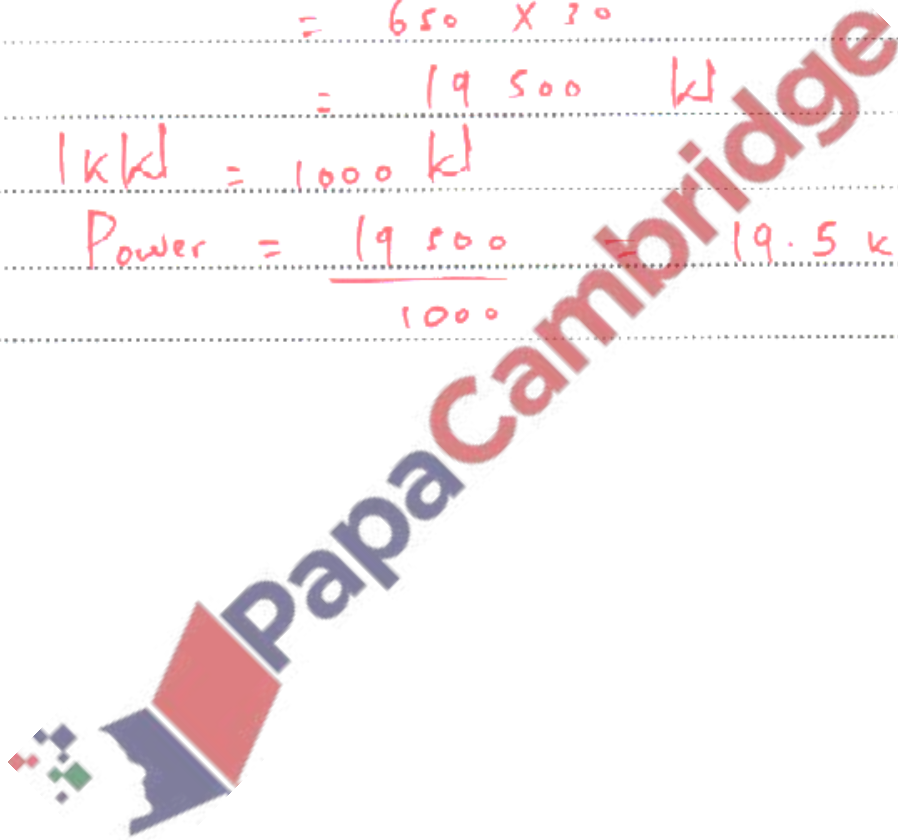
$$\text{Power} = F v$$

$$= 650 \times 30$$

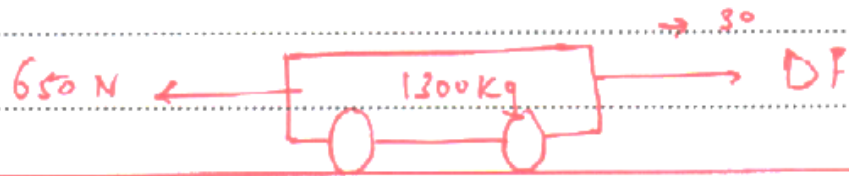
$$= 19500 \text{ W}$$

$$1 \text{ kW} = 1000 \text{ W}$$

$$\text{Power} = \frac{19500}{1000} = 19.5 \text{ kW}$$



- (ii) Given that this power is suddenly increased by 9 kW, find the instantaneous acceleration of the car. [3]



$$\text{Power} = 19.5 + 9 = 28.5 \text{ kW} = 28500 \text{ W}$$

$$\text{Power} = DF \times v \Rightarrow DF = \frac{P}{v} = \frac{28500}{30}$$

$$= 950 \text{ N}$$

Resolving horizontally using Newton's second law of motion, $F = ma$

$$DF - 650 = 1300 a$$

$$950 - 650 = 1300 a \Rightarrow 300 = 1300 a$$

$$\frac{300}{1300} = \frac{1300 a}{1300} \\ \therefore a = \frac{3}{13} \text{ m s}^{-2}$$



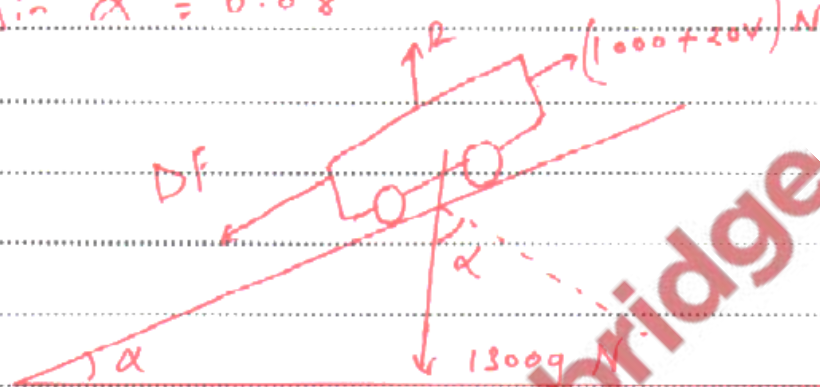
- (b) On a section of the road inclined at $\sin^{-1} 0.08$ to the horizontal, the resistance to the motion of the car is $(1000 + 20v)$ N when the speed of the car is $v \text{ m s}^{-1}$. The car travels downwards along this section of the road at constant speed with the engine working at 11.5 kW.

Find this constant speed.

[4]

Let α be the angle the road makes with the horizontal.

$$\Rightarrow \sin \alpha = 0.08$$



$$DF = \frac{P}{v} = \frac{11500}{v}$$

Resolving parallel to the plane using Newton's second law of motion, $F = ma$

$$DF + 1300g \sin \alpha - (1000 + 20v) = 0$$

(No acceleration down the slope)

$$\frac{11500}{v} + (1300 \times 10 \times 0.08) - 1000 - 20v = 0$$

$$\frac{11500}{v} + 1040 - 1000 - 20v = 0$$

$$\left(\frac{11500}{v} + 40 - 20v = 0 \right) \times v$$

$$11500 + 40v - 20v^2 = 0$$

$$-20(-575 - 2v + v^2) = 0 \Rightarrow v^2 - 2v - 57.5 = 0$$

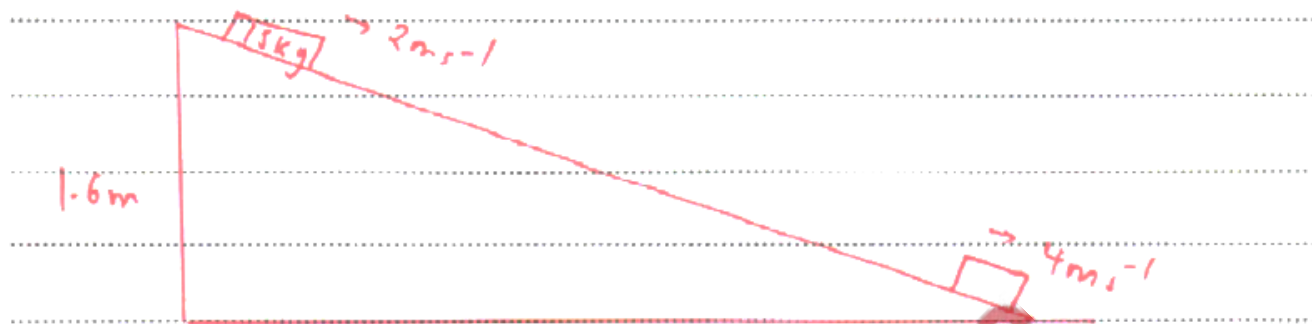
$$(v - 27)(v + 22) = 0 \Rightarrow v = 27, -22$$

But $v > 0$, so $v = 27 \text{ m s}^{-1}$

A block of mass 15 kg slides down a line of greatest slope of an inclined plane. The top of the plane is at a vertical height of 1.6 m above the level of the bottom of the plane. The speed of the block at the top of the plane is 2 m s^{-1} and the speed of the block at the bottom of the plane is 4 m s^{-1} .

Find the work done against the resistance to motion of the block.

[4]



By Conservation of energy:

$$\text{Work done against resistance} = \text{PE lost} - \text{KE gained}$$

$$\begin{aligned} \text{PE lost} &= mgh, \text{ but } g = 10 \text{ m s}^{-2} \\ &= 15 \times 10 \times 1.6 \\ &= 240 \text{ J} \end{aligned}$$

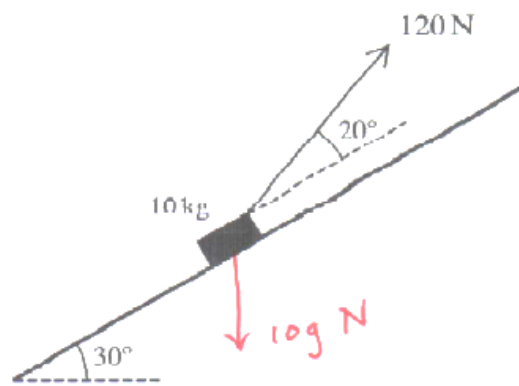
$$\text{KE gained} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= \left(\frac{1}{2} \times 15 \times 4^2 \right) - \left(\frac{1}{2} \times 15 \times 2^2 \right)$$

$$= 120 - 30$$

$$= 90 \text{ J}$$

$$\begin{aligned} \therefore \text{Work done against resistance} &= 240 - 90 \\ &= 150 \text{ J} \end{aligned}$$



A block of mass 10 kg is at rest on a rough plane inclined at an angle of 30° to the horizontal. A force of 120 N is applied to the block at an angle of 20° above a line of greatest slope (see diagram). There is a force resisting the motion of the block and 200 J of work is done against this force when the block has moved a distance of 5 m up the plane from rest.

Find the speed of the block when it has moved a distance of 5 m up the plane from rest.

[5]

By conservation of energy

Work done by an external force = change in PE + change in KE + work done against resistance.

$$\begin{aligned} \text{Work done by 120 N force} &= f \times d \\ &= 120 \times 5 \cos 20^\circ = 563.8 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Change in PE} &= mgh \\ &= 10 \times 10 \times 5 \sin 30^\circ = 250 \end{aligned}$$

$$\text{Change in KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 10 \times v^2 = 5v^2$$

$$\text{Work done against resistance} = 200 \text{ J}$$

$$\Rightarrow 563.8 = 250 + 5v^2 + 200$$

$$563.8 - 250 - 200 = 5v^2$$

$$\frac{113.8}{5} = \frac{5v^2}{5} \Rightarrow v^2 = 22.76$$

$$v = \sqrt{22.76} \quad \therefore v = 4.77 \text{ m s}^{-1} \text{ (3 s.f.)}$$

A car has mass 1600 kg.

- (a) The car is moving along a straight horizontal road at a constant speed of 24 m s^{-1} and is subject to a constant resistance of magnitude 480 N.

Find, in kW, the rate at which the engine of the car is working.

[2]

$$\begin{aligned} \text{Constant } v &\Rightarrow \text{Driving force} = \text{resistance force} = 480 \text{ N} \\ \text{Power} &= DF \times v = 480 \times 24 = 11520 \text{ W} \\ &= 11.52 \text{ kW} \end{aligned}$$

The car now moves down a hill inclined at an angle of θ to the horizontal, where $\sin \theta = 0.09$. The engine of the car is working at a constant rate of 12 kW. The speed of the car is 24 m s^{-1} at the top of the hill. Ten seconds later the car has travelled 280 m down the hill and has speed 32 m s^{-1} .

- (b) Given that the resistance is not constant, use an energy method to find the total work done against the resistance during the ten seconds. [5]

By Conservation of energy

$$W_{\text{work done}} = \text{Gain in KE} - \text{Loss in PE} + W_{\text{work done against resistance}} (W_r)$$

$$W_{\text{work done}} = P \times t = 12000 \times 10 = 120000 \text{ J}$$

$$\text{Gain in KE} = \left(\frac{1}{2} m v^2 - \frac{1}{2} m u^2 \right) = \left(\frac{1}{2} \times 1600 \times 32^2 \right) - \left(\frac{1}{2} \times 1600 \times 24^2 \right)$$

$$= 819200 - 460800 = 358400 \text{ J}$$

$$\begin{aligned}\text{Loss in PE} &= mgh = 1600 \times 9 \times 280 \sin \theta \\ &= 1600 \times 10 \times 280 \times 0.09 \\ &= 403200\end{aligned}$$

$$\Rightarrow 120000 = 358400 - 403200 + W_r$$

$$120000 = -44800 + W_r$$

$$\Rightarrow W_r = 120000 + 44800 = 164800$$

\therefore Work done against resistance = 164800 J

