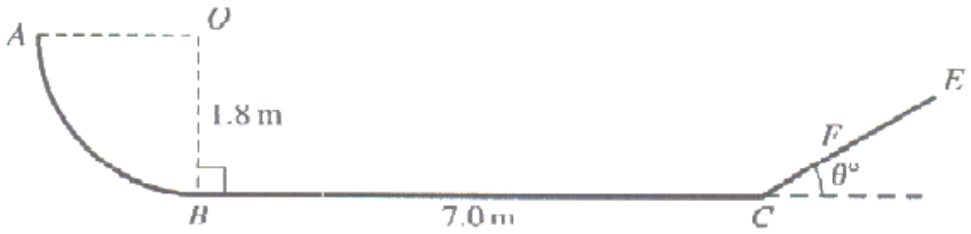
Impulse and Momentum– 2023 March Cambridge AS Level Mathematics

1. March/2023/Paper_9709/42/No.7



The diagram shows a smooth track which lies in a vertical plane. The section AB is a quarter circle of radius 1.8 m with centre O. The section BC is a horizontal straight line of length 7.0 m and OB is perpendicular to BC. The section CFE is a straight line inclined at an angle of $\theta°$ above the horizontal.

A particle P of mass 0.5 kg is released from rest at A. Particle P collides with a particle Q of mass 0.1 kg which is at rest at B. Immediately after the collision, the speed of P is 4 m s^{-1} in the direction BC. You should assume that P is moving horizontally when it collides with Q.

(a) Show that the speed of Q immediately after the collision is 10 m s^{-1}. [4]

Calculate Speed of P before Collision.

By Conservation of energy

KE lost = PE gained

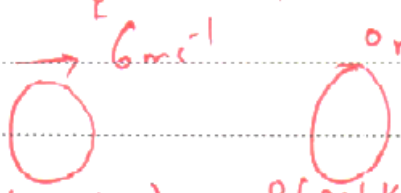
$\frac{1}{2} m v^2 = mgh$

$\Rightarrow \frac{1}{2} \times 0.5 \times v^2 = 0.5 \times 10 \times 1.8$

$0.25 v^2 = 9 \Rightarrow v^2 = \frac{9}{0.25} = 36$

$v = \sqrt{36} = 6 \text{ m s}^{-1}$

Let V_Q = Speed of Q after Collision.



By Conservation of linear momentum:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(0.5 \times 6) + 0 = (0.5 \times 4) + (0.1 \times V_Q)$

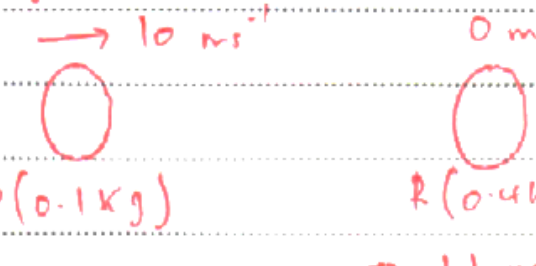
$3 = 2 + 0.1V_Q \Rightarrow \frac{0.1V_Q}{0.1} = \frac{1}{0.1}$

$\therefore V_Q = 10$ m s⁻¹ As required.

When Q reaches C, it collides with a particle R of mass 0.4 kg which is at rest at C. The two particles coalesce. The combined particle comes instantaneously to rest at F. You should assume that there is no instantaneous change in speed as the combined particle leaves C, nor when it passes through C again as it returns down the slope.

(b) Given that the distance CF is 0.4 m, find the value of θ. [4]

Let w be the speed of the combined particle after collision.



By CLM: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow (0.1 \times 10) + (0.4 \times 0) = (0.1 + 0.4)w$

$\frac{1}{0.5} = \frac{0.5w}{0.5}$

$\therefore w = 2\ ms^{-1}$

By Conservation of energy:

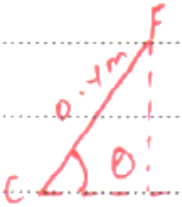
KE lost = PE gained

$\frac{1}{2}mv^2 = mgh$ but $g = 10\ ms^{-2}$

$\Rightarrow \frac{1}{2} \times (0.1 + 0.4) \times 2^2 = (0.1 + 0.4) \times 10 \times h$

$\frac{1}{2} \times 0.5 \times 4 = 0.5 \times 10 \times h$

$\frac{1}{5} = \frac{5h}{5} \Rightarrow h = 0.2\ m$



Using trigonometric ratios:

$\sin\theta = \frac{0.2}{0.4} = 0.5$

$\theta = \sin^{-1}(0.5) \therefore \theta = 30°$

[Question 7 continues on the next page.]

(c) Find the distance from B at which P collides with the combined particle. [5]

Time taken by Q to travel from B to C.

time = $\frac{\text{Distance}}{\text{speed}} = \frac{7}{10} = 0.7$ seconds.

Displacement = Velocity × time.

$s = \left(\frac{u+v}{2}\right)t$, $s = 0.4\text{m}$, $u = 2\text{ms}^{-1}$, $v = 0$

$0.4 = \left(\frac{2+0}{2}\right)t$

$0.4 = t \Rightarrow t = 0.4$ seconds.

Time taken up the slope = 0.4 seconds.

Acceleration up the slope.

$v^2 = u^2 + 2as$

$0 = 2^2 + 2 \times a \times 0.4$

$0 = 4 + 0.8a$

$\frac{-4}{0.8} = \frac{0.8a}{0.8} \Rightarrow a = -5\text{ms}^{-2}$

Time taken on the slope.

$s = ut + \frac{1}{2}at^2 \Rightarrow 0 = 2t - \frac{1}{2}(5)t^2$

$0 = 2t - 2.5t^2 \Rightarrow 0 = t(2 - 2.5t)$

But $t \neq 0$, $2 - 2.5t = 0 \Rightarrow t = \frac{2}{2.5} = 0.8$ seconds.

Distance between P moved = $v \times t = 4 \times (0.7 + 0.8)$

$= 6\text{m}$

$7\text{m} - 6\text{m} = 1\text{m}$

After collision with Q, $v_p = 4\text{ms}^{-1}$

speed of $(Q + P) = 2\text{ms}^{-1}$

Distance = $s \times t$

$4 \times t + 2t = 1 \Rightarrow \frac{6t}{6} = \frac{1}{6} \Rightarrow t = \frac{1}{6}$

Total distance from B $= 6 + \left(4 \times \frac{1}{6}\right)$

$= 6\frac{2}{3}\text{m}$