Algebra – 2020 A2

1. Nov/2020/Paper_9709/21/No.2

The polynomial $p(x)$ is defined by

$$p(x) = x^3 + ax^2 + bx + 16,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$ and that the remainder is 72 when $p(x)$ is divided by $(x - 2)$.

Find the values of a and b. [5]

2. Nov/2020/Paper_9709/21/No.4

(a) Solve the equation $|2x - 5| = |x + 6|$. [3]

(b) Hence find the value of y such that $|2^{1-y} - 5| = |2^{-y} + 6|$. Give your answer correct to 3 significant figures. [2]

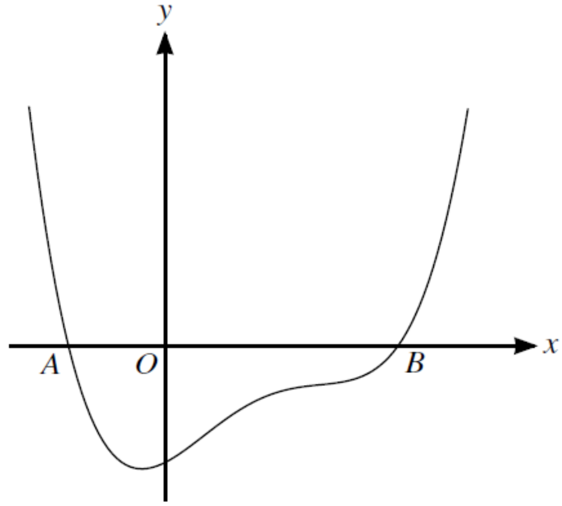
3. Nov/2020/Paper_9709/22/No.3

(a) Sketch, on a single diagram, the graphs of $y = \left|\frac{1}{2}x - a\right|$ and $y = \frac{3}{2}x - \frac{1}{2}a$, where a is a positive constant. [2]

(b) Find the coordinates of the point of intersection of the two graphs. [3]

(c) Deduce the solution of the inequality $\left|\frac{1}{2}x - a\right| > \frac{3}{2}x - \frac{1}{2}a$. [1]

4. Nov/2020/Paper_9709/22/No.7



A curve has equation $y = f(x)$ where $f(x) = x^4 - 5x^3 + 6x^2 + 5x - 15$. As shown in the diagram, the curve crosses the x-axis at the points A and B with coordinates $(a, 0)$ and $(b, 0)$ respectively.

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$. [2]

(b) By first finding the quotient when $f(x)$ is divided by $(x - 3)$, show that

$$a = -\sqrt{\frac{5}{2-a}}.$$ [5]

(c) Use an iterative formula, based on the equation in part **(b)**, to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

5. June/2020/Paper_9709/21/No.2

The polynomial $p(x)$ is defined by

$$p(x) = 6x^3 + ax^2 + 9x + b,$$

where a and b are constants. It is given that $(x - 2)$ and $(2x + 1)$ are factors of $p(x)$.

Find the values of a and b. [5]

6. June/2020/Paper_9709/21/No.4

(a) Sketch, on the same diagram, the graphs of $y = |3x + 2a|$ and $y = |3x - 4a|$, where a is a positive constant.

Give the coordinates of the points where each graph meets the axes. [3]

(b) Find the coordinates of the point of intersection of the two graphs. [3]

(c) Deduce the solution of the inequality $|3x + 2a| < |3x - 4a|$. [1]

7. June/2020/Paper_9709/21/No.7a,7c

(a) Find the quotient when $9x^3 - 6x^2 - 20x + 1$ is divided by $(3x + 2)$, and show that the remainder is 9. [3]

(c) Find the exact root of the equation $9e^{9y} - 6e^{6y} - 20e^{3y} - 8 = 0$. [4]

8. June/2020/Paper_9709/22/No.5

(a) Sketch, on the same diagram, the graphs of $y = |2x - 3|$ and $y = 3x + 5$. [2]

(b) Solve the inequality $3x + 5 < |2x - 3|$. [3]

9. June/2020/Paper_9709/22/No.6

The polynomial p(x) is defined by

$$p(x) = 6x^3 + ax^2 - 4x - 3,$$

where a is a constant. It is given that $(x + 3)$ is a factor of p(x).

(a) Find the value of a. [2]

(b) Using this value of a, factorise p(x) completely. [3]

(c) Hence solve the equation p(cosec θ) = 0 for $0° < \theta < 360°$. [3]

10. March/2020/Paper_9709/22/No.2

(a) Find the quotient when $4x^3 + 17x^2 + 9x$ is divided by $x^2 + 5x + 6$, and show that the remainder is 18. [3]

(b) Hence solve the equation $4x^3 + 17x^2 + 9x - 18 = 0$. [3]

11. March/2020/Paper_9709/22/No.5

(a) Sketch, on the same diagram, the graphs of $y = |x + 2k|$ and $y = |2x - 3k|$, where k is a positive constant.

Give, in terms of k, the coordinates of the points where each graph meets the axes. [3]

(b) Find, in terms of k, the coordinates of each of the two points where the graphs intersect. [4]

(c) Find, in terms of k, the largest value of t satisfying the inequality

$$|2^t + 2k| \geqslant |2^{t+1} - 3k|.$$ [2]