## Numerical Solutions and Equations – 2020 A2
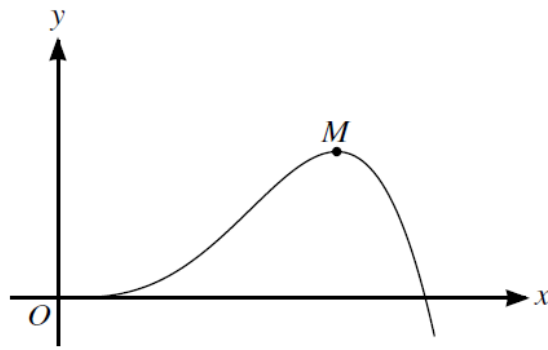
1. Nov/2020/Paper_9709/21/No.5

The sequence of values given by the iterative formula $x_{n+1} = \dfrac{6 + 8x_n}{8 + x_n^2}$ with initial value $x_1 = 2$ converges to α.

(a) Use the iterative formula to find the value of α correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

(b) State an equation satisfied by α and hence determine the exact value of α. [2]
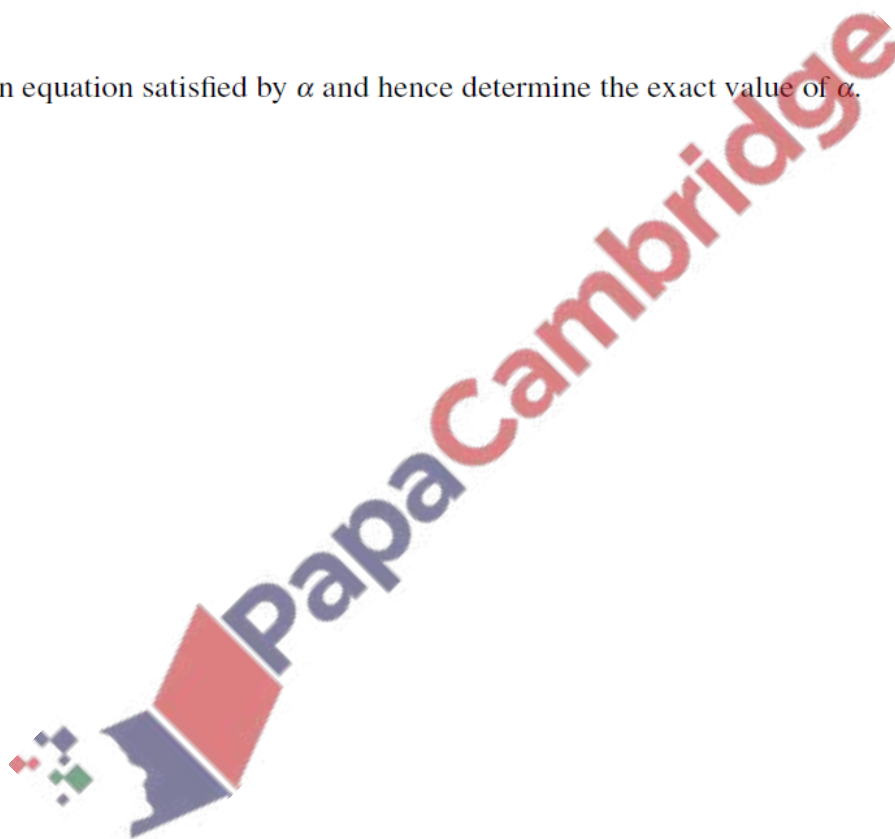
The diagram shows part of the curve with equation $y = x^3 \cos 2x$. The curve has a maximum at the point M.

(a) Show that the x-coordinate of M satisfies the equation $x = \sqrt[3]{1.5x^2 \cot 2x}$. [3]

(b) Use the equation in part **(a)** to show by calculation that the x-coordinate of M lies between 0.59 and 0.60. [2]

(c) Use an iterative formula, based on the equation in part **(a)**, to find the x-coordinate of M correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]
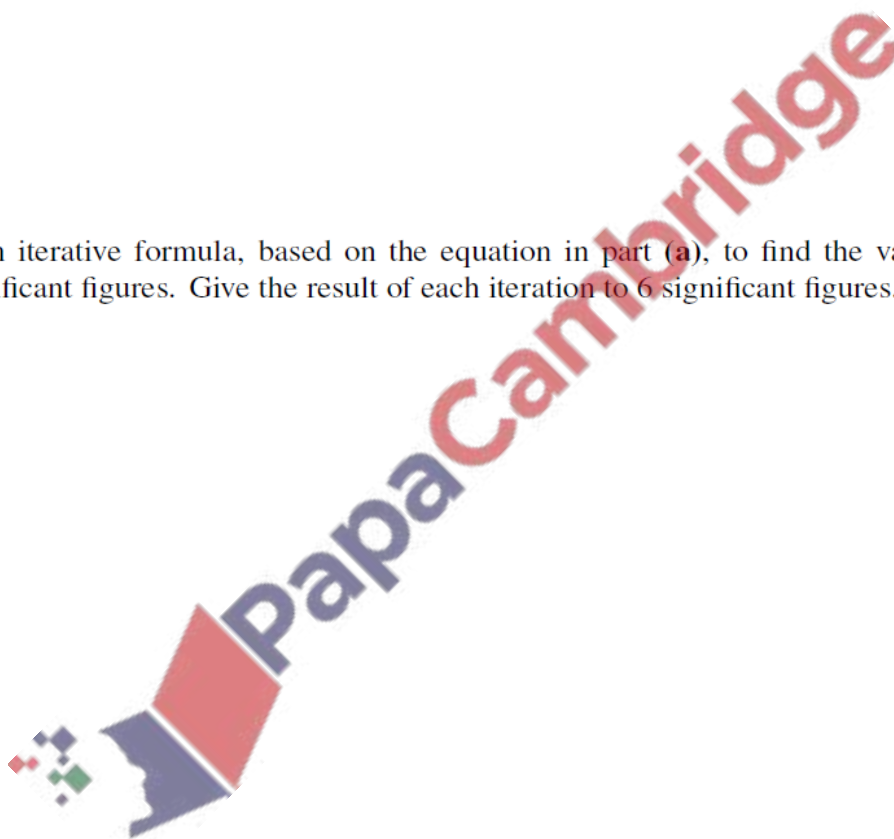
3.

It is given that $\displaystyle\int_0^a \left(\frac{4}{2x+1} + 8x \right) dx = 10$, where a is a positive constant.

(a) Show that $a = \sqrt{2.5 - 0.5\ln(2a+1)}$. [4]

(b) Using the equation in part **(a)**, show by calculation that $1 < a < 2$. [2]

(c) Use an iterative formula, based on the equation in part **(a)**, to find the value of a correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

4.

A curve has equation $y = x^3 e^{0.2x}$ where $x \geqslant 0$. At the point P on the curve, the gradient of the curve is 15.

(a) Show that the x-coordinate of P satisfies the equation $x = \sqrt{\dfrac{75e^{-0.2x}}{15+x}}$. [4]

(b) Use the equation in part **(a)** to show by calculation that the x-coordinate of P lies between 1.7 and 1.8. [2]

(c) Use an iterative formula, based on the equation in part **(a)**, to find the x-coordinate of P correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]