Complex Numbers – 2020 A2

1. Nov/2020/Paper_9709/31/No.7

(a) Verify that $-1 + \sqrt{5}\mathrm{i}$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

(b) Find the other roots of this equation. [4]

2. Nov/2020/Paper_9709/32/No.6

The complex number u is defined by

$$u = \frac{7+\mathrm{i}}{1-\mathrm{i}}.$$

(a) Express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

(b) Show on a sketch of an Argand diagram the points A, B and C representing u, $7 + \mathrm{i}$ and $1 - \mathrm{i}$ respectively. [2]

(c) By considering the arguments of $7 + \mathrm{i}$ and $1 - \mathrm{i}$, show that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = \tan^{-1}\left(\tfrac{1}{7}\right) + \tfrac{1}{4}\pi.$$ [3]

3. June/2020/Paper_9709/31/No.10

(a) The complex number u is defined by $u = \frac{3\mathrm{i}}{a + 2\mathrm{i}}$, where a is real.

(i) Express u in the Cartesian form $x + \mathrm{i}y$, where x and y are in terms of a. [3]

(ii) Find the exact value of a for which $\arg u^* = \frac{1}{3}\pi$. [3]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2\mathrm{i}| \leqslant |z - 1 - \mathrm{i}|$ and $|z - 2 - \mathrm{i}| \leqslant 2$. [4]

(ii) Calculate the least value of $\arg z$ for points in this region. [2]

4. June/2020/Paper_9709/32/No.8

(a) Solve the equation $(1 + 2i)w + iw^* = 3 + 5i$. Give your answer in the form $x + iy$, where x and y are real. [4]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2 - 2i| \leqslant 1$ and $\arg(z - 4i) \geqslant -\frac{1}{4}\pi$. [4]

(ii) Find the least value of $\operatorname{Im} z$ for points in this region, giving your answer in an exact form. [2]

5. June/2020/Paper_9709/33/No.9

(a) The complex numbers u and w are such that

$$u - w = 2\mathrm{i} \quad \text{and} \quad uw = 6.$$

Find u and w, giving your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities

$$|z - 2 - 2\mathrm{i}| \leqslant 2, \quad 0 \leqslant \arg z \leqslant \tfrac{1}{4}\pi \quad \text{and} \quad \operatorname{Re} z \leqslant 3.$$ [5]

6. March/2020/Paper_9709/32/No.10

(a) The complex numbers v and w satisfy the equations

$$v + \mathrm{i}w = 5 \qquad \text{and} \qquad (1 + 2\mathrm{i})v - w = 3\mathrm{i}.$$

Solve the equations for v and w, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

(b) **(i)** On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z - 2 - 3\mathrm{i}| = 1$. [2]

(ii) Calculate the least value of $\arg z$ for points on this locus. [2]