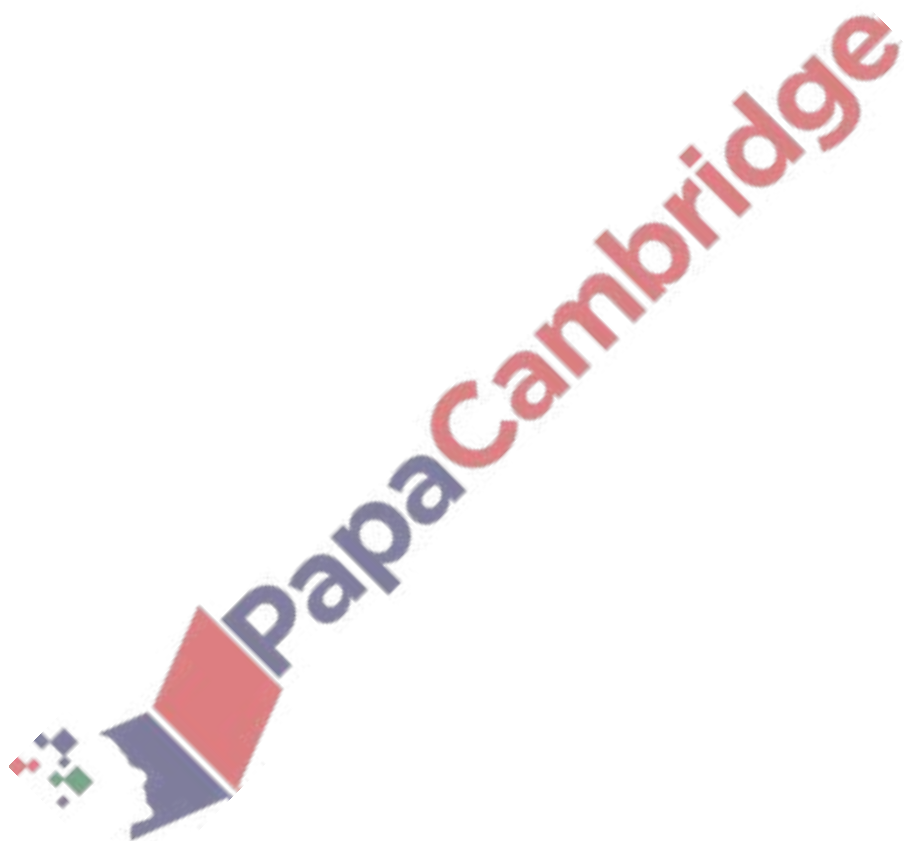


**Complex Numbers – 2020 A2**

1. Nov/2020/Paper\_9709/31/No.7

(a) Verify that  $-1 + \sqrt{5}i$  is a root of the equation  $2x^3 + x^2 + 6x - 18 = 0$ . [3]

(b) Find the other roots of this equation. [4]



The complex number  $u$  is defined by

$$u = \frac{7 + i}{1 - i}.$$

(a) Express  $u$  in the form  $x + iy$ , where  $x$  and  $y$  are real.

[3]

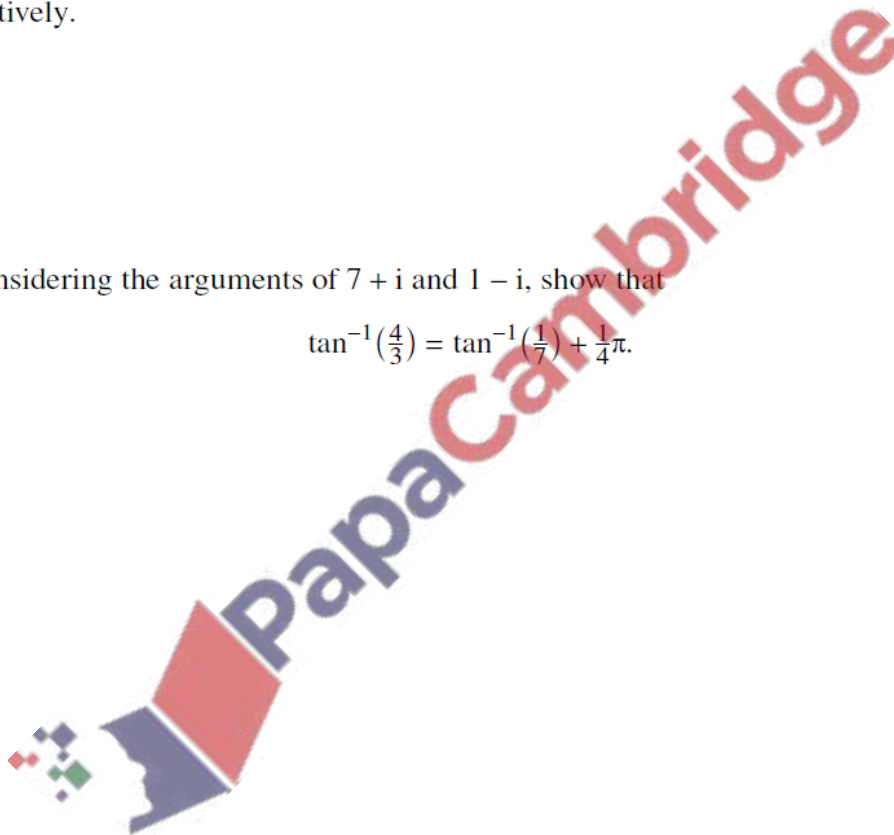
(b) Show on a sketch of an Argand diagram the points  $A$ ,  $B$  and  $C$  representing  $u$ ,  $7 + i$  and  $1 - i$  respectively.

[2]

(c) By considering the arguments of  $7 + i$  and  $1 - i$ , show that

$$\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right) + \frac{1}{4}\pi.$$

[3]



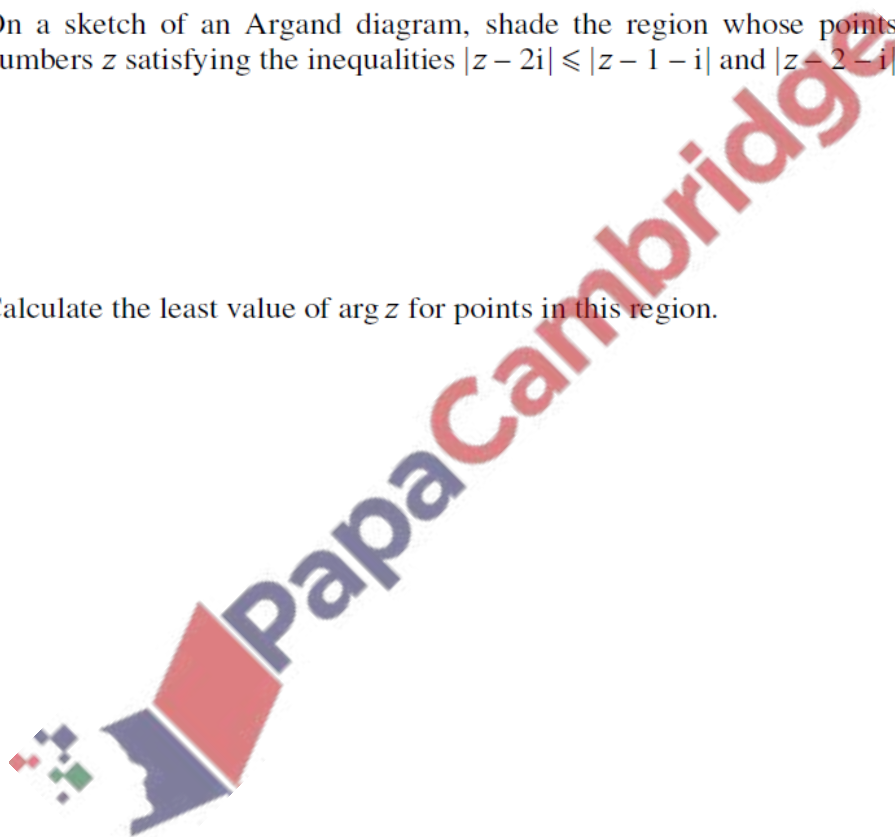
(a) The complex number  $u$  is defined by  $u = \frac{3i}{a + 2i}$ , where  $a$  is real.

(i) Express  $u$  in the Cartesian form  $x + iy$ , where  $x$  and  $y$  are in terms of  $a$ . [3]

(ii) Find the exact value of  $a$  for which  $\arg u^* = \frac{1}{3}\pi$ . [3]

(b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $|z - 2i| \leq |z - 1 - i|$  and  $|z - 2 - i| \leq 2$ . [4]

(ii) Calculate the least value of  $\arg z$  for points in this region. [2]

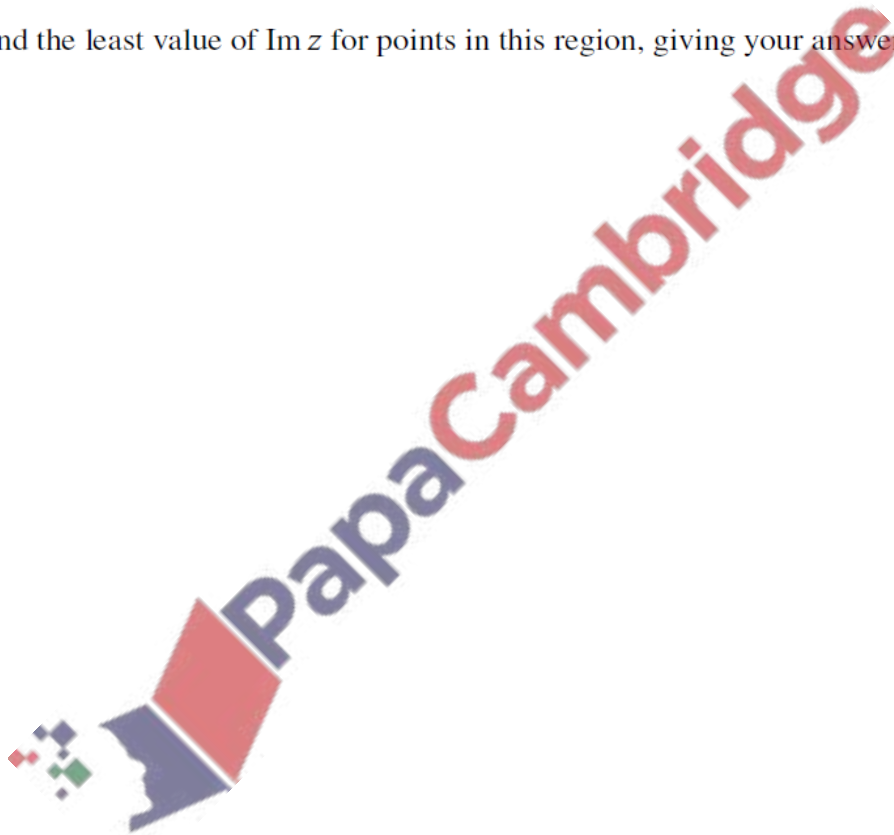


4. June/2020/Paper\_9709/32/No.8

(a) Solve the equation  $(1 + 2i)w + iw^* = 3 + 5i$ . Give your answer in the form  $x + iy$ , where  $x$  and  $y$  are real. [4]

(b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $|z - 2 - 2i| \leq 1$  and  $\arg(z - 4i) \geq -\frac{1}{4}\pi$ . [4]

(ii) Find the least value of  $\text{Im } z$  for points in this region, giving your answer in an exact form. [2]



5. June/2020/Paper\_9709/33/No.9

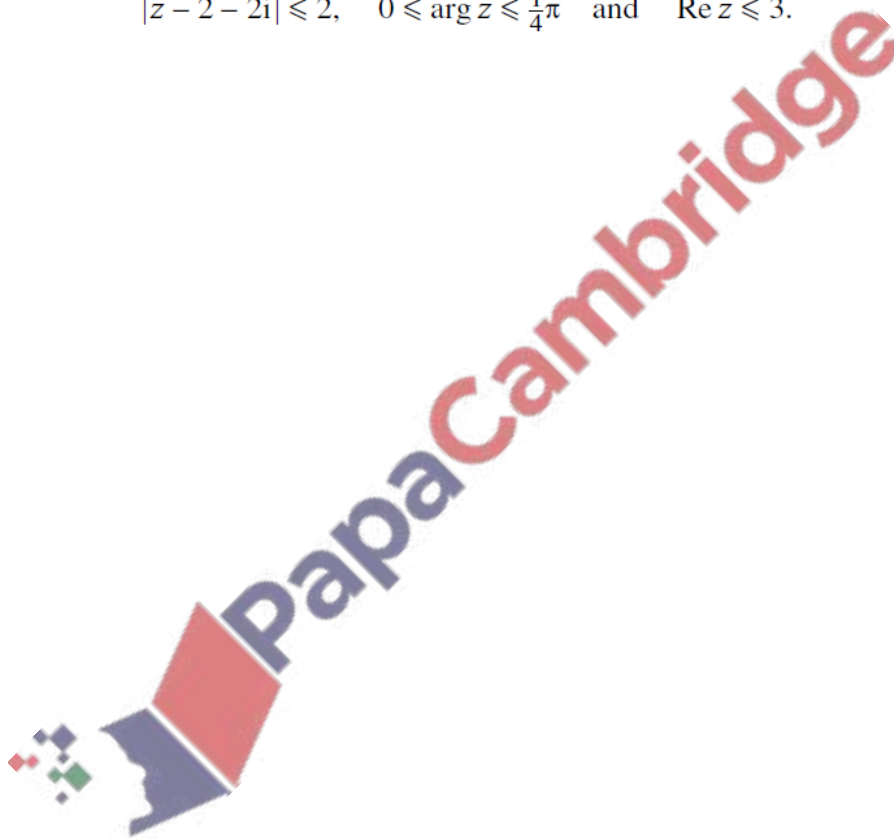
(a) The complex numbers  $u$  and  $w$  are such that

$$u - w = 2i \quad \text{and} \quad uw = 6.$$

Find  $u$  and  $w$ , giving your answers in the form  $x + iy$ , where  $x$  and  $y$  are real and exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities

$$|z - 2 - 2i| \leq 2, \quad 0 \leq \arg z \leq \frac{1}{4}\pi \quad \text{and} \quad \operatorname{Re} z \leq 3. \quad [5]$$



(a) The complex numbers  $v$  and  $w$  satisfy the equations

$$v + iw = 5 \quad \text{and} \quad (1 + 2i)v - w = 3i.$$

Solve the equations for  $v$  and  $w$ , giving your answers in the form  $x + iy$ , where  $x$  and  $y$  are real. [6]

(b) (i) On an Argand diagram, sketch the locus of points representing complex numbers  $z$  satisfying  $|z - 2 - 3i| = 1$ . [2]

(ii) Calculate the least value of  $\arg z$  for points on this locus. [2]

