Numerical Solutions of Equations – 2020 A2

1. Nov/2020/Paper_9709/31/No.5

(a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

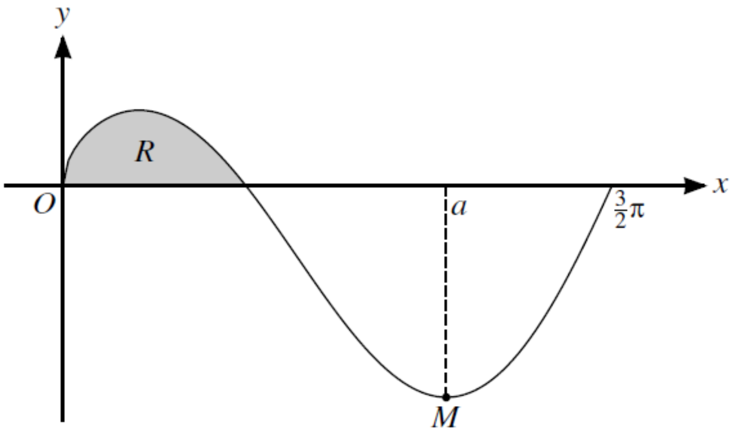
(b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1}\left(\frac{1}{e^{-\frac{1}{2}x_n} + 1}\right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

2. Nov/2020/Paper_9709/32/No.10



The diagram shows the curve $y = \sqrt{x}\cos x$, for $0 \leqslant x \leqslant \frac{3}{2}\pi$, and its minimum point M, where $x = a$. The shaded region between the curve and the x-axis is denoted by R.

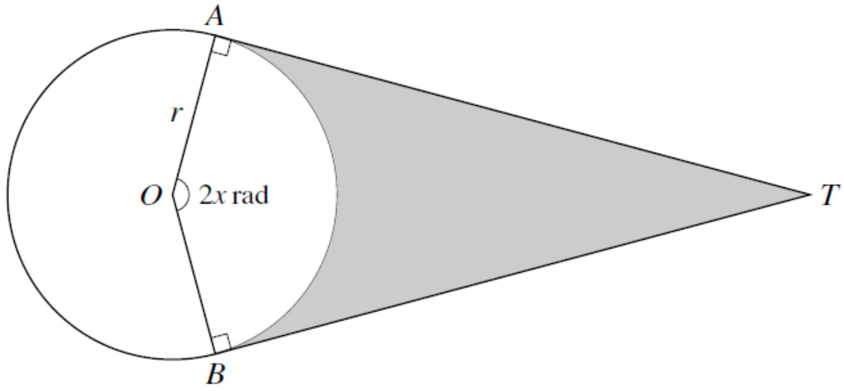
(a) Show that a satisfies the equation $\tan a = \dfrac{1}{2a}$. [3]

(b) The sequence of values given by the iterative formula $a_{n+1} = \pi + \tan^{-1}\left(\dfrac{1}{2a_n}\right)$, with initial value $x_1 = 3$, converges to a.

Use this formula to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(c) Find the volume of the solid obtained when the region R is rotated completely about the x-axis. Give your answer in terms of π. [6]

3. June/2020/Paper_9709/31/No.6



The diagram shows a circle with centre O and radius r. The tangents to the circle at the points A and B meet at T, and angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT, and by the minor arc AB. The area of the shaded region is equal to the area of the circle.

(a) Show that x satisfies the equation $\tan x = \pi + x$. [3]

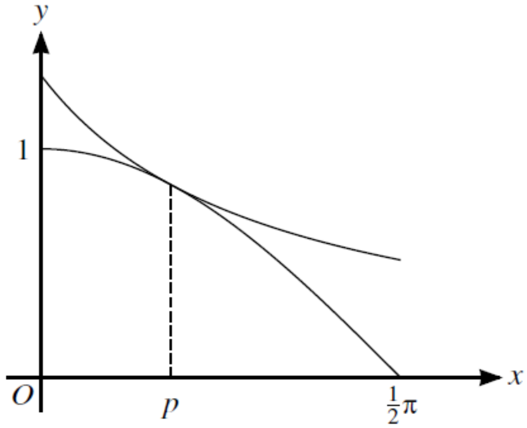
(b) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.4. [2]

(c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4. June/2020/Paper_9709/32/No.9



The diagram shows the curves $y = \cos x$ and $y = \dfrac{k}{1+x}$, where k is a constant, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curves touch at the point where $x = p$.

(a) Show that p satisfies the equation $\tan p = \dfrac{1}{1+p}$. [5]

(b) Use the iterative formula $p_{n+1} = \tan^{-1}\left(\dfrac{1}{1+p_n}\right)$ to determine the value of p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(c) Hence find the value of k correct to 2 decimal places. [2]

5. June/2020/Paper_9709/33/No.6

(a) By sketching a suitable pair of graphs, show that the equation $x^5 = 2 + x$ has exactly one real root. [2]

(b) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{4x_n^5 + 2}{5x_n^4 - 1}$$

converges, then it converges to the root of the equation in part **(a)**. [2]

(c) Use the iterative formula with initial value $x_1 = 1.5$ to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

6. March/2020/Paper_9709/32/No.3

(a) By sketching a suitable pair of graphs, show that the equation $\sec x = 2 - \frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \frac{1}{2}\pi$. [2]

(b) Verify by calculation that this root lies between 0.8 and 1. [2]

(c) Use the iterative formula $x_{n+1} = \cos^{-1}\left(\frac{2}{4 - x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]