Sampling and Estimation – 2020 A2

1. Nov/2020/Paper_9709/61/No.2

In a survey, a random sample of 250 adults in Fromleigh were asked to fill in a questionnaire about their travel.

(a) It was found that 102 adults in the sample travel by bus. Find an approximate 90% confidence interval for the proportion of all the adults in Fromleigh who travel by bus. [3]

(b) The survey included a question about the amount, x dollars, spent on travel per year. The results are summarised as follows.

$$n = 250 \qquad \Sigma x = 50\,460 \qquad \Sigma x^2 = 19\,854\,200$$

Find unbiased estimates of the population mean and variance of the amount spent per year on travel. [3]

A councillor wanted to select a random sample of houses in Fromleigh. He planned to select the first house on each of the 143 streets in Fromleigh.

(c) Explain why this would not provide a random sample. [1]

2. Nov/2020/Paper_9709/62/No.2

A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws.

(a) Calculate an approximate 96% confidence interval for the probability that the die shows a six on one throw. [3]

(b) Maroulla claims that the die is biased.

Use your answer to part **(a)** to comment on this claim. [1]

3. June/2020/Paper_9709/61/No.1

The lengths, X centimetres, of a random sample of 7 leaves from a certain variety of tree are as follows.

5.2 4.8 5.5 6.1 4.8 3.9 4.4

(a) Calculate unbiased estimates of the population mean and variance of X. [3]

It is now given that the true value of the population variance of X is 0.55, and that X has a normal distribution.

(b) Find a 95% confidence interval for the population mean of X. [3]

4. June/2020/Paper_9709/62/No.4

The score on one spin of a 5-sided spinner is denoted by the random variable X with probability distribution as shown in the table.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

(a) Show that $\text{Var}(X) = 1.2$. [2]

The spinner is spun 200 times. The score on each spin is noted and the mean, $\overline{X}$, of the 200 scores is found.

(b) Given that $P(\overline{X} > a) = 0.1$, find the value of a. [4]

(c) Explain whether it was necessary to use the Central Limit theorem in your answer to part **(b)**. [1]

(d) Johann has another, similar, spinner. He suspects that it is biased so that the mean score is less than 2. He spins his spinner 200 times and finds that the mean of the 200 scores is 1.86.

Given that the variance of the score on one spin of this spinner is also 1.2, test Johann's suspicion at the 5% significance level. [5]

5. June/2020/Paper_9709/63/No.1

A random sample of 100 values of a variable X is taken. These values are summarised below.

$$n = 100 \qquad \Sigma x = 1556 \qquad \Sigma x^2 = 29\,004$$

Calculate unbiased estimates of the population mean and variance of X. [3]

6. June/2020/Paper_9709/63/No.5

Sunita has a six-sided die with faces marked 1, 2, 3, 4, 5, 6. The probability that the die shows a six on any throw is p. Sunita throws the die 500 times and finds that it shows a six 70 times.

(a) Calculate an approximate 99% confidence interval for p. [4]

(b) Sunita believes that the die is fair. Use your answer to part **(a)** to comment on her belief. [1]

(c) Sunita uses the result of her 500 throws to calculate an α% confidence interval for p. This interval has width 0.04.

Find the value of α. [5]

7. March/2020/Paper_9709/62/No.2

Lengths of a certain species of lizard are known to be normally distributed with standard deviation 3.2 cm. A naturalist measures the lengths of a random sample of 100 lizards of this species and obtains an $\alpha\%$ confidence interval for the population mean. He finds that the total width of this interval is 1.25 cm.

Find α. [5]

8. March/2020/Paper_9709/62/No.3

In the past, the mean time taken by Freda for a particular daily journey was 39.2 minutes. Following the introduction of a one-way system, Freda wishes to test whether the mean time for the journey has decreased. She notes the times, t minutes, for 40 randomly chosen journeys and summarises the results as follows.

$$n = 40 \qquad \Sigma t = 1504 \qquad \Sigma t^2 = 57\,760$$

(a) Calculate unbiased estimates of the population mean and variance of the new journey time. [3]

(b) Test, at the 5% significance level, whether the population mean time has decreased. [5]