Complex Numbers – 2021 A2

1. June/2021/Paper_9709/31/No.5

(a) Solve the equation $z^2 - 2piz - q = 0$, where p and q are real constants. [2]

In an Argand diagram with origin O, the roots of this equation are represented by the distinct points A and B.

(b) Given that A and B lie on the imaginary axis, find a relation between p and q. [2]

(c) Given instead that triangle OAB is equilateral, express q in terms of p. [3]

2. June/2021/Paper_9709/32/No.2

On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 1 - \mathrm{i}| \leqslant 1$ and $\arg(z - 1) \leqslant \frac{3}{4}\pi$. [4]

3. June/2021/Paper_9709/32/No.5

The complex number u is given by $u = 10 - 4\sqrt{6}\text{i}$.

Find the two square roots of u, giving your answers in the form $a + \text{i}b$, where a and b are real and exact. [5]

4. June/2021/Paper_9709/33/No.10

(a) Verify that $-1 + \sqrt{2}\mathrm{i}$ is a root of the equation $z^4 + 3z^2 + 2z + 12 = 0$. [3]

(b) Find the other roots of this equation. [7]

5. March/2021/Paper_9709/32/No.8

The complex numbers u and v are defined by $u = -4 + 2\text{i}$ and $v = 3 + \text{i}$.

(a) Find $\frac{u}{v}$ in the form $x + \text{i}y$, where x and y are real. [3]

(b) Hence express $\frac{u}{v}$ in the form $r\text{e}^{\text{i}\theta}$, where r and θ are exact. [2]

In an Argand diagram, with origin O, the points A, B and C represent the complex numbers u, v and $2u + v$ respectively.

(c) State fully the geometrical relationship between OA and BC. [2]

(d) Prove that angle $AOB = \frac{3}{4}\pi$. [2]