Integration – 2021 A2

1. June/2021/Paper_9709/31/No.4

(a) Prove that $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} \equiv \tan^2\theta$. [2]

(b) Hence find the exact value of $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1-\cos 2\theta}{1+\cos 2\theta}\,\mathrm{d}\theta$. [4]

2. June/2021/Paper_9709/31/No.9b

The equation of a curve is $y = x^{-\frac{2}{3}} \ln x$ for $x > 0$. The curve has one stationary point.

(b) Show that $\int_1^8 y \, dx = 18 \ln 2 - 9$. [5]

3. June/2021/Paper_9709/32/No.4

Using integration by parts, find the exact value of $\int_0^2 \tan^{-1}(\frac{1}{2}x)\,dx$. [5]

4. June/2021/Paper_9709/32/No.6

(a) Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan \theta$. [3]

(b) Hence show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} (\operatorname{cosec} 2\theta - \cot 2\theta)\, d\theta = \frac{1}{2} \ln 2$. [4]

5. June/2021/Paper_9709/33/No.4b

Let $f(x) = \dfrac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence find $\int_1^2 f(x)\,dx$, giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a and b are integers. [4]

6. June/2021/Paper_9709/33/No.8b

(b) By using integration by parts, show that for all $a > 1$, $\int_1^a \frac{\ln x}{x^4}\,\mathrm{d}x < \frac{1}{9}$. [6]

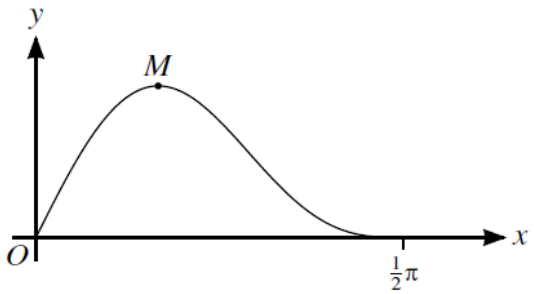
7. March/2021/Paper_9709/32/No.6

Let $\mathrm{f}(x) = \dfrac{5a}{(2x-a)(3a-x)}$, where a is a positive constant.

(a) Express $\mathrm{f}(x)$ in partial fractions. [3]

(b) Hence show that $\displaystyle\int_a^{2a} \mathrm{f}(x)\,\mathrm{d}x = \ln 6$. [4]

8. March/2021/Paper_9709/32/No.10



The diagram shows the curve $y = \sin 2x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Using the substitution $u = \sin x$, find the exact area of the region bounded by the curve and the x-axis. [5]

(b) Find the exact x-coordinate of M. [6]