1. Nov/2021/Paper_9709/31/No.10

The complex number $1 + 2i$ is denoted by u. The polynomial $2x^3 + ax^2 + 4x + b$, where a and b are real constants, is denoted by p(x). It is given that u is a root of the equation p(x) = 0.

(a) Find the values of a and b. [4]

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(b) State a second complex root of this equation. [1]

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(c) Find the real factors of p(x). [2]

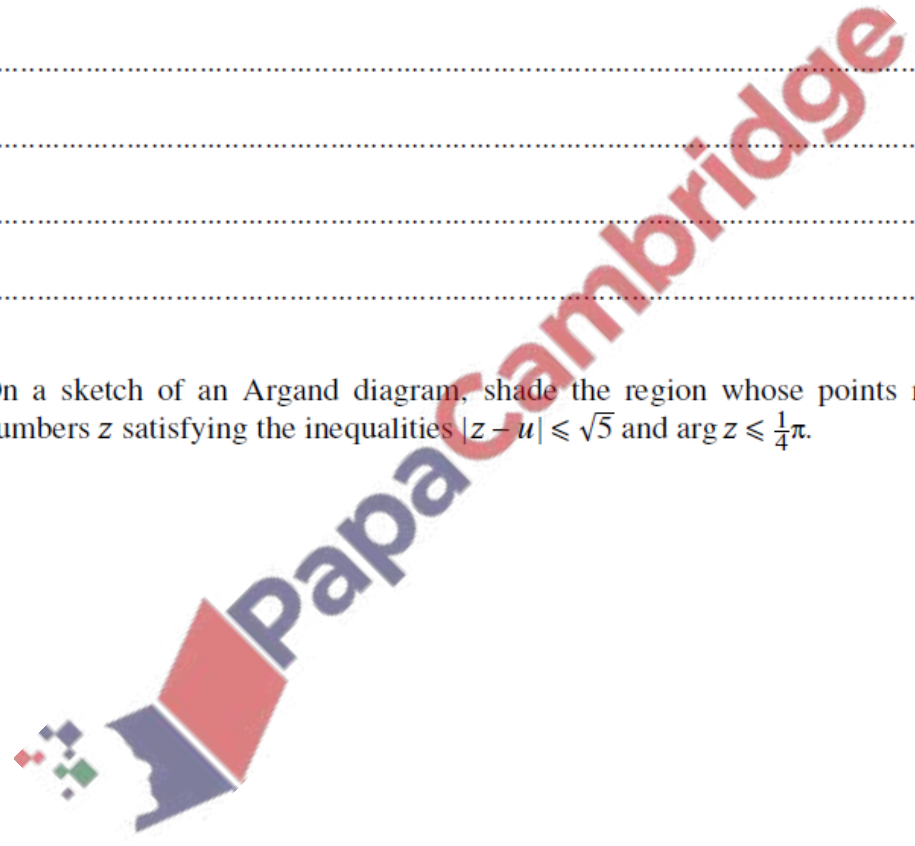...

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(d) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leqslant \sqrt{5}$ and $\arg z \leqslant \frac{1}{4}\pi$. [4]

(ii) Find the least value of $\operatorname{Im} z$ for points in the shaded region. Give your answer in an exact form. [1]

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2.

(a) Given the complex numbers $u = a + ib$ and $w = c + id$, where a, b, c and d are real, prove that $(u + w)^* = u^* + w^*$. [2]

(b) Solve the equation $(z + 2 + i)^* + (2 + i)z = 0$, giving your answer in the form $x + iy$ where x and y are real. [4]

(a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - 2i| \leqslant 1$ and $\operatorname{Im} z \geqslant 2$. [4]

(b) Find the greatest value of $\arg z$ for points in the shaded region, giving your answer in degrees. [3]

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4.

The complex number $-\sqrt{3} + i$ is denoted by u.

(a) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ.   [2]

(b) Hence show that u^6 is real and state its value.   [2]

(c) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $0 \leqslant \arg(z-u) \leqslant \frac{1}{4}\pi$ and $\operatorname{Re} z \leqslant 2$. [4]

(ii) Find the greatest value of $|z|$ for points in the shaded region. Give your answer correct to 3 significant figures. [2]

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