**Differential Equations – 2021 A2 Nov P3**

**1.** Nov/2021/Paper_9709/31/No.7

**(a)** Given that $y = \ln(\ln x)$, show that

$$\frac{dy}{dx} = \frac{1}{x \ln x}.$$ [1]

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The variables $x$ and $t$ satisfy the differential equation

$$x \ln x + t\frac{dx}{dt} = 0.$$

It is given that $x = e$ when $t = 2$.

**(b)** Solve the differential equation obtaining an expression for $x$ in terms of $t$, simplifying your answer. [7]

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**(c)** Hence state what happens to the value of $x$ as $t$ tends to infinity. [1]

**2.** Nov/2021/Paper_9709/32/No.7

The variables $x$ and $y$ satisfy the differential equation

$$e^{2x}\frac{dy}{dx} = 4xy^2,$$

and it is given that $y = 1$ when $x = 0$.

Solve the differential equation, obtaining an expression for $y$ in terms of $x$. [7]

**3.** Nov/2021/Paper_9709/33/No.10

A large plantation of area 20 $km^2$ is becoming infected with a plant disease. At time $t$ years the area infected is $x\,km^2$ and the rate of increase of $x$ is proportional to the ratio of the area infected to the area not yet infected.

When $t = 0$, $x = 1$ and $\frac{dx}{dt} = 1$.

**(a)** Show that $x$ and $t$ satisfy the differential equation

$$\frac{dx}{dt} = \frac{19x}{20 - x}.$$ [2]

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**(b)** Solve the differential equation and show that when $t = 1$ the value of $x$ satisfies the equation $x = e^{0.9+0.05x}$. [5]

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(c) Use an iterative formula based on the equation in part **(b)**, with an initial value of 2, to determine $x$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(d) Calculate the value of $t$ at which the entire plantation becomes infected. [1]