

**1. Nov/2021/Paper\_9709/31/No.2**

- (a) Express  $5 \sin x - 3 \cos x$  in the form  $R \sin(x - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . Give the exact value of  $R$  and give  $\alpha$  correct to 2 decimal places. [3]

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- (b) Hence state the greatest and least possible values of  $(5 \sin x - 3 \cos x)^2$ . [2]

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(a) Show that the equation

$$\cot 2\theta + \cot \theta = 2$$

can be expressed as a quadratic equation in  $\tan \theta$ .

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(b) Hence solve the equation  $\cot 2\theta + \cot \theta = 2$ , for  $0 < \theta < \pi$ , giving your answers correct to 3 decimal places. [3]

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3. Nov/2021/Paper\_9709/32/No.8

(a) By first expanding  $(\cos^2 \theta + \sin^2 \theta)^2$ , show that

$$\cos^4 \theta + \sin^4 \theta \equiv 1 - \frac{1}{2} \sin^2 2\theta. \quad [3]$$

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(b) Hence solve the equation

$$\cos^4 \theta + \sin^4 \theta = \frac{5}{9},$$

for  $0^\circ < \theta < 180^\circ$ .

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4. Nov/2021/Paper\_9709/33/No.5

Solve the equation  $\sin \theta = 3 \cos 2\theta + 2$ , for  $0^\circ \leq \theta \leq 360^\circ$ .

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5. **Nov/2021/Paper\_9709/33/No.6**

(a) By first expanding  $\cos(x - 60^\circ)$ , show that the expression

$$2 \cos(x - 60^\circ) + \cos x$$

can be written in the form  $R \cos(x - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . Give the exact value of  $R$  and the value of  $\alpha$  correct to 2 decimal places. [5]

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(b) Hence find the value of  $x$  in the interval  $0^\circ < x < 360^\circ$  for which  $2 \cos(x - 60^\circ) + \cos x$  takes its least possible value. [2]

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