Complex Numbers – 2022 A2 June

1. March/2022/Paper_9709/32/No.2

On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 2 - 3\mathrm{i}| \leqslant 2$ and $\arg z \leqslant \frac{3}{4}\pi$. [4]

2. March/2022/Paper_9709/32/No.6

Find the complex numbers w which satisfy the equation $w^2 + 2\mathrm{i}w^* = 1$ and are such that $\operatorname{Re} w \leqslant 0$. Give your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

3. June/2022/Paper_9709/31/No.7

The complex number u is defined by $u = \dfrac{\sqrt{2} - a\sqrt{2}\mathrm{i}}{1 + 2\mathrm{i}}$, where a is a positive integer.

(a) Express u in terms of a, in the form $x + \mathrm{i}y$, where x and y are real and exact. [3]

It is now given that $a = 3$.

(b) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [2]

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(c) Using your answer to part **(b)**, find the two square roots of u. Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [3]

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4. June/2022/Paper_9709/32/No.10

The complex number $-1 + \sqrt{7}\,\text{i}$ is denoted by u. It is given that u is a root of the equation

$$2x^3 + 3x^2 + 14x + k = 0,$$

where k is a real constant.

(a) Find the value of k. [3]

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(b) Find the other two roots of the equation. [4]

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(c) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the equation $|z - u| = 2$. [2]

(d) Determine the greatest value of $\arg z$ for points on this locus, giving your answer in radians. [2]

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5. June/2022/Paper_9709/33/No.5

The complex number $3 - \mathrm{i}$ is denoted by u.

(a) Show, on an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively.

State the type of quadrilateral formed by the points O, A, B and C. [3]

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(b) Express $\dfrac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

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(c) By considering the argument of $\dfrac{u^*}{u}$, or otherwise, prove that $\tan^{-1}\left(\frac{3}{4}\right) = 2\tan^{-1}\left(\frac{1}{3}\right)$. [2]