1. March/2022/Paper_9709/32/No.7

(a) By sketching a suitable pair of graphs, show that the equation $4 - x^2 = \sec \frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 2. [2]

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(c) Use the iterative formula $x_{n+1} = \sqrt{4 - \sec \frac{1}{2}x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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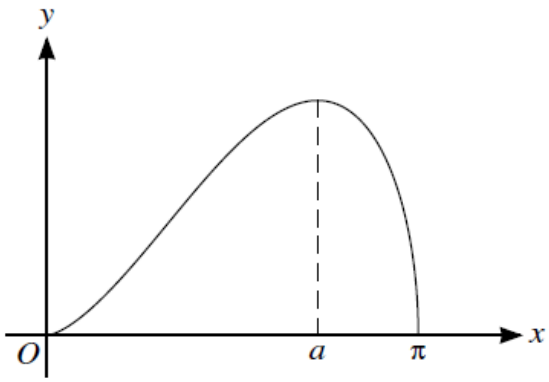
2. June/2022/Paper_9709/31/No.10



The curve $y = x\sqrt{\sin x}$ has one stationary point in the interval $0 < x < \pi$, where $x = a$ (see diagram).

(a) Show that $\tan a = -\frac{1}{2}a$. [4]

(b) Verify by calculation that a lies between 2 and 2.5. [2]

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(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula $x_{n+1} = \pi - \tan^{-1}\left(\frac{1}{2}x_n\right)$ converges, then it converges to a, the root of the equation in part **(a)**. [2]

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(d) Use the iterative formula given in part **(c)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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3. June/2022/Paper_9709/32/No.5

(a) By sketching a suitable pair of graphs, show that the equation $\ln x = 3x - x^2$ has one real root. [2]

(b) Verify by calculation that the root lies between 2 and 2.8. [2]

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(c) Use the iterative formula $x_{n+1} = \sqrt{3x_n - \ln x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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4. June/2022/Paper_9709/33/No.10

The constant a is such that $\int_1^a x^2 \ln x \, dx = 4$.

(a) Show that $a = \left(\frac{35}{3 \ln a - 1}\right)^{\frac{1}{3}}$. [5]

(b) Verify by calculation that a lies between 2.4 and 2.8. [2]

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(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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