Hypothesis Tests – 2022 A2 June

1.

Harry has a five-sided spinner with sectors coloured blue, green, red, yellow and black. Harry thinks the spinner may be biased. He plans to carry out a hypothesis test with the following hypotheses.

$$H_0: \text{P(the spinner lands on blue)} = \tfrac{1}{5}$$

$$H_1: \text{P(the spinner lands on blue)} \neq \tfrac{1}{5}$$

Harry spins the spinner 300 times. It lands on blue on 45 spins.

Use a suitable approximation to carry out Harry's test at the 5% significance level. [5]

In the past the time, in minutes, taken by students to complete a certain challenge had mean 25.5 and standard deviation 5.2. A new challenge is devised and it is expected that students will take, on average, less than 25.5 minutes to complete this challenge. A random sample of 40 students is chosen and their mean time for the new challenge is found to be 23.7 minutes.

(a) Assuming that the standard deviation of the time for the new challenge is 5.2 minutes, test at the 1% significance level whether the population mean time for the new challenge is less than 25.5 minutes. [5]

(b) State, with a reason, whether it is possible that a Type I error was made in the test in part (a). [1]

Arvind uses an ordinary fair 6-sided die to play a game. He believes he has a system to predict the score when the die is thrown. Before each throw of the die, he writes down what he thinks the score will be. He claims that he can write the correct score more often than he would if he were just guessing. His friend Laxmi tests his claim by asking him to write down the score before each of 15 throws of the die. Arvind writes the correct score on exactly 5 out of 15 throws.

Test Arvind's claim at the 10% significance level. [5]

In the past, the mean time for Jenny's morning run was 28.2 minutes. She does some extra training and she wishes to test whether her mean time has been reduced. After the training Jenny takes a random sample of 40 morning runs. She decides that if the sample mean run time is less than 27 minutes she will conclude that the training has been effective. You may assume that, after the training, Jenny's run time has a standard deviation of 4.0 minutes.

(a) State suitable null and alternative hypotheses for Jenny's test. [1]

(b) Find the probability that Jenny will make a Type I error. [3]

(c) Jenny found that the sample mean run time was 27.2 minutes.

Explain briefly whether it is possible for her to make a Type I error or a Type II error or both.
[2]

5.

In the past, the mean height of plants of a particular species has been 2.3 m. A random sample of 60 plants of this species was treated with fertiliser and the mean height of these 60 plants was found to be 2.4 m. Assume that the standard deviation of the heights of plants treated with fertiliser is 0.4 m.

Carry out a test at the 2.5% significance level of whether the mean height of plants treated with fertiliser is greater than 2.3 m. [5]

The number of cars arriving at a certain road junction on a weekday morning has a Poisson distribution with mean 4.6 per minute. Traffic lights are installed at the junction and a council officer wishes to test at the 2% significance level whether there are now fewer cars arriving. He notes the number of cars arriving during a randomly chosen 2-minute period.

(a) State suitable null and alternative hypotheses for the test. [1]

(b) Find the critical region for the test. [4]
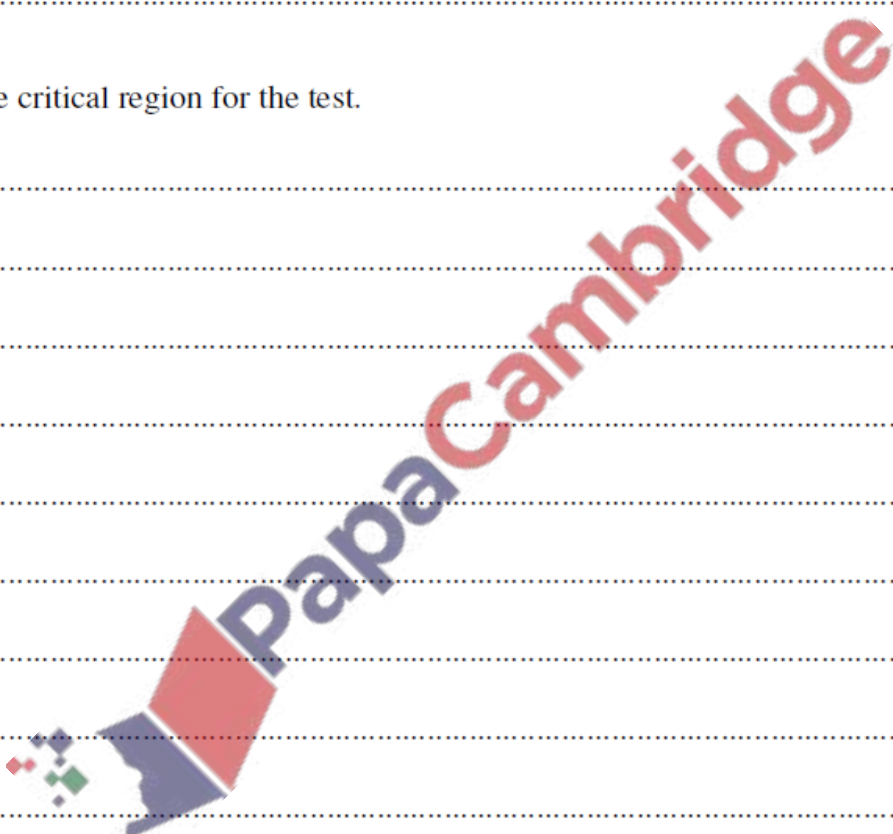
The officer notes that, during a randomly chosen 2-minute period on a weekday morning, exactly 5 cars arrive at the junction.

(c) Carry out the test. [2]

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(d) State, with a reason, whether it is possible that a Type I error has been made in carrying out the test in part **(c)**. [1]

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The number of cars arriving at another junction on a weekday morning also has a Poisson distribution with mean 4.6 per minute.

(e) Use a suitable approximating distribution to find the probability that more than 300 cars will arrive at this junction in an hour. [3]

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Anton believes that 10% of students at his college are left-handed. Aliya believes that this is an underestimate. She plans to carry out a hypothesis test of the null hypothesis $p = 0.1$ against the alternative hypothesis $p > 0.1$, where p is the actual proportion of students at the college that are left-handed. She chooses a random sample of 20 students from the college. She will reject the null hypothesis if at least 5 of these students are left-handed.

(a) Explain what is meant by a Type I error in this context. [1]

(b) Find the probability of a Type I error in the test. [3]

(c) Given that the true value of p is 0.3, find the probability of a Type II error in the test. [2]

Batteries of type A are known to have a mean life of 150 hours. It is required to test whether a new type of battery, type B, has a shorter mean life than type A batteries.

(a) Give a reason for using a sample rather than the whole population in carrying out this test. [1]

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A random sample of 120 type B batteries are tested and it is found that their mean life is 147 hours, and an unbiased estimate of the population variance is 225 hours^2.

(b) Test, at the 2% significance level, whether type B batteries have a shorter mean life than type A batteries. [5]

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(c) Calculate a 94% confidence interval for the population mean life of type B batteries. [3]

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