The Poisson distribution – 2022 A2 June

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   (a) Two ponds, A and B, each contain a large number of fish. It is known that 2.4% of fish in pond A are carp and 1.8% of fish in pond B are carp. Random samples of 50 fish from pond A and 60 fish from pond B are selected.

   Use appropriate Poisson approximations to find the following probabilities.

   (i) The samples contain at least 2 carp from pond A and at least 2 carp from pond B. [3]

   (ii) The samples contain at least 4 carp altogether. [3]

(b) The random variables X and Y have the distributions $\text{Po}(\lambda)$ and $\text{Po}(\mu)$ respectively. It is given that

- $P(X = 0) = [P(Y = 0)]^2$,

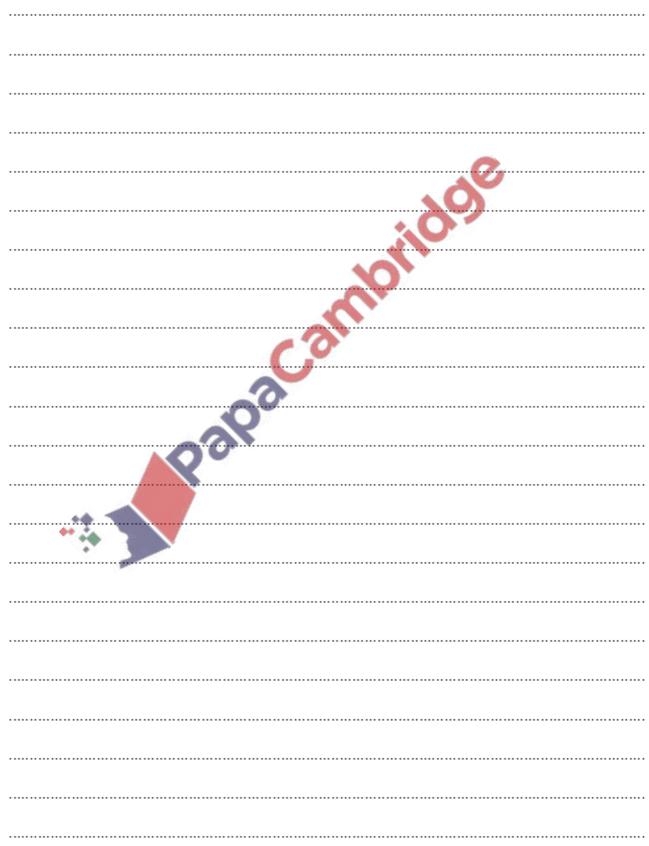- $P(X = 2) = k[P(Y = 1)]^2$, where k is a non-zero constant.

Find the value of k. [4]

Cars arrive at a fuel station at random and at a constant average rate of 13.5 per hour.

(a) Find the probability that more than 4 cars arrive during a 20-minute period. [3]

(b) Use an approximating distribution to find the probability that the number of cars that arrive during a 12-hour period is between 150 and 160 inclusive. [4]

Independently of cars, trucks arrive at the fuel station at random and at a constant average rate of 3.6 per 15-minute period.

(c) Find the probability that the total number of cars and trucks arriving at the fuel station during a 10-minute period is more than 3 and less than 7. [3]

3.

X is a random variable with distribution Po(2.90). A random sample of 100 values of X is taken.

Find the probability that the sample mean is less than 2.88. [5]

The number of clients who arrive at an information desk has a Poisson distribution with mean 2.2 per 5-minute period.

(a) Find the probability that, in a randomly chosen 15-minute period, exactly 6 clients arrive at the desk. [3]

(b) If more than 4 clients arrive during a 5-minute period, they cannot all be served.

Find the probability that, during a randomly chosen 5-minute period, not all the clients who arrive at the desk can be served. [2]

(c) Use a suitable approximating distribution to find the probability that, during a randomly chosen 1-hour period, fewer than 20 clients arrive at the desk. [4]