Differential Equations – 2022 A2 Nov Math

1. Nov/2022/Paper_9709_31/No.8

In a certain chemical reaction the amount, x grams, of a substance is increasing. The differential equation satisfied by x and t, the time in seconds since the reaction began, is

$$\frac{dx}{dt} = kxe^{-0.1t},$$

where k is a positive constant. It is given that $x = 20$ at the start of the reaction.

(a) Solve the differential equation, obtaining a relation between x, t and k. [5]

(b) Given that $x = 40$ when $t = 10$, find the value of k and find the value approached by x as t becomes large. [3]

2. Nov/2022/Paper_9709_32/No.7

The variables x and θ satisfy the differential equation

$$x\sin^2\theta\frac{dx}{d\theta} = \tan^2\theta - 2\cot\theta,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 2$ when $\theta = \frac{1}{4}\pi$.

(a) Show that $\frac{d}{d\theta}(\cot^2\theta) = -\frac{2\cot\theta}{\sin^2\theta}$.

(You may assume without proof that the derivative of $\cot\theta$ with respect to θ is $-\operatorname{cosec}^2\theta$.) [1]

....

....

....

....

....

....

....

....

(b) Solve the differential equation and find the value of x when $\theta = \frac{1}{6}\pi$. [7]

....

....

....

....

....

....

....

....

3. Nov/2022/Paper_9709_33/No.10

A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time t minutes after filling begins the volume of water in the pool is V litres. The pool has a small leak and loses water at a rate of $0.01V$ litres per minute.

The differential equation satisfied by V and t is of the form $\dfrac{dV}{dt} = a - bV$.

(a) Write down the values of the constants a and b. [1]

....

....

....

....

(b) Solve the differential equation and find the value of t when $V = 1000$. [6]

....

....

....

....

....

....

....

....

....

....

....

....

....

(c) Obtain an expression for V in terms of t and hence state what happens to V as t becomes large. [2]