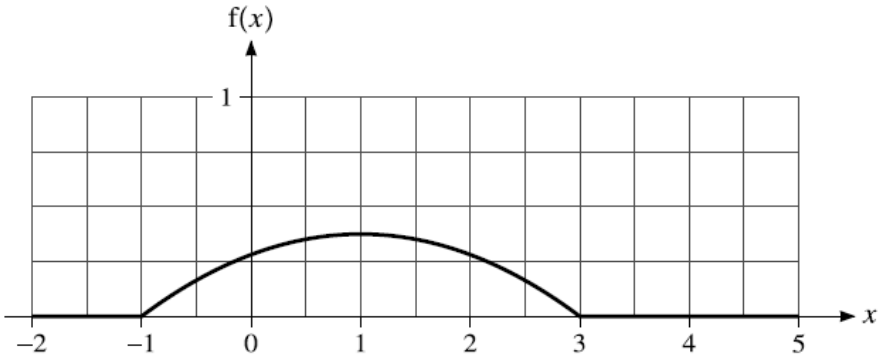
Continuous Random Variables – 2022 A2 Nov Math

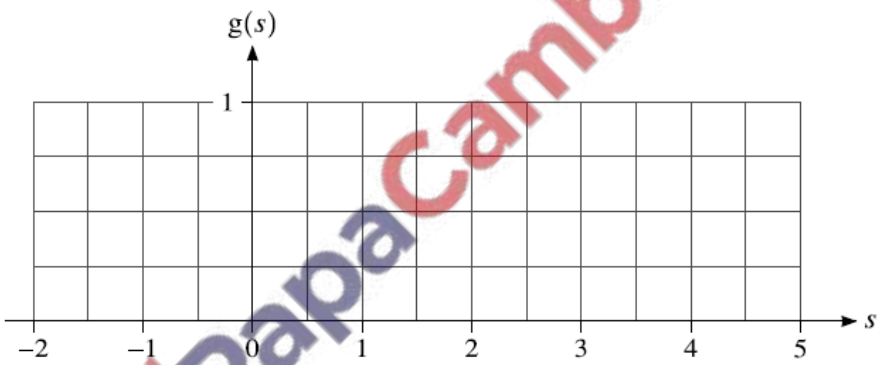
1. Nov/2022/Paper_9709_61/No.6



The diagram shows the graph of the probability density function of a random variable X that takes values between -1 and 3 only. It is given that the graph is symmetrical about the line $x = 1$. Between $x = -1$ and $x = 3$ the graph is a quadratic curve.

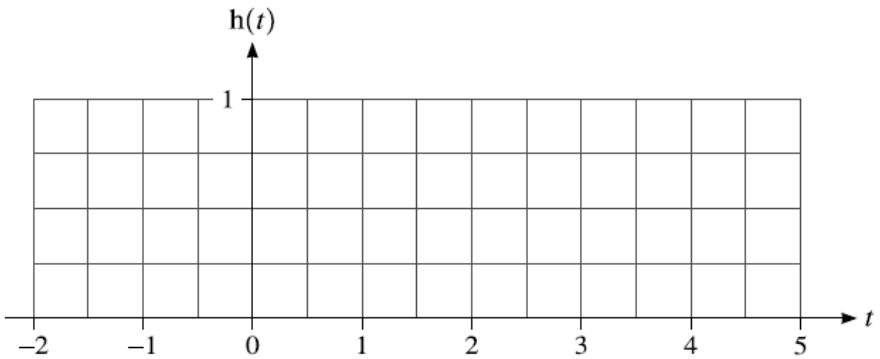
The random variable S is such that $\mathrm{E}(S) = 2 \times \mathrm{E}(X)$ and $\mathrm{Var}(S) = \mathrm{Var}(X)$.

(a) On the grid below, sketch a quadratic graph for the probability density function of S. [1]



The random variable T is such that $\mathrm{E}(T) = \mathrm{E}(X)$ and $\mathrm{Var}(T) = \frac{1}{4}\mathrm{Var}(X)$.

(b) On the grid below, sketch a quadratic graph for the probability density function of T. [2]



It is now given that

$$f(x) = \begin{cases} \frac{3}{32}(3 + 2x - x^2) & -1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $P(1 - a < X < 1 + a) = 0.5$, show that $a^3 - 12a + 8 = 0$. [3]

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(d) Hence verify that $0.69 < a < 0.70$. [1]

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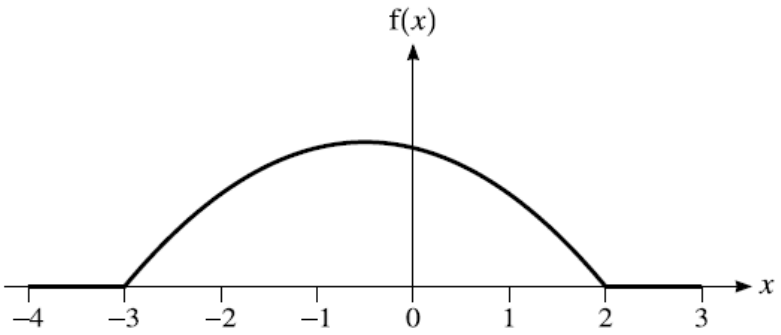
2. Nov/2022/Paper_9709_62/No.7



The diagram shows the graph of the probability density function, f, of a random variable X which takes values between -3 and 2 only.

(a) Given that the graph is symmetrical about the line $x = -0.5$ and that $P(X < 0) = p$, find $P(-1 < X < 0)$ in terms of p. [2]

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(b) It is now given that the probability density function shown in the diagram is given by

$$f(x) = \begin{cases} a - b(x^2 + x) & -3 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(i) Show that $30a - 55b = 6$. [3]

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(ii) By substituting a suitable value of x into $f(x)$, find another equation relating a and b and hence determine the values of a and b. [3]