Hypothesis Tests – 2022 A2 Nov Math

1. Nov/2022/Paper_9709_61/No.2

A spinner has five sectors, each printed with a different colour. Susma and Sanjay both wish to test whether the spinner is biased so that it lands on red on fewer spins than it would if it were fair. Susma spins the spinner 40 times. She finds that it lands on red exactly 4 times.

(a) Use a binomial distribution to carry out the test at the 5% significance level. [5]

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Sanjay also spins the spinner 40 times. He finds that it lands on red r times.

(b) Use a binomial distribution to find the largest value of r that lies in the rejection region for the test at the 5% significance level. [3]

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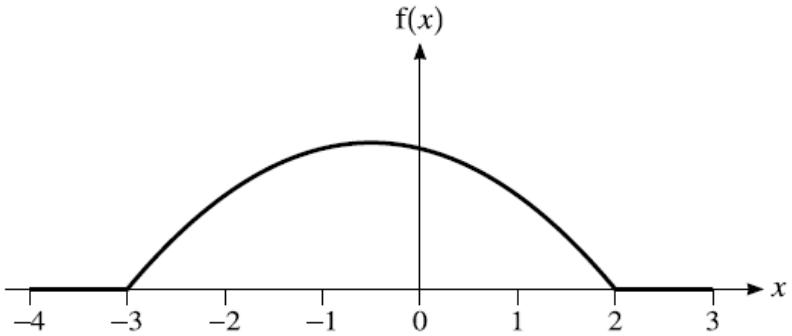
2. Nov/2022/Paper_9709_61/No.7



The diagram shows the graph of the probability density function, f, of a random variable X which takes values between -3 and 2 only.

(a) Given that the graph is symmetrical about the line $x = -0.5$ and that $P(X < 0) = p$, find $P(-1 < X < 0)$ in terms of p. [2]

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(b) It is now given that the probability density function shown in the diagram is given by

$$f(x) = \begin{cases} a - b(x^2 + x) & -3 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(i) Show that $30a - 55b = 6$. [3]

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(ii) By substituting a suitable value of x into $f(x)$, find another equation relating a and b and hence determine the values of a and b. [3]

3. Nov/2022/Paper_9709_62/No.2

In the past, the mean length of a particular variety of worm has been 10.3 cm, with standard deviation 2.6 cm. Following a change in the climate, it is thought that the mean length of this variety of worm has decreased. The lengths of a random sample of 100 worms of this variety are found and the mean of this sample is found to be 9.8 cm.

Assuming that the standard deviation remains at 2.6 cm, carry out a test at the 2% significance level of whether the mean length has decreased. [5]

4. Nov/2022/Paper_9709_62/No.4

The number of faults in cloth made on a certain machine has a Poisson distribution with mean 2.4 per $10\,m^2$. An adjustment is made to the machine. It is required to test at the 5% significance level whether the mean number of faults has decreased. A randomly selected $30\,m^2$ of cloth is checked and the number of faults is found.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the probability of a Type I error. [3]

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Exactly 3 faults are found in the randomly selected 30 m^2 of cloth.

(c) Carry out the test at the 5% significance level. [2]

Later a similar test was carried out at the 5% significance level, using another randomly selected 30 m^2 of cloth.

(d) Given that the number of faults actually has a Poisson distribution with mean 0.5 per 10 m^2, find the probability of a Type II error. [2]