Differentiation – 2023 June A2 Math 9709

1. June/2023/Paper_9709/31/No.5

The equation of a curve is $x^2y - ay^2 = 4a^3$, where a is a non-zero constant.

(a) Show that $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$. [4]

(b) Hence find the coordinates of the points where the tangent to the curve is parallel to the y-axis. [4]

2. June/2023/Paper_9709/32/No.7

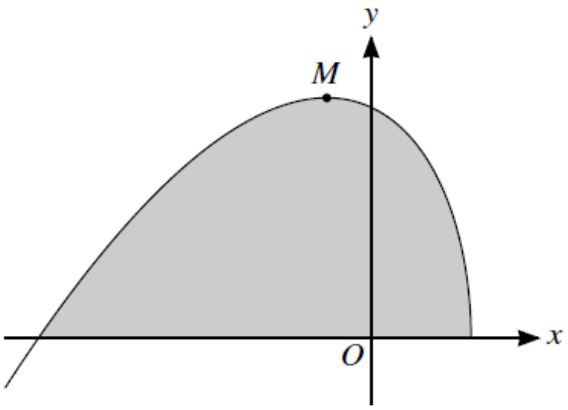
The variables x and y satisfy the differential equation

$$\cos 2x \frac{dy}{dx} = \frac{4 \tan 2x}{\sin^2 3y},$$

where $0 \leqslant x < \frac{1}{4}\pi$. It is given that $y = 0$ when $x = \frac{1}{6}\pi$.

Solve the differential equation to obtain the value of x when $y = \frac{1}{6}\pi$. Give your answer correct to 3 decimal places. [8]

3. June/2023/Paper_9709/32/No.10(a)



The diagram shows the curve $y = (x + 5)\sqrt{3 - 2x}$ and its maximum point M.

(a) Find the exact coordinates of M. [5]

(b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x-axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

4. June/2023/Paper_9709/33/No.4

The parametric equations of a curve are

$$x = \frac{\cos\theta}{2 - \sin\theta}, \qquad y = \theta + 2\cos\theta.$$

Show that $\frac{dy}{dx} = (2 - \sin\theta)^2$. [5]